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Research Article

Some New Results, Structures and Operations on Soft Multigroup Theory

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Abstract— In this paper, we introduce the notion of commutators in soft mgroups, soft m-monoid, soft semi mgroup, soft mgroupoid, conjugate soft mgroups, and exemplify some important operations in soft mgroups. We show that the nth power of the union of two soft mgroups is the same as the union of nth power of each of the mgroups, that the root set of the union of two soft mgroups is closed under arithmetic power. We examine some algebraic properties of soft mgroups. We obtain some results on commutators in soft mgroups, normal soft mgroups and conjugate soft mgroups.

Keywords— soft sets, mgroups, soft msets, soft mgroups, regular soft mgroups, soft mmonoids, commutators in soft mgroup, soft normal submgroups, soft mgroupoids

1. Introduction

A mathematical theory called Soft Set was initiated by Molodtsov [1] to handle issues involving uncertainties and was later found applicable in many areas of mathematics and other sciences. It addressed the existing difficulties of previous theories of fuzzy sets [2], rough sets [3], intuitionistic fuzzy [4], and many others in dealing with uncertainties. Multiset which is seen to be of two folds and unified [5] is among the theories that played an important role in the development of Soft Sets as many researchers have combined the concept of soft set theory with other theories of set, multiset, and group to generate hybrid structures like fuzzy soft sets [6], soft group [7] and soft multigroup [8] among others.

Aktas and Cagman [7] first introduced the notion of a soft group and studied soft subgroups, normal soft subgroups, and soft homomorphisms. Jebalily et. al. [12], investigated some properties of algebraic substructures of soft fields and soft submodules and discussed the correlation coefficient between them. Feng et.al [13] pioneered the work on soft semirings, soft sub semirings, soft ideals, idealistic soft semirings, and soft semiring homomorphisms.

Nazmul and Samanta [8] initiated the study on soft multigroups (mgroups, for short), in which identity soft mgroups, absolute soft mgroups, soft abelian mgroups, and soft factor mgroups were defined and some of their important properties studied.

This paper is a further attempt to broaden the theoretical aspects of soft mgroups. To further develop, the work of [8], we have developed more propositions on soft mgroups, expanded on the properties, characterizations, and exemplifications of some of the algebraic properties, and obtained some important results.

The rest of this paper is organized as follows. In section II, we review the basic definitions of soft sets, Multisets, soft multisets, Soft groups, and Multigroups. In section III, we present novel formulations and propositions on soft mgroups and related structures. Also, more algebraic properties of soft mgroups are investigated. In IV, we give the conclusion.

2. Preliminaries

In this section, we present basic definitions of some basic concepts like of soft set, soft msets and mgroups.

Definition 2.1 Soft set [1]

Let U be an initial universe set, E a set of parameters, $A \subseteq E$ and P(U) be the power set of U. A pair (F, A) is called a soft set over U iff F: A→P(U). Hence, we can write (F, A) as: (F, A) = {F(e) ∈ P(U): e ∈A, F(e) = ∅ if e ∉A)}. For e ∈ A, F(e) may be considered as the set of e - approximate elements of the soft set (F, A). A soft set over U can be written as (F, A), FA or (FA, E) over U, where A ⊆ E.

Definition 2.2 Multiset [9]

A mset A drawn from the set X is represented by a count function $C_A(x)$ and is defined as $C_A: X \longrightarrow \mathbb{N}$, where $\mathbb{N}$ represents the set of natural numbers including zero.

Let $M \in \mathfrak{M}(X)$. The support set of M denoted by M^* is a subset of X given by
$M^* = \{x \in X: C_M(x) > 0\}$. M^* is also called root set.
Let's denote the set of all multisets over X by $\mathfrak{M}(X)$.

Definition 2.3 Soft Multiset (Soft Mset) [10]
Let X be a universal mset, E a set of parameters and A$\subseteq$ E. Let the set of all submsets of X be denoted by P^*(X) and called the power set of X.
Then a pair (F, A) is called a soft mset over X where F: $A \to P^*$(X). For all $\alpha \in A$, the mset $F(\alpha)$ is represented by a count function $C_{F(\alpha)}: X^* \longrightarrow \mathbb{N}$.
Denote the set of all finite soft msets over X by S$\mathfrak{M}(X)$.

Definition 2.4 Soft Group [7]
Let G be a group and A be a non-empty set. Let (F, A) be a soft set over G. Then (F, A) is called a soft group over G if and only if F (α) is a subgroup of G $\forall$ $\alpha \in$ A, that is, For the soft set (F, A) over G, it is said that (F, A) is a soft group over G if and only if $F(\alpha) <$ G for all $\alpha \in$ A. From the definition, it is easy to see that the soft group (F, A) is a parameterized family of subgroups of the group G.

