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Review Paper

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Improved Ratio-Cum-Product Type Exponential Estimator of Population Mean in Stratified Random Sampling

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Abstract- In this chapter suggests dual to ratio-cum-product type exponential estimator in stratified random sampling. In fact, Srivenkataramana (1980) transformation has been used on Tailor and Chouhan (2013) estimator. The bias and mean squared error of the suggested estimator have been obtained upto the first degree of approximation. The suggested estimator has been compared with usual unbiased estimator in stratified random sampling, combined ratio and product estimators, dual to combined ratio and product estimators, Singh et al. (2008) ratio and product type exponential estimators and dual to ratio and product type exponential estimators given by Tailor et al. (2013). An empirical study shows the performance of the suggested estimator.

Keywords-Population Mean, Ratio and product estimator, Correlation coefficient, Bias , Mean squared error.

I. INTRODUCTION

Bahl and Tuteja (1991) envisaged ratio and product type exponential estimators using exponential function. These estimators were studied in stratified random sampling by Singh et al. (2008). Using Srivenkataramana (1980) transformation on auxiliary variates Tailor et al. (2013) obtained dual to Singh et al. (2008) ratio and product type exponential estimators. Singh et al. (2009) envisaged ratio-cum-product type exponential estimator for population mean. Tailor and Chouhan (2013) suggested ratio-cum-product type exponential estimator of population mean in stratified random sampling.

Let us consider a finite population $U=\{U_1,U_2,...,U_N\}$ of size N which is divided into L strata of sizes $N_h(h=1, 2, ..., L)$. Let y be the study variate and x and z be two auxiliary variables taking values y_{hi} , x_{hi} and z_{hi} respectively where $i=1, 2, ..., N_h$. Here the auxiliary variate x and the study variate y are assumed to be positively correlated while the auxiliary variate z is assumed to be negatively correlated with the study variate y. To estimate population mean of the study variate y, a sample of size n_h is drawn using simple random sampling without replacement from each stratum that constitutes the stratified random sample of size n such that $n=\sum_{h=1}^{L} n_h$. Then we define

$\bar{Y}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}y_{hi}=\frac{1}{N}\sum_{h=1}^{L}N_h\bar{Y}_h=\sum_{h=1}^{L}W_h\bar{Y}_h$: Population mean of the study variate y ,

$\bar{X}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}x_{hi}=\sum_{h=1}^{L}W_h\bar{X}_h$: Population mean of the auxiliary variate *x*,

$\bar{Z}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}z_{hi}=\sum_{h=1}^{L}W_h\bar{Z}_h$: Population mean of the auxiliary variate z.

Our problem is to estimate population mean of $\bar{Y}$ of the study variate y using stratified random sampling.

Usual unbiased estimator of population of population mean $\bar{Y}$ in stratified random sampling is defined as $\bar{y}_{st}=\sum_{h=1}^{L} W_h \bar{y}_h$.

Similarly, unbiased estimators of population mean $\bar{X}$ of the auxiliary variate x and population mean $\bar{Z}$ of the auxiliary variate z are defined respectively as

$\bar{x}_{st}=\sum_{h=1}^{L} W_h \bar{x}_h$ and $\bar{z}_{st}=\sum_{h=1}^{L} W_h \bar{z}_h$ respectively,

where

$\bar{x}_h=\frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$ and $\bar{z}_h=\frac{1}{n_h}\sum_{i=1}^{n_h} z_{hi}$ are means of sample taken from h^{th} stratum for the auxiliary variate x and z respectively.

Hansen et al. (1946) envisaged combined ratio estimator for population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_{RC}=\bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right). \tag{5.1.1}$$

For negative correlation between the study variate y and the auxiliary variate z, combined product estimator is defined as

$$\hat{\bar{Y}}_{PC}=\bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right). \tag{5.1.2}$$

