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Research Paper

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Performance and Analysis of Flow between Annular Space Surrounded by a Rotating Coaxial cylinder with Co-axial Cylindrical Porous Medium

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Abstract— In this paper we consider the flow of second order fluid in the annular space between an impervious solid circular cylinder rotating with constant angular velocity and surrounded by the co-axial cylinder in the presence of porous medium. The flow in the annular free flow region is taken to be governed by the Navier- Stokes equations and in the porous region by the Brinkman equations. The case namely when the porous medium is finite region, has been considered when outer cylinder rotates with velocity (w). The solutions are obtained by using boundary and matching conditions at the surface of the interface. In case of finite region of porous medium, the rotation of cylinder decreases of the permeability**.**

Keywords— Brinkman equations; Porous medium; Inner cylinder; Navier- Stokes equations; Matching Condition

I. INTRODUCTION

The fluid flow through porous medium has been a topic of longstanding interest for researchers from last six decades, due to its numerous applications in bio-mechanics, physical sciences, chemical engineering, and industries etc. Several conceptual models have been developed for describing fluid flow in porous medium. The problem of the flow of co-axial flow through porous medium has becomes quite popular due to immense importance and continuing interest of scientists and mathematicians in the many engineering, technological and medical fields. Application of the porous medium has been realized as an elegant device for flow control in fluid flows. Raptis **[**16**]** has studied mathematically the case of time varying two dimensional natural convective heat transfer of an incompressible, electrically conducting viscous fluid via a highly porous medium bounded by an infinite vertical porous plate. Acrivos and Taylor [1] investigated a heat and mass transfer from single spheres in Stokes flow and their application. Kumar [4] studied an analytical study of effect of disorder on dispersion in steady inertial flows in porous effect. Kumar et al. [6] studied MHD free convective fluctuating flow through a porous effect with variable permeability parameter. Kumar [7] investigated finite difference technique for reliable MHD steady flow through channels permeable boundaries. Kumar [8] developed a finite element Galerkin's approach for viscous incompressible fluid flow through a porous medium in coaxial cylinders. Takhar and Beg [20] have studied the effects of Hall currents on hydro-magnetic free convection boundary layer via a porous medium past a plate using harmonic analysis. Purohit and Patidar [21] studied the flow of viscous incompressible fluid through an annulus. Kaur et al. [11] made an analysis of heat transfer in hydrodynamic rotating flow of viscous fluid through a non homogenious porous medium with constant heat source / sin**k.** Kumar [12] investigated mathematical study of thermosolutal convection in heterogeneous vscoelastic fluid in the presence of porous medium Padmavati and Amaranath [14] have considered the problem of general non axi-symmetric Stokes flow past a porous sphere in a viscous, incompressible fluid. They have considered the flow inside the sphere governed by Brinkman's equation. Al-Nimr et al. [2,3] have investigated numerically convection in the entrance region of either tube of an annulus, when a time- wise step change of wall temperature is imposed, for Darcy and non-Darcy models. Senoy [19] has reviewed studies of flow in non-Newtonian fluids in porous media, with attention concentrated on Power law fluids. The study of the Darcian porous MHD flow is much more complex. Barman [5] analysed the steady incompressible flow past an impervious sphere embedded in a porous medium of constant porosity based on the Brinkman model. Sharma et al.[17] studied how non-zero bulk flow is affected with transfer of the solute. They had also considered non-mixing on diffusion. A great number of Darcian porous MHD studies have been performed examining the effects of magnetic field on hydrodynamic- flow without heat transfer in various configurations, e.g., in channels and over plates, wedges, etc. [20]. Kumar and Agrawal [14] investigated mathematical study on the MHD squeeze flow between two parallel disks with suction or injection via HAM and HPM and its applications.

Patidar and Purohit [15] investigated free convection flow of an incompressible fliud in the porous medium between two long vertical wavy walls. El-Hakien [9] made the study of the effect of the thermal dispersion, viscous and Joule heating on the flow of an incompressible, electrically conducting micro polar fluid past a semi-infinite plate whose surface temperature varies linearly with the distance from the leading edges. Kim [12] has considered the case of a semi-infinite moving porous plate in a porous medium with the presence of pressure gradient and constant velocity in the flow direction when the magnetic field is imposed transverse to the plate. Feng and Michaelides[10] studied numerically the transient heat transfer from a sphere at high Reynolds and peclet numbers. Sekhar [18] investigated a MHD flow past a sphere at low and moderate Reynolds numbers. Sheny [19] made a Newtonian fluid heat transfer in porous media. Raptis[16] studied the local similarity transformation for the boundary layer flow through a homogeneous porous medium with high porosity and semi-infinite horizontal plate in the presence of heat transfer. Kumar et al.[22] investigated perturbation technique to unsteady MHD periodic flow of viscous fluid through a planer channel.