Definition 2.5 Mgroup [11]
Let X be a group. A multiset M over X is said to be a mgroup over X iff the Count function C_M satisfies the following two conditions.

(i) $C_M(xy) \geq C_M(x) \wedge C_M(y) \forall x, y \in$X;
(ii) $C_M(x^{-1}) \geq C_M(x) \forall\, x \in$ X;

Here we denote the set of all finite mgroups over X by $MG(X)$

3. Soft Mgroup

Definition 3.1 Soft Mgroup [8]
Let $M \in MG(X)$ be a universal mgroup and $A \subseteq E$ be a set of parameters. A soft mset (F, A) where F: A $\to P^*(M)$is said to be a soft mgroup over M if F(α) is a submgroup of M, $\forall\, \alpha \in$ A, $i.e.$
If (i) $C_{F_{(\alpha)}}(xy) \geq C_{F_{(\alpha)}}(x) \wedge C_{F(\alpha)}(y)$ and
(ii) $C_{F(\alpha)}(x^{-1}) \geq C_{F_{(\alpha)}}(x) \forall \alpha \in$A, $\forall x, y \in$ X.
We denote the set of all soft mgroups over X by S$MG(X)$.

Remark: 3.2 Consequently,
$C_{F_{(\alpha)}}(x) = C_{F_{(\alpha)}}(x^{-1})$ since
$C_{F_{(\alpha)}}(x) = C_{F_{(\alpha)}}((x^{-1})^{-1}) \geq C_{F_{(\alpha)}}(x^{-1})$.

Remark: 3.2.1
Following from (i),
$C_{F_{(\alpha)}}(y) = C_{F_{(\alpha)}}(ey) \geq C_{F_{(\alpha)}}(e) \wedge C_{F(\alpha)}(y)$

Example 3.3
Let the set X be a group of modulo 5 such that X = $Z_5 = \{0, 1, 2, 3, 4\}$ with respect to addition. Let M be a mgroup drawn from X, such that M = $[0,1,2,3,4]_{3,2,2,2,2}$ and A = $\{\alpha_1, \alpha_2, \alpha_3\}$. Let F: A $\to P^*(M)$ be defined by
F(α_1) = $[0,1,2,3,4]_{3,1,1,1,1}$.
F(α_2) = $[0,1,2,3,4]_{2,1,1,1,1}$.
F(α_3) = $[0,1,2,3,4]_{2,2,2,2,2}$.
Clearly, F(α_i), i = 1, 2, 3 are submgroups of M.
Hence (F, A) is a soft mgroup over M.

Proposition 3.4: Let M $\in MG(X)$be a universal mgroup and$(F, A) \in SMG(X)$. Then
$C_{F_{(\alpha)}}(x^n) \geq C_{F_{(\alpha)}}(x)$ $\forall x \in$ X and $n \in \mathbb{N}$.
Proof:
For $\forall$ $\alpha \in$ A, and $\forall x, y \in$ X we have
$C_{F_{(\alpha)}}(x^n) = C_{F_{(\alpha)}}(x^{n-1}.x)$.
$\geq C_{F(\alpha)}(x^{n-1}) \wedge C_{F(\alpha)}(x)$
$\geq C_{F(\alpha)}(x) \wedge \ldots \wedge C_{F(\alpha)}(x)$
$= C_{F(\alpha)}(x)$
Therefore $C_{F_{(\alpha)}}(x^n) \geq C_{F_{(\alpha)}}(x)$

Proposition 3.5: Let M $\in$ $MG(X)$ be a universal mgroup and $e \in X$ be the identity. Then for any (F, A) $\in SMG(X)$ such that $C_{F_{(\alpha)}}(xy^{-1}) = C_{F_{(\alpha)}}(e)$, we have $C_{F_{(\alpha)}}(x) = C_{F_{(\alpha)}}(y)\ \forall\, x, y \in X$.