Variance of unbiased estimators $\bar{y}_{st}$, $\bar{x}_{st}$ and $\bar{z}_{st}$ are respectively given as

$$V(\bar{y}_{st})=\sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2, \tag{5.1.3}$$

$$V(\bar{x}_{st})=\sum_{h=1}^{L} W_h^2\gamma_h S_{xh}^2 . \tag{5.1.4}$$

and

$$V(\bar{z}_{st})=\sum_{h=1}^{L} W_h^2\gamma_h S_{zh}^2 . \tag{5.1.5}$$

The biases and mean squared error of the combined ratio estimator $\hat{\bar{Y}}_{RC}$ and combined product estimator $\hat{\bar{Y}}_{PC}$ are

$$B\left(\hat{\bar{Y}}_{RC}\right)=\frac{1}{\bar{X}}\sum_{h=1}^{L} W_h^2\gamma_h\left(R_1S_{xh}^2-S_{yxh}\right) , \tag{5.1.6}$$

$$B\left(\hat{\bar{Y}}_{PC}\right)=\frac{1}{\bar{Z}}\sum_{h=1}^{L} W_h^2\gamma_h\left(R_2^2S_{zh}^2+S_{yzh}\right), \tag{5.1.7}$$

$$MSE\left(\hat{\bar{Y}}_{RC}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+R_1^2S_{xh}^2-2R_1S_{yxh}\right), \tag{5.1.8}$$

$$MSE\left(\hat{\bar{Y}}_{PC}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+R_2^2S_{zh}^2+2R_2S_{yzh}\right) , \tag{5.1.9}$$

where

$R_1=\frac{\bar{Y}}{\bar{X}}$ and $R_2=\frac{\bar{Y}}{\bar{Z}}$.

Bahl and Tuteja (1991) developed ratio and product type exponential estimators for population mean $\bar{Y}$ in simple random sampling as

$$\hat{\bar{Y}}_{\text{Re}} = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right), \quad (5.1.10)$$

and

$$\hat{\bar{Y}}_{Pe} = \bar{y} \exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}} \right). \quad (5.1.11)$$

Singh et al. (2008) studied Bahl and Tuteja (1991) estimators in stratified random sampling as

$$\hat{\bar{Y}}_{\text{Re}}^{ST} = \bar{y}_{st} \exp\left[\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h - \bar{x}_h\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \bar{x}_h\right)} \right]. \quad (5.1.12)$$

and

$$\hat{\bar{Y}}_{Pe}^{ST} = \bar{y}_{st} \exp\left[\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h - \bar{Z}_h\right)}{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \bar{Z}_h\right)} \right]. \quad (5.1.13)$$

Tailor et al. (2013) used Srivenkataramana (1980) transformation and obtained dual to Singh et al. (2008) ratio and product type exponential estimators $\hat{\bar{Y}}_{\text{Re}}^{ST}$ and $\hat{\bar{Y}}_{Pe}^{ST}$ as

$$\hat{\bar{Y}}_{\text{Re}}^{*ST} = \bar{y}_{st} \exp\left[\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_h^* - \bar{X}_h\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h^* + \bar{X}_h\right)} \right], \quad (5.1.14)$$

and

$$\hat{\bar{Y}}_{Pe}^{*ST} = \bar{y}_{st} \exp\left[\frac{\sum_{h=1}^{L} W_h\left(\bar{Z}_h - \bar{z}_h^*\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \bar{z}_h^*\right)} \right]. \quad (5.1.15)$$

where $\bar{x}_h^* = \frac{\bar{X}_h N_h - \bar{x}_h n_h}{N_h - n_h}$ and $\bar{z}_h^* = \frac{\bar{Z}_h N_h - \bar{z}_h n_h}{N_h - n_h}$ are unbiased estimators of population mean $\bar{X}$ and $\bar{Z}$ respectively.

Singh (1967) utilized information on population mean of two auxiliary variates i.e. $\bar{X}$ and $\bar{Z}$ and suggested ratio-cum-product estimator for population mean $\bar{Y}$ in simple random sampling as

$$\hat{\bar{Y}}_{RP} = \bar{y}\left(\frac{\bar{X}}{\bar{x}} \right)\left(\frac{\bar{z}}{\bar{Z}} \right). \quad (5.1.16)$$

Tailor et al. (2012) defined Singh (1967) estimator $\hat{\bar{Y}}_{RP}$ in stratified random sampling as

$$\hat{\bar{Y}}_{RP}^{ST} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}} \right)\left(\frac{\bar{z}_{st}}{\bar{Z}} \right). \quad (5.1.17)$$

Singh et al. (2009) defined ratio-cum-product type exponential estimator in simple random sampling as

$$\hat{\hat{Y}}_{RPe} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\exp\left(\frac{\bar{z}-\bar{Z}}{\bar{z}+\bar{Z}}\right). \quad (5.1.18)$$

