International Journal of Scientific Research in Mathematical and Statistical Sciences
Volume-5, Issue-3, pp.69-72, June (2018)

Research Paper

E-ISSN: 2348-4519

On Weakly m-semi-I-Open Sets in Minimal Spaces

R. Mariappan[1*], M. Murugalingam[2]

[1] Department of Mathematics, Dr. Mahalingam College of Engineering and Technology, Pollachi, India
[2] Department of Mathematics, Sri Sarada College for Women, Tirunelveli, India

[*]Corresponding Author: mariappanbagavathi@gmail.com

Available online at: www.isroset.org

Received: 11/May/2018, Revised: 23/May/2018, Accepted: 18/Jun/2018, Online: 30/Jun2018

Abstract— In this paper, we introduce and study the notions of weakly m-semi-I-open sets and weakly m-semi-I-continuity, and their related notions in ideal minimal spaces. Also we investigate the decomposition of weakly m-semi-I-open set.
2000 Mathematics Subject Classification: 54D10

Keywords— *Ideal minimal space, m-semi-I-open sets, m-semi-I-closed sets, m-semi-I-continuity.*

I. INTRODUCTION

In 2001, Popa and Noiri introduced the notions of minimal structure and m-continuous function as a function defined between a minimal structure and a topological space *[7]*. A minimal structure m on a nonempty set X is a collection of subsets of X such that $\phi \in m$ and $X \in m$ *[7]*. By (X, m), we denote a nonempty set X with minimal structure m on X. The members of the minimal structure m are called m-open sets and the complement of m-open set is said to be m-closed *[7]*. The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. An ideal is defined as a nonempty collection I of subsets of X satisfying the following two conditions: (i) If $A \in I$ and $B \subset A$, then $B \in I$; (ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$ *[3]*. For a subset $A \subset X$, $A^*_m(m, I) = \{x \in X : U \cap A \notin I$ for each m neighborhood U of x$\}$ is called the local minimal function of A with respect to I and m *[3]*. We simply write A^*_m instead of $A^*_m(I, m)$ in case there is no chance for confusion. For every ideal topological space (X, I, m), there exists a minimal structure m*(m, I) called the ∗-minimal, finner than m. Additionally $mCl^*(A) = A \cup A^*_m$ for every $A \subset X$.

II. PRELIMINARIES

Definition 2.1 A subset A of a minimal space (X, m) is said to be βm-open*[1]* (resp. m-semiopen *[5]*, m-preopen *[6]*, αm-open *[7]*, m-b-open *[11]*) if $A \subset mCl(mInt(mCl(A)))$ (resp. $A \subset mCl(mInt(A))$, $A \subset mInt(mCl(A))$, $A \subset mCl(mInt(mCl(A)))$, $A \subset mInt(mCl(A)) \cup mCl(mInt(A))$).

Definition 2.2 Let (X, m) be a minimal space. For a subset A of X, the m-closure of A and m-interior of A are defined in *[9]* as follows:
(a) $mCl(A) = \cap\{F : A \subset F, X - F \in m\}$,
(b) $mInt(A) = \cup\{U : U \subset A, U \in m\}$.

Definition 2.3 A function f:(X, m) → (Y, τ) is said to be βm-continuous *[1]* (resp. m-semicontinuous *[5]*, m-semicontinuous *[6]*, αm-continuous *[7]*, m-b-continuous *[11]*) if the inverse image of every open set of Y is βm-open (resp. m-semiopen, αm-open, m-preopen, m-b-open) in (X, m).

Definition 2.4 *[11]* A subset A of a minimal space (X, m, I) is said to be m-semi-I-open (resp. m-pre-I-open, m-α-I-open, m-β-I-open, strongly m-β-I-open, m-δ-I-open) if $A \subset mCl^*(mInt(A))$ (resp. $A \subset mInt(mCl^*(A))$, $A \subset mInt(mCl^*(mInt(A)))$, $A \subset mCl(mInt(mCl^*(A)))$, $A \subset mCl^*(mInt(mCl^*(A)))$, $mInt(mCl^*(A)) \subset mCl^*(mInt(A))$).