Nomenclature

K	Permeability parameter
φ	Porosity of the porous medium
μ	Viscosity of the viscous fluid
λ	Positive constant
τ	Stress
a_i	Acceleration
V_i	velocity
$\mu 1, \mu 2, \mu 3$	Material constant
p	hydrostatic pressure
ω, ω'	Angular velocities
u ,w	Velocity component in the direction of r and z respectively
ρ	Density of the fluid
p	hydrostatic pressure
X1, Y1	Modified Bessel's function of first order.

II. DESCRIPTION OF THE MODEL

$$\tau_{i,j} = -p\delta_{i,j} + \mu_1 A(1)_{i,j} + \mu_2 A(2)_{i,j} + \mu_3 A(1)_{i,k} A(1)_{k,j} \quad (1)$$

where A(1)i,j=vi,j+vj,j,A(2)i,j=ai,j+aj,i+2vm,ivm,j, (2)

$\tau_{i,j}$ is the stress tensor μ_1, μ_2, μ_3 are the material constants. Following conditions had been proposed by Dunn and Fosdick (1974).

$$\mu_2 + \mu_3 = 0,\ \mu_1 \geq 0,\ \mu_2 \geq 0 \quad (3)$$

For the steady flow of a second order fluid between an impervious rotating circular cylinder of radius (a) and another co-axial cylinder of radius b (b>a) surrounded by the cylindrical porous medium, the relevant equations are as given below:

$$\rho v_j v_{i,j} = \tau_{i,j} \quad (4)$$

$$v_{i,j} = 0 \quad (5)$$

The cylinders are rotating about their axes with constantangular velocities ω and ω' respectively.

The velocity components for annular space are:

u=0, v = v(r), w = 0 (6)

Substituting (1), (2), and (6) in equation (4) and using the boundary condition (3) we get

$$\frac{\partial p}{\partial r} = \rho \frac{v^2}{r} + \mu_2 \left(\frac{d}{dr} \left\{ \frac{dv}{dr} - \frac{v}{r} \right\}^2 + \frac{2}{r} \left\{ \frac{dv}{dr} - \frac{v}{r} \right\}^2 \right), \quad (7)$$

$$\frac{d^2 v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{v}{r} = 0 \quad (8)$$

And $$\frac{dp}{dz} = 0 \quad (9)$$

In the porous region $r \geq b$ the equation of motion (4) is:

$$\rho v_j v_{i,j} = \tau_{ij.j} - \frac{\mu_1}{k} v_j, \quad (10)$$

and the equation of continuity remains unchanged.

We are taking

$$\bar{u} ==, \quad \bar{v} = \bar{v}(r), \qquad \bar{w} = 0 \tag{11}$$

$$\frac{\partial \bar{p}}{\partial r} = \rho \frac{\bar{v}^2}{r} + \mu_2 \left(\frac{d}{dr} \left\{ \frac{d\bar{v}}{dr} - \frac{\bar{v}}{r} \right\}^2 + \frac{2}{r} \left\{ \frac{d\bar{v}}{dr} - \frac{\bar{v}}{r} \right\}^2 \right), . \tag{12}$$

$$\frac{d^2 \bar{v}}{dr^2} + \frac{1}{r}\frac{d\bar{v}}{dr} - \frac{\bar{v}}{r} = 0 \tag{13}$$

$$\frac{d\bar{p}}{dz} = 0 \tag{14}$$

The matching conditions at the interface, r=b are as given below:

$$v = \phi \bar{v} \tag{15}$$

$$\frac{dv}{dz} - \frac{v}{r} = \lambda \varphi \left(\frac{d\bar{v}}{dr} - \frac{\bar{v}}{r} \right), \tag{16}$$