Tailor and Chouhan (2013) studied $\hat{\hat{Y}}_{RPe}$ in stratified random sampling as

$$\hat{\hat{Y}}_{RPe}^{ST} = \bar{y}_{st}\exp\left(\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right)\exp\left(\frac{\bar{z}_{st}-\bar{Z}}{\bar{z}_{st}+\bar{Z}}\right), \quad (5.1.19)$$

or

$$\hat{\hat{Y}}_{RPe}^{ST} = \bar{y}_{st}\exp\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h-\bar{x}_h\right)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h+\bar{x}_h\right)}\right)\exp\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h-\bar{Z}_h\right)}{\sum_{h=1}^{L} W_h\left(\bar{z}_h+\bar{Z}_h\right)}\right). \quad (5.1.20)$$

II. SUGGESTED IMPROVED RATIO-CUM-PRODUCT ESTIMATOR

Using the transformation $x_i^* = \dfrac{N\bar{X}-n\bar{x}}{N-n}$, Srivenkataramana (1980) obtained dual to classical ratio estimator as

$$\hat{\hat{Y}}_R^* = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right). \quad (5.2.1)$$

Using the same transformation i.e. $\bar{x}_h^* = \dfrac{N_h\bar{X}_h - n_h\bar{x}_h}{N_h-n_h}$ and $\bar{z}_h^* = \dfrac{N_h\bar{Z}_h - n_h\bar{z}_h}{N_h-n_h}$ on auxiliary variates x and z dual to Tailor and Chouhan (2013) estimator is suggested as

$$\hat{\hat{Y}}_{RPe}^{*ST} = \left(\sum_{h=1}^{L} W_h\bar{y}_h\right)\exp\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{x}_h^*-\bar{X}_h\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h^*+\bar{X}_h\right)}\right)\exp\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{Z}_h-z_h^*\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h+z_h^*\right)}\right). \quad (5.2.2)$$

$$\hat{\hat{Y}}_{RPe}^{*ST} = \sum_{h=1}^{L} W_h\bar{y}_h \exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)-\sum_{h=1}^{L} W_h\bar{X}_h}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)+\sum_{h=1}^{L} W_h\bar{X}_h}\right]$$

$$\exp\left[\frac{\sum_{h=1}^{L} W_h\bar{Z}_h-\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\bar{Z}_h+\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)}\right]$$

$$=\left(\sum_{h=1}^{L} W_h\bar{y}_h\right)\exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h-N_h\bar{X}_h+n_h\bar{X}_h}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h+N_h\bar{X}_h-n_h\bar{X}_h}{N_h-n_h}\right)}\right]$$

$$\exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h - n_h\bar{Z}_h - N_h\bar{Z}_h + n_h\bar{z}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h - n_h\bar{Z}_h + N_h\bar{Z}_h - n_h\bar{z}_h}{N_h - n_h}\right)}\right]$$

$$=\left(\sum_{h=1}^{L} W_h\bar{y}_h\right)\exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{n_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{X}_h - n_h\bar{X}_h - n_h\bar{x}_h}{N_h - n_h}\right)}\right]$$

$$\exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{n_h\bar{z}_h - n_h\bar{Z}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{Z}_h - n_h\bar{z}_h - n_h\bar{Z}_h}{N_h - n_h}\right)}\right].$$

To obtain the bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{RPe}^{*ST}$, we write

$\bar{y}_h = \bar{Y}_h(1+e_{0h})$, $\bar{x}_h = \bar{X}_h(1+e_{1h})$ and $\bar{z}_h = \bar{Z}_h(1+e_{2h})$ such that

$E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = 0$,

$E(e_{0h}^2) = \gamma_h C_{yh}^2$,

$E(e_{1h}^2) = \gamma_h C_{xh}^2$,

$E(e_{2h}^2) = \gamma_h C_{zh}^2$,

$E(e_{0h}e_{1h}) = \gamma_h \rho_{yxh} C_{yh} C_{xh}$,

$E(e_{0h}e_{2h}) = \gamma_h \rho_{yzh} C_{yh} C_{zh}$,

and

$E(e_{1h}e_{2h}) = \gamma_h \rho_{xzh} C_{xh} C_{zh}$.