Remark 2.1 Let (X, τ) be a topological space. The families αIO(X, m), SIO(X, m), PIO(X, m) and βIO(X, m) are all minimal structures on X.

Definition 2.5 *[11]* A function f :(X, m, I) → (Y, τ) is said to be m-pre-I-continuous (resp. m-semi-I-continuous, m-α-I-continuous, m-δ-I-continuous) if the inverse image of every open set of (Y, τ) is m-pre-I-open (resp. m-semi-I-open, m-α-I-open, m-δ-I-open) in (X, m, I).

Lemma 2.1 *[4]* Let (X, m) be a minimal space and A, B subsets of X. Then $x \in mCl(A)$ if and only if $U \cap A = \emptyset$ for every $U \in m$ containing *x*. And satisfying the following properties:
(a) $mCl(mCl(A)) = mCl(A)$.
(b) $mInt(mInt(a)) = mCl(A)$.
(c) $mInt(X-A) = X-mCl(A)$.
(d) $mCl(X-A) = X-mInt(A)$.
(e) If $A \subset B$, then $mCl(A) \subset mCl(B)$.
(f) $mCl(A \cap B) \subset mCl(A) \cup mcl(B)$.

(g) A ⊂ mCl(A) and mInt(A) ⊂ A.

Lemma 2.2 *[9]* Let (X, m) be a minimal space and A be subset of X. Then x ∈ mCl(A) if and only if U ∩ A = ∅ for each U ∈ m containing x.

III. WEAKLY M-SEMI-I-OPEN SET

Definition 3.1 Let (X, m, I) be a minimal space. A subset A of X is said to be a weakly m-semi-I-open set if A ⊂ mCl* (mInt(mCl(A))). A subset A of X is said to be a m-semi-I-closed set if its complement is a weakly m-semi-I-open set.

Proposition 3.1 In a minimal space (X, m, I), the following hold.
(a) Every m-α-I-open set is a weakly m-semi-I-open set.
(b) Every strongly m-β-I-open set is a weakly m-semi-I-open set.
(c) Every m-semi-I-open set is a weakly m-semi-I-open set.
(d) Every m-pre-I-open set is a weakly m-semi-I-open set.
Proof. (a) A is a m-α-I-open set, then A ⊂ mInt(mCl* (mInt(A))) ⊂ mCl*(mInt(A)) ⊂mCl*(mInt(mCl(A))). Hence A is a weakly m-semi-I-open set.
(b) If A is a strongly m-β-I-open set, then A ⊂ mCl* (mInt(mCl*(A))) ⊂ mCl*(mInt(mCl(A))). Hence A is a weakly m-semi-I-open set.
(c) If A is a m-semi-I-open set, then A ⊂ mCl*(mInt(A)) ⊂ mCl(mInt(A)) ⊂ mCl(mInt(mCl(A))). Hence A is a weakly m-semi-I-open set.
(d) If A is a m-pre-I-open set, then A ⊂ mInt(mCl*(A)) ⊂ mInt(mCl(A)) ⊂ mCl*(mInt(mCl(A))). Hence A is a weakly m-semi-I-open set.

Remark 3.1 The converse of Proposition 3.1 need not be true as shown in the following examples.

Example 3.1 Let X = {a, b, c}, m = {∅, {a}, X} and I = {∅, {a}}. Then
(i) A = {a, b} is a weakly m-semi-I-open set but not m-α-I-open set.
(ii) A = {a} is a weakly m-semi-I-open set but not strongly m-β-I-open set.
(iii) A = {a, c} is a weakly m-semi-I-open set but not m-semi-I-open set.

Example 3.2 Let X = {a, b, c}, m = {∅, {a, b}, {a, c}, X} and I = {∅, {a}}. Then A = {a} is a weakly m-semi-I-open set but not m-pre-I-open set.