III. RESERCH METHODOLOGY

In the present communication of investigation we consider the problem of finite region; we are discussing the case, when the porous medium is infinite region. Following Barman and Sharma [1993] the results in brief are as follows. Because the problem is symmetric, the numerical computation is performed in a circular cylindrical domain. In the surrounding magnetic medium the boundary conditions are given by:

$$v = r\omega, \quad at \quad r = a \tag{17}$$

$$\bar{v} \ is \ \ finite \quad at \ r = \infty \tag{18}$$

along with the matching conditions given by (15) and (16). Using the conditions (15), (16), (17) and (18) in solving the (8) and (13) we get the solution for free and porous medium respectively,

$$V = \frac{v}{a\omega} = C_1 \beta + \frac{C_2}{\beta} \tag{19}$$

$$\bar{V} = \frac{\bar{v}}{a\omega} = C_3 X_1(\sigma\beta) + C_4 Y_1(\sigma\beta) \tag{20}$$

Where $\beta = r/a$, $\sigma = a/\sqrt{k}$, X_1 and Y+ are modified Bessel 's functions of the first order . We calculate constant in (19) , (20) satisfying the boundary conditions (17) and (18)
and using matching conditions at the interface (r=b). It is observed that $v \to \infty$ as $\beta \to \infty$. Hence

$C_3 = 0$.

And

$$C_1 = \frac{[(2-\lambda)Y_1(\sigma\beta_1) + \lambda\sigma\beta_1 Y_1'(\sigma\beta_1)]}{[\{2+\lambda(\beta^2{}_1 - 1)\}Y_1(\sigma\beta_1) - \lambda\sigma\beta_1(\beta^2{}_1 - 1)Y_1'(\sigma\beta_1)]}$$

$$C_2 = \frac{\lambda\beta_1{}^2 [Y_1(\sigma\beta_1) - (\sigma\beta_1)Y_1'(\sigma\beta_1)]}{[\{2+\lambda(\beta^2{}_1 - 1)\}Y_1(\sigma\beta_1) - \lambda\sigma\beta_1(\beta^2{}_1 - 1)Y_1'(\sigma\beta_1)]}$$

$$C_4 = \frac{2\beta_1}{\phi[\{2+\lambda(\beta^2{}_1 - 1)\}Y_1(\sigma\beta_1) - \lambda\sigma\beta_1(\beta^2{}_1 - 1)Y_1'(\sigma\beta_1)]}$$

Where $\beta_1 = b/a$

Using (20) in (8) we get the result

$$P = \frac{C_1{}^2}{2}\beta^2 + C_1 C_2 \log\beta - \frac{C_1{}^2}{2}\frac{1}{\beta_1{}^2} + 2C_2\frac{\alpha}{\beta^4} + d$$

Where $P = \frac{p}{a^2\Omega^2\rho}$, $\alpha = \frac{\mu_2}{\rho a^2}$

And d is a constant of integration.
The torque on the surface of the inner cylinder is

$$\frac{\tau \ \ at \ r = a}{4L\lambda\mu_1 a^2} = \frac{\beta_1{}^2 [Y_1(\sigma\beta_1) - \sigma\beta_1 Y'_1(\sigma\beta_1)]}{[\{2+\lambda(\beta_1^2 - 1\}Y_1(\sigma\beta_1) - \lambda\sigma\beta_1(\beta_1^2 - 1\}Y_1'(\sigma\beta_1)} \tag{21}$$

Now we proceed to discuss the desired case when the porous medium is the finite region. The porous region is bounded by an impermeable cylindrical surface of radius c (c>b). And the boundary conditions (15), (16) and (17) remains the same except (18). We have

$$\bar{v} = r\omega', \quad at \quad r = b \tag{22}$$

$$\bar{v} = 0 \quad at \quad r = c \tag{23}$$

Solving equations (8) and (14) by using boundary conditions (15), (16), (17) and (22) we get the following results

$$v' = K_1\beta + \frac{K_2}{\beta} \tag{24}$$

$$\bar{v}' = K_3 X_1(\sigma\beta_1) + K_4 Y_1(\sigma\beta_1) \qquad (25)$$

Where $v' = \dfrac{v}{c\omega'}$, $\bar{v}' = \dfrac{\bar{v}}{c\omega'}$, $\beta = \dfrac{r}{c}$, $\sigma = \dfrac{c}{\sqrt{k}}$, X_1 and Y_1 are modified Bessel's functions of order. We calculate the constants in equation (24), (25) satisfying the boundary conditions (22) and (23) and also matching conditions at the interface. And the constants are given below

$$K_3 = \frac{\sigma\beta_1 Y_1(\sigma\beta_1) X_1'(\sigma\beta_1)(\beta_2^2 - \beta_1^2)(\lambda - 1) \left[(\beta_2^2 - \beta_1^2)(\lambda - 2) Y_1(\sigma\beta_1) X_1(\sigma\beta_1)\right]}{\sigma\phi\lambda\beta_1^2 X_1(\sigma\beta_1)(\beta_2^2 - 1)[Y_1(\sigma\beta_1) X_1'(\sigma\beta_1) - Y_1'(\sigma\beta_1) X_1(\sigma\beta_1)]}$$