Now the suggested dual to ratio-cum-product type exponential estimator in terms of e_i's are expressed as

$$\hat{\bar{Y}}_{RPe}^{*ST} = \left(\sum_{h=1}^{L} W_h\bar{Y}_h(1+e_{0h})\right)\exp\left[\frac{\sum_{h=1}^{L} W_h g_h\left(\bar{X}_h - \bar{X}_h(1+e_{1h})\right)}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{X}_h - n_h\left(\bar{X}_h + \bar{X}_h(1+e_{1h})\right)}{N_h - n_h}\right)}\right]$$

$$\exp\left[\frac{\sum_{h=1}^{L} W_h n_h\left(\frac{\bar{Z}_h(1+e_{2h}) - \bar{Z}_h}{N_h - n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{Z}_h - n_h\left(\bar{Z}_h(1+e_{2h}) + \bar{Z}_h\right)}{N_h - n_h}\right)}\right]$$

$$=\bar{Y}\left(1+\frac{\sum_{h=1}^{L} W_h\bar{Y}_h e_{0h}}{\bar{Y}}\right)\exp\left[\frac{-\sum_{h=1}^{L} W_h\bar{X}_h g_h e_{1h}}{2\bar{X}-\sum_{h=1}^{L} W_h\bar{X}_h g_h e_{1h}}\right]\exp\left[\frac{\sum_{h=1}^{L} W_h\bar{Z}_h g_h e_{2h}}{2\bar{Z}-\sum_{h=1}^{L} W_h\bar{Z}_h g_h e_{2h}}\right]$$

$$\bar{Y}\left(1+\frac{\sum_{h=1}^{L} W_h\bar{Y}_h e_{0h}}{\bar{Y}}\right)\exp\left[\frac{-\frac{\sum_{h=1}^{L} W_h\bar{X}_h g_h e_{1h}}{\bar{X}}}{2-\frac{\sum_{h=1}^{L} W_h\bar{X}_h g_h e_{1h}}{\bar{X}}}\right]\exp\left[\frac{\frac{\sum_{h=1}^{L} W_h\bar{Z}_h g_h e_{2h}}{\bar{Z}}}{2-\frac{\sum_{h=1}^{L} W_h\bar{Z}_h g_h e_{2h}}{\bar{Z}}}\right]$$

$$\hat{\bar{Y}}_{RPe}^{*ST}=\bar{Y}\left(1+e_0\right)\exp\left[\frac{-e_1}{2-e_1}\right]\exp\left[\frac{e_2}{2-e_2}\right]$$

where

$$g_h=\frac{n_h}{N_h-n_h}$$

$$e_0=\frac{\sum_{h=1}^{L} W_h\bar{Y}_h e_{0h}}{\bar{Y}},\; e_1=\frac{\sum_{h=1}^{L} W_h\bar{X}_h g_h e_{1h}}{\bar{X}} \text{ and } e_2=\frac{\sum_{h=1}^{L} W_h\bar{Z}_h g_h e_{2h}}{\bar{Z}}$$

such that

$$E(e_0)=E(e_1)=E(e_2)=0,$$

$$E(e_0^2)=\frac{\sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2}{\bar{Y}^2},$$

$$E(e_1^2)=\frac{\sum_{h=1}^{L} W_h^2\gamma_h g_h^2 S_{xh}^2}{\bar{X}^2},$$

$$E(e_2^2)=\frac{\sum_{h=1}^{L} W_h^2\gamma_h g_h^2 S_{zh}^2}{\bar{Z}^2},$$

$$E(e_0e_1)=\frac{1}{\bar{Y}\,\bar{X}}\sum_{h=1}^{L} W_h^2\gamma_h g_h S_{yxh},$$

$$E(e_0e_2)=\frac{1}{\bar{Y}\,\bar{Z}}\sum_{h=1}^{L} W_h^2\gamma_h g_h S_{yzh}$$

and

$$E(e_1e_2)=\frac{1}{\bar{X}\,\bar{Z}}\sum_{h=1}^{L} W_h^2\gamma_h g_h S_{xzh}.$$