Proposition 3.2 Let (X, m, I) be a minimal space and A ⊂ X. Then A is a weakly m-semi-I-open set if and only if A is a m-δ-I-open set.
Proof.
Necessity: If A is a weakly m-semi-I-open set, by Proposition 3.1, then A is a weakly m-semi-I-open set. Now we prove that mInt(mCl*(A)) ⊂ mCl*(mInt(A)). Since A is a m-semi-I-open set implies that A ⊂ mCl*(mInt(A)), so, mInt(mCl*(A)) ⊂ mInt(mCl*(mCl*(mInt(A)))) ⊂ mInt(mCl* (mInt(A))) ⊂ mCl* (mInt(A)). Hence A is a m-δ-I-open set.
Sufficiency: If A is a m-δ-I-open set, then mInt(mCl*(A)) ⊂ mCl* (mInt(A)) and A is a weakly m-semi-I-open set, then A ⊂ mCl*(mInt(mCl(A))). Then we have A ⊂ mCl* (mInt(mCl(A))) = mCl*(mInt(mCl*(A))) ⊂ mCl* (mCl*(mInt(A))) ⊂ mCl*(mInt(A)). Hence A is a m-semi-I-open set.

Proposition 3.3 Let (X, m, I) be a minimal space. Then any arbitrary union of weakly m-semi-I-open sets is a weakly m-semi-I-open set.
Proof. Let U_α be a weakly m-semi-I-open set for every $\alpha \in \Delta$, we have $U_\alpha \subset mCl_*(mInt(mCl(U_\alpha)))$ for every $\alpha \in \Delta$.
Then $\cup_{\alpha\in\Delta} U_\alpha \subset \cup_{\alpha\in\Delta} mCl_*(mInt(mCl(U_\alpha)))$
$= \cup_{\alpha\in\Delta} ((mInt(mCl(U_\alpha)))_* \cup mInt(mCl(U_\alpha)))$
$= \cup_{\alpha\in\Delta} (mInt(mCl(U_\alpha)))_* \cup \cup_{\alpha\in\Delta}(mInt(mCl(U_\alpha)))$
$\subset (\cup_{\alpha\in\Delta} mInt(mCl(U_\alpha)))_* \cup \cup_{\alpha\in\Delta}(mInt(mCl(U_\alpha)))$
$\subset (mInt(\cup_{\alpha\in\Delta} mCl(U_\alpha)))_* \cup (mInt(\cup_{\alpha\in\Delta} mCl(U_\alpha)))$
$= (mInt(mCl(\cup_{\alpha\in\Delta} U_\alpha)))_* \cup (mInt(mCl(\cup_{\alpha\in\Delta} U_\alpha)))$
$= mCl_*(mInt(mCl(\cup_{\alpha\in\Delta} U_\alpha)))$.
Hence $\cup_{\alpha\in\Delta} U_\alpha$ is a weakly m-semi-I-open set.

Proposition 3.4 Let (X, m, I) be a minimal space and A, B ⊂ X. If A is a weakly m-semi-I-open set and B ∈ m, then A∩B is a weakly m-semi-I-open set.
Proof. If A is a weakly m-semi-I-open set, then A ⊂ mCl*(mInt(mCl(A))) and B is m-open, then mInt(B) = B.
Now A∩B ⊂ mCl*(mInt(mCl(A)))∩B
= ((mInt(mCl(A)))* ∪ mInt(mCl(A)))∩B
= ((mInt(mCl(A)))*∩B) ∪ (mInt(mCl(A))∩B)
⊂ (mInt(mCl(A))∩B)* ∪ (mInt(mCl(A))∩B)
= (mInt(mCl(A))∩mInt(B))* ∪ (mInt(mCl(A))∩mInt(B))
⊂ (mInt(mCl(A∩mInt(B))))* ∪ (mInt(mCl(A∩mInt(B))))
⊂ (mInt(mCl(A∩B)))* ∪ (mInt(mCl(A∩B)))
= mCl*(mInt(mCl(A∩B))).
Hence A∩B is a weakly m-semi-I-open set.