Proof:
Let (F, A) $\in SMG(X)$ and
$C_{F(\alpha)}(xy^{-1}) = C_{F(\alpha)}(e)\ \forall x, y \in X,\ \alpha \in A$,
then we have
$C_{F_{(\alpha)}}(x) = C_{F_{(\alpha)}}(x\, y^{-1}y) = C_{F_{(\alpha)}}((x\, y^{-1})y)$
$\geq C_{F_{(\alpha)}}(x\, y^{-1}) \wedge C_{F(\alpha)}(y) = C_{F_{(\alpha)}}(e) \wedge C_{F_{(\alpha)}}(y)$
$= C_{F_{(\alpha)}}(y)\ i.e. C_{F_{(\alpha)}}(x) \geq C_{F_{(\alpha)}}(y)$ ………….. (1)
Also $C_{F_{(\alpha)}}(y) = C_{F_{(\alpha)}}(y^{-1}) = C_{F_{(\alpha)}}(x^{-1}xy^{-1})$
$= C_{F_{(\alpha)}}(x^{-1})(xy^{-1})$
$\geq C_{F_{(\alpha)}}(x^{-1}) \wedge C_{F_{(\alpha)}}(xy^{-1})$
$= C_{F_{(\alpha)}}(x) \wedge C_{F_{(\alpha)}}(e)$
$= C_{F_{(\alpha)}}(x)$
$i.e. C_{F_{(\alpha)}}(y) \geq C_{F_{(\alpha)}}(x)$ …………………… (2)
(1) And (2) $\Rightarrow$
$C_{F_{(\alpha)}}(y) = C_{F_{(\alpha)}}(x) \forall x, y \in X$ and $\alpha \in A$.

Definition 3.6
Intersection of soft mgroups:
Let $(F_1, \text{A}), (F_2, \text{B})$ be two soft mgroups over X, then their intersection, denoted by
$(F_1$, A) $\tilde{\cap}$ $(F_2$, B) = $(H,\ \text{A} \cap \text{B})$ where

$H(\alpha) = min\{F_1(\alpha), F_2(\alpha)\} \forall \alpha \in A \cap B$.

Theorem 3.7 The intersection of two soft mgroups is also a soft mgroup **(**Nazmul and Samanta (2015))

Alternative proof:

Let $(F_1, A), (F_2, B) \in SMG(X)$.

Now let $(F_1, A) \tilde{\cap} (F_2, B) = (H, A \cap B)$

Such that $H(\alpha) = F_1(\alpha) \cap F_2(\alpha)$

For any $\alpha \in A \cap B$

Clearly $F_1(\alpha)$ and $F_2(\alpha)$ are msubgroups of M. (by definition)

and $F_1(\alpha) \cap F_2(\alpha)$ is a msubgroup of M

thus $H(\alpha)$ is a msubgroup of M, $\forall \alpha \in A \cap B$.

in particular, $(H, A \cap B)$ is a soft mgroup.

Thus $(F_1, A) \tilde{\cap} (F_2, B)$ is a soft mgroup.

Definition 3.8

Union of soft mgroups

Let (F_1, A) and (F_2, B) be two soft mgroups over X, then their union is denoted by $(F_1, A) \tilde{\cup} (F_2, B)$ where:

$(F_1, A) \tilde{\cup} (F_2, B) = (H, A \cup B)$ defined by:

$H(\alpha) = \max\{F_1(\alpha), F_2(\alpha)\}) \ \forall \alpha \in A \cup B$

Remark 3.9 The union of two or more soft mgroups may not be a soft mgroup.

Definition 3.10

Root set of a soft mgroup

Let $(F, A) \in SMG(X)$. The root set of (F, A) denoted by $(F, A)^*$ is defined by

$(F, A)^* = (F^*, A)$ where $F^*: A \rightarrow P(X^*)$ given by $F^*(\alpha) = (F(\alpha))^*$.

Proposition 3.11 The root set of a soft mgroup is a soft group.

Proof:

Let $(F, A)^*$ be the root set of $(F, A) \in SMG(X)$

$\Rightarrow F^*(\alpha) \in (F, A)^* \forall \alpha \in A$

But $F(\alpha)$ is a mgroup

Let $x, y \in F(\alpha)^* \forall \alpha \in A$.

$\Rightarrow C_{F(\alpha)}(xy) \geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)$ and $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x)$

Now $C_{F(\alpha)}(xy^{-1}) \geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y^{-1})$ and $C_{F(\alpha)}(y^{-1}) = C_{F(\alpha)}(y)$

But $C_{F(\alpha)}(x) = 1 \ \forall x \in (F(\alpha))^*, \forall \alpha \in A$.

$\Rightarrow xy^{-1} \in (F(\alpha))^*, \forall \alpha \in A$

$\Rightarrow (F(\alpha))^*$ is a subgroup $\forall \alpha \in A$.

Therefore $(F, A)^*$ is a soft group.