Now

$$\hat{\hat{Y}}_{RPe}^{*ST} = \bar{Y}\left(1+e_0\right) \exp\left[\frac{-e_1}{2}\left(1-\frac{e_1}{2}\right)^{-1}\right] \exp\left[\frac{e_2}{2}\left(1-\frac{e_2}{2}\right)^{-1}\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1-\frac{e_1}{2}\left(1-\frac{e_1}{2}\right)^{-1}+\frac{e_1^2}{8}\left(1-\frac{e_1}{2}\right)^{-2}\right] \left[1+\frac{e_2}{2}\left(1-\frac{e_2}{2}\right)^{-1}+\frac{e_2^2}{8}\left(1-\frac{e_2}{2}\right)^{-2}\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1-\frac{e_1}{2}\left(1+\frac{e_1}{2}+\frac{e_1^2}{4}\right)+\frac{e_1^2}{8}\left(1+e_1+\frac{3e_1^2}{4}\right)\right]$$

$$\left[1+\frac{e_2}{2}\left(1+\frac{e_2}{2}+\frac{e_2^2}{4}\right)+\frac{e_2^2}{8}\left(1+e_2+\frac{3e_2^2}{4}\right)\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1-\frac{e_1}{2}-\frac{e_1^2}{4}+\frac{e_1^2}{8}\right]\left[1+\frac{e_2}{2}+\frac{e_2^2}{4}+\frac{e_2^2}{8}\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1-\left(\frac{e_1}{2}\right)-\left(\frac{2e_1^2-e_1^2}{8}\right)\right]\left[1+\frac{e_2}{2}+\frac{3e_2^2}{8}\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1-\frac{e_1}{2}-\frac{e_1^2}{8}\right]\left[1+\frac{e_2}{2}+\frac{3e_2^2}{8}\right]$$

$$= \bar{Y}\left(1+e_0\right)\left[1-\frac{e_1}{2}-\frac{e_1^2}{8}+\frac{e_2}{2}-\frac{e_1e_2}{4}+\frac{3e_2^2}{8}\right]$$

$$= \bar{Y}\left[1-\frac{e_1}{2}-\frac{e_1^2}{8}+\frac{e_2}{2}-\frac{e_1e_2}{2}+\frac{3e_2^2}{8}+e_0-\frac{e_0e_1}{2}+\frac{e_0e_2}{2}\right]$$

$$\hat{\hat{Y}}_{RPe}^{*ST} - \bar{Y} = \bar{Y}\left[e_0-\frac{e_1}{2}+\frac{e_2}{2}-\frac{e_1^2}{8}+\frac{3e_2^2}{8}-\frac{e_1e_2}{2}-\frac{e_0e_1}{2}+\frac{e_0e_2}{2}\right]. \qquad (5.2.3)$$

Taking expectation of both sides of (5.2.3), the bias of the suggested dual to ratio-cum-product estimator $\hat{\hat{Y}}_{RPe}^{*ST}$ upto the first degree of approximation is obtained as

$$B\left(\hat{\hat{Y}}_{RPe}^{*ST}\right) = \bar{Y}\sum_{h=1}^{L} W_h^2\gamma_h\left[-\frac{1}{8}S_{xh}^2g_h^2+\frac{3}{8}S_{zh}^2g_h^2-\frac{1}{2}S_{yxh}g_h+\frac{1}{2}S_{yzh}g_h-\frac{1}{2}S_{xzh}g_h\right],$$

or

$$B\left(\hat{\hat{Y}}_{RPe}^{*ST}\right) = \bar{Y}\,\frac{\sum_{h=1}^{L} W_h^2\gamma_h g_h}{8}\left[-g_h\left(S_{xh}^2-3S_{zh}^2\right)-4\left(S_{yxh}+S_{yzh}-S_{xzh}\right)\right]. \qquad (5.2.4)$$

Taking square and expectation of both sides of (5.2.3), mean squared error of the suggested dual to ratio-cum-product estimator $\hat{\hat{Y}}_{RPe}^{*ST}$, upto the first degree of approximation is obtained as

$$MSE\left(\hat{\hat{Y}}_{RPe}^{*ST}\right) = \bar{Y}^2\,E\left[e_0-\frac{e_1}{2}+\frac{e_2}{2}\right]^2$$

$$MSE\left(\hat{\hat{Y}}_{RPe}^{*ST}\right) = \sum_{h=1}^{L} W_h^2\gamma_h\left[S_{yh}^2+\frac{1}{4}R_1^2S_{xh}^2g_h^2+\frac{1}{4}R_2^2S_{zh}^2g_h^2-R_1S_{yxh}g_h\right.$$

$$+R_2 S_{yzh} g_h - \frac{R_1 R_2 g_h^2 S_{xzh}}{2} \Bigg]$$

$$MSE\left(\hat{\hat{Y}}_{RPe}^{*ST}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left[S_{yh}^2+\frac{1}{4}\left\{R_1^2S_{xh}^2g_h^2+R_2^2S_{zh}^2g_h^2-4\left(R_1S_{yxh}g_h-R_2S_{yzh}g_h-2R_1R_2g_h^2S_{xzh}\right)\right\}\right] \text{or}$$

$$MSE\left(\hat{\hat{Y}}_{RPe}^{*ST}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left[S_{yh}^2+\frac{g_h^2}{4}\left(\left(R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{xzh}\right)-g_h\left(R_1S_{yxh}-R_2S_{yzh}\right)\right)\right] \quad (5.2.5)$$