Proposition 3.5 Let (X, m, I) be a minimal space and A, B ⊂ X. If A is a weakly m-semi-I-open set and B is an m-α-I-open set, then A∩B is a weakly m-semi-I-open set.
Proof. Since A is an m-α-I-open set, then A ⊂ mInt(mCl* (mInt(A))) and B is a weakly m-semi-I-open set, then B ⊂ mCl*(mInt(mCl(B))).
Now
A∩B⊂mCl*(mInt(mCl(A)))∩mInt(mCl*(mInt(B)))
=((mInt(mCl(A)))*∪(mInt(mCl(A))))∩mInt(mCl*(mInt(B)))
=((mInt(mCl(A)))*∩mInt(mCl*mInt(B))))∪(mInt(mCl(A))∩ mInt(mCl*(mInt(B))))
⊂(mInt(mCl(A))∩mInt(mCl*(mInt(B))))*∪mInt(mInt(mCl(A))∩mCl*(mInt(B)))
=(mInt(mInt(mCl(A)))∩mCl*(mInt(B)))*∪mInt(mInt(mCl(A))∩mCl*(mInt(B)))
⊂(mInt(mCl*(mInt(mCl(A))∩mInt(B))))*∪mInt(mCl*(mInt (mCl(A))∩mInt(B)))
=(mInt(mCl*(mInt(mCl(A)∩mInt(B)))))*∪mInt(mCl*(mInt(

mCl(A)∩mInt(B))))
⊂(mInt(mCl*(mInt(mCl(A∩mInt(B))))))*∪mInt(mCl*(mInt (mCl(A∩mInt(B)))))
⊂(mInt(mCl*(mInt(mCl(A∩B)))))*∪mInt(mCl*(mInt(mCl(A∩B))))
⊂(mInt(mCl(mInt(mCl(A∩B)))))*∪mInt(mCl(mInt(mCl(A ∩B))))
=(mInt(mCl(A∩B)))*∪mInt(mCl(A∩B))
= mCl*(mInt(mCl(A∩B))).
Hence A∩B is a weakly m-semi-I-open set.

Proposition 3.6 Let (X, m, I) be a minimal space and A, B ⊂ X. Then
(a) If A ⊂ B ⊂ mCl* (A) and A is a weakly m-semi-I-open set, then B, A* and B* are weakly m-semi-I-open sets.
(b) If A ⊂ B ⊂ mCl*(A) and A is a m-pre-I-open set, then B is a strong m-β-I-open set.
(c) If A ⊂ B ⊂ mCl(A) and A is a m-pre-I-open set, then B is a m-β-I-open set.
Proof. (a) Suppose that A ⊂ B ⊂ mCl*(A) and A is a weakly m-semi-I-open set implies that A ⊂ mCl* (mInt(mCl(A))). Since B ⊂ mCl*(A) ⊂ mCl*(mInt(mCl(A))) ⊂ mCl* (mInt(mCl(B))). Hence B is a weakly m-semi-I-open set. Since A ⊂ B ⊂ A*, we have A* and B* are weakly m-semi-I-open sets.
(b) Suppose A ⊂ B ⊂ mCl*(A) and A is a m-pre-I-open set implies that A ⊂ mInt(mCl*(A)). Now B ⊂ mCl*(A) ⊂ mCl*(mInt(mCl*(A))) ⊂ mCl*(mInt(mCl*(B))). Hence B is a strongly m-β-I-open set.
(c) Suppose A ⊂ B ⊂ mCl(A) and A is a m-pre-I-open set implies that A ⊂ mInt(mCl*(A)). Then we have B ⊂ mCl(A) ⊂ mCl(mInt(mCl*(A))) ⊂ mCl(mInt(mCl*(B))). Hence B is a m-β-I-open set.