$$K_4 = \frac{(\beta_2^2 - \beta_1^2)\sigma\beta X_1'(\sigma\beta_1) - [(\lambda - 2)(\beta_2^2 - \beta_1^2) X_1(\sigma\beta_1)}{\sigma\phi\lambda\beta_1^2(\beta_2^2 - 1)[X_1'(\sigma\beta_1) Y_1(\sigma\beta_1) - Y_1'(\sigma\beta_1) X_1(\sigma\beta_1)]}$$

Where $\beta_1 = \dfrac{b}{a}, \beta_2 = \dfrac{c}{a}$

IV. RESULTS AND DISCUSSION

The results have been numerically worked out for various combinations of the parameters involved in the solutions (K=1). For many applications these findings may be beneficial. The graph of the velocity profiles has been shown in fig. (1) and (2) for various parameters.. Table (1) shows the torque on the surface of the inner cylinder for different value of σ and β_1. It is observed that the torque on the surface of the inner cylinder decreases with the increases of the outer cylindrical surface of the magnetic effects and increases with the decrease of the permeability of the porous medium.

V. CONCLUSION

The magnitude of surface density is greater on the inner cylinder compared to the outer cylinder. In general, skin-friction, mass flux of fluid and induced current density can be increased by increasing the gap between the cylinders. The effect of porous can be useful in cholesterol, biomedical engineering for modeling a system. We can easily obtain a suitable value of permeability parameter for which the value of the cholesterol and fluid flow control will be optimum. It is clear that the velocity decreases with the increase of permeability of the inner rotating cylinder in the presence of magnetic medium. It has been observed that the torque on the surface of the inner cylinder decreases with the increase of the width of the annular region and increases with the decrease of the permeability of the porous medium.

Table 1. Torque on the Surface of the Cylinder in presence of porous medium

σ \ β1	1.3	1.7	2.1	2.5	2.9	3.3
1.0	**-0.989**	**-0.841**	**-0.780**	**-0.747**	**-0.727**	**-0.714**
1.5	**-1.060**	**-0.871**	**-0.796**	**-0.757**	**-0.734**	**-0.719**
2.0	**-1.118**	**-0.893**	**-0.807**	**-0.764**	**-0.738**	**-0.722**
2.5	**-1.166**	**-0.909**	**-0.815**	**-0.768**	**-0.741**	**-0.724**
3.0	**-1.207**	**-0.921**	**-0.821**	**-0.771**	**-0.743**	**-0.725**

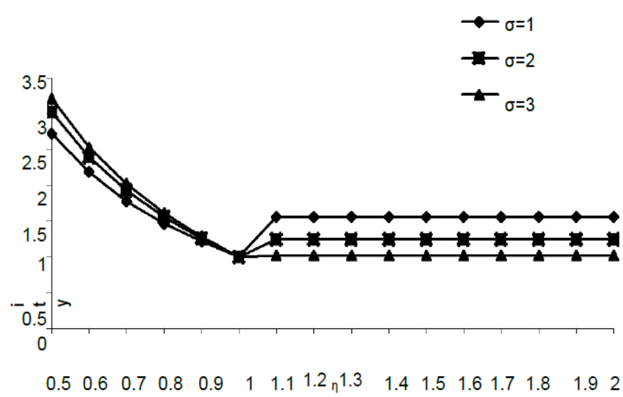

Fig. 1 Velocity against the distance from rotating cylinder in the presence of porous effects in infinite extent

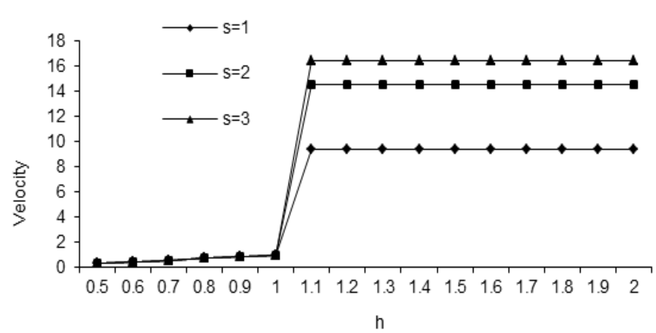

Fig. 2. Velocity against the distance from the rotating cylinder when the porous medium is of finite extent

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