III. EFFICIENCY COMPARISONS

This section compares the efficiency of the suggested estimator $\hat{\hat{Y}}_{RPe}^{*ST}$, with respect to usual unbiased estimator $\bar{y}_{st}$, combined ratio and product estimators $\hat{\hat{Y}}_{RC}$ and $\hat{\hat{Y}}_{PC}$, dual to combined ratio and product estimators $\hat{\hat{Y}}_{RC}^{*}$ and $\hat{\hat{Y}}_{PC}^{*}$, Singh et al. (2008) ratio and product type exponential estimators $\hat{\hat{Y}}_{\mathrm{Re}}^{ST}$ and $\hat{\hat{Y}}_{Pe}^{ST}$, dual to Singh et al. (2008) estimators given by Tailor et al. (2013) $\hat{\hat{Y}}_{\mathrm{Re}}^{*ST}$ and $\hat{\hat{Y}}_{Pe}^{*ST}$ and Tailor and Chouhan (2013) estimator $\hat{\hat{Y}}_{RPe}^{*ST}$.

Variance of usual unbiased estimator $\bar{y}_{st}$ is expressed as

$$V\left(\bar{y}_{st}\right)=\sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2. \quad (5.3.1)$$

Mean squared errors of the estimators $\hat{\hat{Y}}_{RC}$, $\hat{\hat{Y}}_{PC}, \hat{\hat{Y}}_{RC}^{*}$, $\hat{\hat{Y}}_{PC}^{*}, \hat{\hat{Y}}_{\mathrm{Re}}^{ST}$, $\hat{\hat{Y}}_{Pe}^{ST}$ $\hat{\hat{Y}}_{\mathrm{Re}}^{*ST}$, $\hat{\hat{Y}}_{Pe}^{*ST}$ and $\hat{\hat{Y}}_{RPe}^{*ST}$ are expressed as

$$MSE\left(\hat{\hat{Y}}_{RC}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+R_1^2S_{xh}^2-2R_1S_{yxh}\right), \quad (5.3.2)$$

$$MSE\left(\hat{\hat{Y}}_{PC}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+R_2^2S_{zh}^2+2R_2S_{yzh}\right), \quad (5.3.3)$$

$$MSE\left(\hat{\hat{Y}}_{RC}^{*}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+R_1^2g_h^2S_{xh}^2-2R_1g_hS_{yxh}\right), \quad (5.3.4)$$

$$MSE\left(\hat{\hat{Y}}_{PC}^{*}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+R_2^2g_h^2S_{zh}^2+2R_2g_hS_{yzh}\right), \quad (5.3.5)$$

$$MSE\left(\hat{\hat{Y}}_{\mathrm{Re}}^{ST}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+\frac{1}{4}R_1^2S_{xh}^2-R_1S_{yxh}\right) \quad (5.3.6)$$

$$MSE\left(\hat{\hat{Y}}_{Pe}^{ST}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+\frac{1}{4}R_2^2S_{zh}^2+R_2S_{yzh}\right), \quad (5.3.7)$$

$$MSE\left(\hat{\hat{Y}}_{\mathrm{Re}}^{*ST}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+\frac{1}{4}R_1^2g_h^2S_{xh}^2-R_1g_hS_{yxh}\right), \quad (5.3.8)$$

$$MSE\left(\hat{\hat{Y}}_{Pe}^{*ST}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+\frac{1}{4}R_2^2g_h^2S_{zh}^2-R_2g_hS_{yzh}\right). \quad (5.3.9)$$

$$MSE\left(\hat{\hat{Y}}_{RPe}^{ST}\right)=\sum_{h=1}^{L} W_h^2\gamma_h\left(S_{yh}^2+\frac{1}{4}(R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{zzh})+R_2S_{yzh}-R_1S_{yxh}\right). \quad (5.3.10)$$