Corollary 3.1 Let (X, m, I) be a minimal space and A ⊂ X. Then the following hold.
(a) If A is a weakly m-semi-I-open set, then mCl*(A) and mCl*(mInt(mCl*(A))) are weakly m-semi-I-open sets.
(b) If A is m-pre-I-open set, then mCl*(A) and mCl* (mInt(mCl*(A))) are strongly m-β-I-open sets.

Proposition 3.7 Let (X, m, I) be a minimal space and A ⊂ X be a weakly m-semi-I-open set. Then the following hold.
(a) If A ⊂ A*, then A* is a weakly m-semi-I-open set.
(b) If A = A*, then every subset containing A is a strongly m-β-I-open set.

Proposition 3.8 Let (X, m, I) be a minimal space and A ⊂ X. Then A is a weakly m-semi-I-closed set if and only if mInt* (mCl(mInt(A))) ⊂ A.
Proof. If A is a weakly m-semi-I-closed set, then X-A is a weakly m-semi-I-open set and hence X-A⊂mCl* (mInt(mCl(X-A)))=X-mInt*(mCl(mInt(A))). Therefore mInt*(mCl(mInt(A)))⊂A. Conversely, let mInt* (mCl(mInt(A)))⊂A. Then X-A ⊂ mCl*(mInt(mCl(X−A))). Hence X-A is a m-weakly semi-I-open set. Thus A is a weakly m-semi-I-closed set.

Proposition 3.9 Let (X, m, I) be a minimal space and A ⊂ X. If A is a weakly m-semi-I-closed set, then mInt(mCl* (mInt(A))) ⊂ A.
Proof. If A is a weakly m-semi-I-closed set, then mInt* (mCl(mInt(A))) ⊂ A. We have mInt(mCl*(mInt(A))) ⊂ mInt*(mCl*(mInt(A))) ⊂ mInt*(mCl(mInt(A))) ⊂ A.

IV. WEAKLY M-SEMI-I-CONTINUITY

Definition 4.1 A function f :(X, m, I) → (Y, τ) is said to be a m-semi-I-continuity if f^{-1} (V) is a weakly m-semi-I-open set in (X, m, I) for every open V of (Y, τ).

Proposition 4.1 For a function f:(X, m, I) → (Y, τ), the following hold.
(a) Every m-α-I-continuous function is a weakly m-semi-I-continuous function.
(b) Every m-semi-I-continuous function is a weakly m-semi-I-continuous function.
(c) Every m-pre-I-continuous function is a weakly m-semi-I-continuous function.
Proof. It follows from Theorem 3.1.

Proposition 4.2 Let f :(X, m, I) → (Y, τ) be a function. Then f is a weakly m-semi-I-continuous function if and only if f is a m-δ-I-continuous function.
Proof: The proof follows from Proposition 3.4.

Proposition 4.3 For a function f:(X, m, I) → (Y, τ), the following are equivalent.
(a) f is a weakly m-semi-I-continuous function.
(b) For each x ∈ X and each open V containing f (x), there exists a weakly m-semi-I-open set U such that f (U) ⊂ V.
Proof. Let x ∈ X and V be a open set of Y containing f (x). Take W = f^{-1}(V), then by definition, W is a weakly m-semi-I-open set containing x and f (W) ⊂ V.
Conversely, let F be a closed set of Y. Take V = Y−F, then V is a open set in Y. Let x ∈ f^{-1}(V), by hypothesis, there exists a weakly m-semi-I-open set W of X containing x such that f(W) ⊂ V. Thus, we obtain x ∈ W ⊂ mCl*(mInt(mCl(W))) ⊂ mCl*(mInt(mCl(f^{-1}(V)))) and hence f^{-1}(V) ⊂ mCl* (mInt(mCl(V))). This shows that f^{-1}(V) is a weakly m-semi-I-open set in X. Hence f^{-1}(F) = X−f^{-1}(Y−F) = X−f^{-1}(V) is a weakly m-semi-I-closed set in X.