Comparisons of (5.2.5), (5.3.1), (5.3.2), (5.3.3), (5.3.4), (5.3.5), (5.3.6), (5.3.7), (5.3.8), (5.3.9) and (5.3.10), shows that the suggested dual to ratio-cum-product type exponential estimator of population mean $\hat{\bar{Y}}^{*ST}_{RPe}$ would be more efficient than

(i) $\bar{y}_{st}$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[\frac{g_h^2}{4}\left(R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{xzh}\right)-g_h\left(R_1S_{yxh}-R_2S_{yzh}\right)\right]<0, \quad (5.3.11)$$

(ii) $\hat{\bar{Y}}_{RC}$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[\frac{1}{4}R_1^2S_{xh}^2\left(g_h^2-4\right)+\frac{g_h^2}{4}\left(R_2^2S_{zh}^2-2R_1R_2S_{xzh}\right)-R_1S_{yxh}\left(g_h-2\right)+g_hR_2S_{yzh}\right]<0, \quad (5.3.12)$$

(iii) $\hat{\bar{Y}}_{PC}$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[\frac{1}{4}R_2^2S_{zh}^2\left(g_h^2-4\right)+\frac{g_h^2}{4}\left(R_1^2S_{xh}^2-2R_1R_2S_{xzh}\right)-R_2S_{yzh}\left(g_h-2\right)-g_hR_1S_{yxh}\right]<0, \quad (5.3.13)$$

(iv) $\hat{\bar{Y}}^{*}_{RC}$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[\frac{g_h^2}{4}\left(-3R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{xzh}\right)+g_h\left(R_1S_{yxh}+R_2S_{yzh}\right)\right]<0, \quad (5.3.14)$$

(v) $\hat{\bar{Y}}^{*}_{PC}$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[\frac{g_h^2}{4}\left(R_1^2S_{xh}^2-3R_2^2S_{zh}^2-2R_1R_2S_{xzh}\right)-g_h\left(R_1S_{yxh}+R_2S_{yzh}\right)\right]<0, \quad (5.3.15)$$

(vi) $\hat{\bar{Y}}^{ST}_{\mathrm{Re}}$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[\frac{1}{4}R_1^2S_{xh}^2\left(g_h^2-1\right)+\frac{g_h^2}{4}\left(R_2^2S_{zh}^2-2R_1R_2S_{xzh}\right)-R_1S_{yxh}\left(g_h-1\right)+g_hR_2S_{yzh}\right]<0, \quad (5.3.16)$$

(vii) $\hat{\bar{Y}}^{ST}_{Pe}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{1}{4} R_2^2 S_{zh}^2 \left(g_h^2 - 1\right) + \frac{g_h^2}{4} \left(R_1^2 S_{xh}^2 - 2R_1 R_2 S_{xzh}\right) + R_2 S_{yzh}\left(g_h - 1\right) - g_h R_1 S_{yxh} \right] < 0, \quad (5.3.17)$$

(viii) $\hat{\hat{Y}}_{\text{Re}}^{*ST}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{g_h^2}{4} \left(R_2^2 S_{zh}^2 - 2R_1 R_2 S_{xzh}\right) + g_h R_2 S_{yzh} \right] < 0 \ , \quad (5.3.18)$$

(ix) $\hat{\hat{Y}}_{Pe}^{*ST}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{g_h^2}{4} \left(R_1^2 S_{xh}^2 - 2R_1 R_2 S_{xzh}\right) - g_h R_1 S_{yxh} \right] < 0 \ . \quad (5.3.19)$$

(x) $\hat{\hat{Y}}_{RP}^{ST}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{1}{4} \left(R_1^2 S_{xh}^2 - 2R_1 R_2 S_{xzh}\right)\left(g_h^2 - 1\right) - \left(g_h - 1\right)\left(R_1 S_{yxh} - R_2 S_{yzh}\right) \right] < 0. \quad (5.3.20)$$

Expressions (5.3.11) to (5.3.20) provide the conditions under which suggested dual to ratio-cum-product type exponential estimator has less mean squared error as compared to $\bar{y}_{st}$ $\hat{\hat{Y}}_{RC}$, $\hat{\hat{Y}}_{PC}$, $\hat{\hat{Y}}_{RC}^{*}$, $\hat{\hat{Y}}_{PC}^{*}$, $\hat{\hat{Y}}_{\text{Re}}^{ST}$, $\hat{\hat{Y}}_{Pe}^{ST}$, $\hat{\hat{Y}}_{\text{Re}}^{*ST}$, $\hat{\hat{Y}}_{Pe}^{*ST}$ and $\hat{\hat{Y}}_{RPe}^{ST}$.

IV. EMPIRICAL STUDY

To see the performance of the suggested estimator $\hat{\hat{Y}}_{RPe}^{*ST}$, a natural population data set is being considered. Description of the populations is given below :

Population I [Source: Murthy (1967), p. 228]
y : Output ,
x : Fixed capital ,
z : Number of workers .

N= 10 n= 4	n_1=2	n_2=2	N_1=5	N_2=5
	$\bar{Z}_1$=51.80	$\bar{Z}_2$=60.60	$\bar{X}_1$=214.40	$\bar{X}_2$=333.80
	$\bar{Y}_1$=1925.80	$\bar{Y}_2$= 315.60	S_{z_1}=0.75	S_{z_2}=4.84
	S_{x_1}= 74.87	S_{x_2}=66.35	S_{y_1}=615.92	S_{y_2}=340.38
	S_{zx_1} = 38.08	S_{zx_2}=287.92	S_{yz_1}=-411.16	S_{yz_2}=-1536.24
	S_{yx_1}=39360.68	S_{yx_2}=22356.50		

Table 5.4.1: Percent relative efficiencies of $\bar{y}_{st}$, $\hat{\bar{Y}}_{RC}$, $\hat{\bar{Y}}_{PC}$, $\hat{\bar{Y}}^{*}_{RC}$, $\hat{\bar{Y}}^{*}_{PC}$, $\hat{\bar{Y}}^{ST}_{\mathrm{Re}}$, $\hat{\bar{Y}}^{ST}_{Pe}$, $\hat{\bar{Y}}^{*ST}_{\mathrm{Re}}$, $\hat{\bar{Y}}^{*ST}_{Pe}$, $\hat{\bar{Y}}^{ST}_{RPe}$ and $\hat{\bar{Y}}^{*ST}_{RPe}$ with respect to $\bar{y}_{st}$

Estimator	$\bar{y}_{st}$	$\hat{\bar{Y}}_{RC}$	$\hat{\bar{Y}}_{PC}$	$\hat{\bar{Y}}^{*}_{RC}$	$\hat{\bar{Y}}^{*}_{PC}$
PREs	100.00	313.75	115.95	432.03	123.74
Estimator	$\hat{\bar{Y}}^{ST}_{\mathrm{Re}}$	$\hat{\bar{Y}}^{ST}_{Pe}$	$\hat{\bar{Y}}^{*ST}_{\mathrm{Re}}$	$\hat{\bar{Y}}^{*ST}_{Pe}$	$\hat{\bar{Y}}^{ST}_{RPe}$
PREs	173.94	107.94	234.96	111.95	244.86
Estimator	$\hat{\bar{Y}}^{*ST}_{RPe}$				
PREs	454.56				

V. CONCLUSION

In this chapter dual to ratio-cum-product type exponential estimator has been suggested using Srivenkataramana (1980) transformation. In section 5.3 conditions under which the suggested estimator would be more efficient than other considered estimators have been obtained. Table 5.4.1 reflects the percent relative efficiencies of all considered estimators. This table shows that the suggested dual to ratio-cum-product estimator $\hat{\bar{Y}}^{*ST}_{RPe}$ has highest percent relative efficiency as compared to usual unbiased estimator $\bar{y}_{st}$, combined ratio estimator $\hat{\bar{Y}}_{RC}$, combined product estimator $\hat{\bar{Y}}_{PC}$, dual to combined ratio estimator $\hat{\bar{Y}}^{*}_{RC}$, dual to combined product estimator $\hat{\bar{Y}}^{*}_{PC}$, , ratio type exponential estimator $\hat{\bar{Y}}^{ST}_{\mathrm{Re}}$, dual to ratio type exponential estimator $\hat{\bar{Y}}^{*ST}_{\mathrm{Re}}$, product type exponential estimator $\hat{\bar{Y}}^{ST}_{Pe}$, dual to product type exponential estimator $\hat{\bar{Y}}^{*ST}_{Pe}$ and ratio-cum-product type exponential estimator $\hat{\bar{Y}}^{ST}_{RPe}$. Thus the suggested dual to ratio-cum-product type exponential estimator $\hat{\bar{Y}}^{*ST}_{RPe}$ is recommended for the estimation of population mean when conditions obtained in Section 5.3 are satisfied.

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