Proposition 4.4 A function f:(X, m, I) → (Y, τ) is a weakly m-semi-I-continuous function if and only if f^{-1}(U) is a weakly m-semi-I-closed set in (X, m, I), for every closed set U in (Y, τ).
Proof. Let f be a weakly m-semi-I-continuous function and F be a closed set in (Y, τ). Then Y−F is open in (Y, τ). Since f is a weakly m-semi-I-continuous function, f^{-1}(Y−F) is a weakly m-semi-I-open set in (X, m, I). But f^{-1}(Y−F) = X−f^{-1}(F) and so f^{-1}(F) is a weakly m-semi-I-closed set in (X, m, I). Conversely, assume that f^{-1}(F) is a weakly m-semi-I-closed set in (X, m, I) for every closed in F in (Y, τ). Let V be open in (Y, τ). Then Y−V is closed in (Y, τ) and by

hypothesis $f^{-1}(Y-F)$ is a weakly m-semi-I-closed set in (X, m, I). Since $f^{-1}(Y-V) = X-f^{-1}(V)$, we have $f^{-1}(V)$ is a weakly m-semi-I-open set in (X, m, I), and so f is a weakly m-semi-I-continuous function.

V. References

[1] Boonpok C, "*Almost and weakly M -continuous functions in m-spaces*", East J. Math Sci, Vol.43, pp.41-58, 2010.

[2] S. Jafari, N. Rajesh and R. Saranya, "*Semiopen sets in Ideal mimimal spaces*", Vasile Alecsandri University of Bacau Faculty of Sciences Scientific Studies and Research Series Mathematics and Informatics, Vol.27, Issue.1, pp.33-48, 2017, .

[3] D. Jankovic and T.R. Hamlett, "*New topologies from old via ideals*", American Math.Monthly, Vol.97, pp.295-310, 1990.

[4] Maki. H, K.C. rao and A. Nagoor, "*On generalizing semi-open and preopen sets*", Pure Appl. Math. Sc., Vol.42, pp.13-21, 1993.

[5] W. K. Min, "*m-Semiopen sets and M-Semicontinuous functions on Spaces with minimal structures*", Honam Math. J., Vol.31, Issue.2, pp.239-245, 2009.

[6] W. K. Min, Kim Y. K, "*m-preopen sets and M -precontinuous functions on Spaces with minimal structures*", Advances in Fuzzy Sets and Systems, Vol.4, Issue.3, pp.237-245, 2009.

[7] W. K. Min, "*αm-open sets and αM-continuous functions*", Commun. Korean Math. Soc., Vol.25, Issue.2, pp.251-256, 2010.

[8] O.B. Ozbakir and E.E. Yildirim, "*On some closed sets in ideal minimal spaces*", Acta Math. Hungar., Vol.125, Issue.3, pp.227-235, 2009.

[9] V. Popa and T.Noiri, "*On the definition of some generalized forms of continuity under minimal condition*", Mem. Fac. Sci. Kochi. Uni. Ser. Math., Vol.22, pp.9-19,2001.

[10] V. Popa and T.Noiri, "*On M-continuous functions"*, Anal Uni. "Durarea de Jos", Galati,Ser. Mat. Fis. Me. Teor., Vol.18, Issue.23, No.2, pp.31-41, 2000.

[11] Takashi Noiri and Veleriu Popa, "*On Iterate Minimal Structures and m-Iterate Continuous Functions*", Fasciculi Mathematici, Vol.50, pp.109-118, 2013.

AUTHORS PROFILE

Dr. R. Mariappan has completed his M.Sc, M.Phil and Ph.D. from Manonmaniam Sundaranar Univesity, Tirunelveli, India. He is currently working as Assistant Professor in Department of Mathematics, Dr. Mahalingam College of Engineering and Technology, Pollachi, India. His main research work focuses on Topology and graph theory.

Dr. M. Murugalingam is currently working in Sri Sarada College for Women, Tirunelveli, India. His main research work focuses on Topology.