Definition 3.12

Let (F, A) $(G, B) \in SMG(X)$ then

(i) $((F, A) \tilde{\cup} (G, B))^* = (H, A \cup B)^* = (H^*, A \cup B)$ with $H^*(\alpha) = (H(\alpha))^* = ((F(\alpha))^* \cup ((G(\alpha))^* \ \forall \alpha \in A \cup B$

(ii) $((F, A) \tilde{\cap} (G, B))^* = (K, A \cap B)^* = (K^*, A \cap B)$ with $K^*(\alpha) = (K(\alpha))^* = ((F(\alpha))^* \cap ((G(\alpha))^* \ \forall \alpha \in A \cap B$

Proposition 3.13 Let (F, A) $(G, B) \in SMG(X)$. Then

(i) $((F, A) \tilde{\cup} (G, B))^* = (F, A)^* \tilde{\cup} (G, B)^*$

(ii) $((F, A) \tilde{\cap} (G, B))^* = (F, A)^* \tilde{\cap} (G, B)^*$

Proof:

Let (F, A), $(G, B) \in SMG(X)$ and

$(H(\alpha))^* \in ((F, A) \tilde{\cup} (G, B))^*$

$\Longrightarrow$ for each $x \in (H(\alpha))^*, \alpha \in A \cup B$

We have $x \in (F(\alpha))^*, \forall \alpha \in A$ or $x \in (G(\alpha))^*, \forall \alpha \in B$

$\Longrightarrow (F(\alpha))^* \in (F, A)^*$ or $(G(\alpha))^* \in (G, B)^*$

$(F(\alpha))^* \tilde{\cup} (G(\alpha))^* \in (F, A)^* \tilde{\cup} (G, B)^*$

$(H(\alpha))^* \in (F, A)^* \cup (G, B)^*$

Thus, $((F, A) \tilde{\cup} (G, B))^* \subseteq (F, A)^* \tilde{\cup} (G, B)^*$………. (1)

Now let $y \in (F(\alpha))^* \tilde{\cup} (G(\alpha))^*$ for any $\alpha \in A \cup B$

$that$ is $y \in (F(\alpha))^*$ or $y \in (G(\alpha))^*$

$\Longrightarrow y \in ((F, A) \tilde{\cup} (G, B))^* = (H(\alpha))^* \ \forall \alpha \in A \cup B$

$(F, A)^* \tilde{\cup} (G, B)^* \subseteq ((F, A) \tilde{\cup} (G, B))^*$…………….. (2)

By (1) and (2)

$((F, A) \tilde{\cup} (G, B))^* = (F, A)^* \tilde{\cup} (G, B)^*$

ii) Let (F, A), $(G, B) \in SMG(X)$ and

$(K(\alpha))^* \in ((F, A) \tilde{\cap} (G, B))^*$

$\Longrightarrow$ for each $y \in (K(\alpha))^*, \alpha \in A \cap B$

We have $y \in (F(\alpha))^*, \forall \alpha \in A$ and $y \in (G(\alpha))^*, \forall \alpha \in B$

$\Longrightarrow (F(\alpha))^* \in (F, A)^*$ and $(G(\alpha))^* \in (G, B)^*$

$(F(\alpha))^* \tilde{\cap} (G(\alpha))^* \in (F, A)^* \tilde{\cap} (G, B)^*$

$(K(\alpha))^* \in (F, A)^* \cup (G, B)^*$

Thus, $((F, A) \tilde{\cap} (G, B))^* \subseteq (F, A)^* \tilde{\cap} (G, B)^*$……… (1)

Now let $y \in (F(\alpha))^* \tilde{\cap} (G(\alpha))^*$ for any $\alpha \in A \cap B$

$that$ is $y \in (F(\alpha))^*$ and $y \in (G(\alpha))^*$

$\Longrightarrow y \in ((F, A) \tilde{\cap} (G, B))^* = (H(\alpha))^* \forall \alpha \in A \cap B$

$(F, A)^* \tilde{\cap} (G, B)^* \subseteq ((F, A) \tilde{\cap} (G, B))^*$……………. (2)

By (1) and (2)

$((F, A) \tilde{\cap} (G, B))^* = (F, A)^* \tilde{\cap} (G, B)^*$

Proposition 3.14 Let (F, A) $(G, B) \in SMG(X)$ such that $((F, A) \tilde{\subseteq} (G, B)$

Then $(F, A)^* \tilde{\subseteq} (G, B)^*$

Proof:

Let (F, A) $(G, B) \in SMG(X)$ and $((F, A) \tilde{\subseteq} (G, B)$

$(F, A) \tilde{\subseteq} (G, B)$ implies $A \subseteq B$

But $(F,A)^* = (F^*, A)$ where $F^*(\alpha) = (F(\alpha))^*$ (from definition)
For any $x \in (F(\alpha))^*$ $\forall \alpha \in A$, we have $x \in (G(\alpha))^*$ since $A \subseteq B$ and $F(\alpha) \subseteq G(\alpha)$
Imply $(F(\alpha))^* \subseteq (G(\alpha))^* \forall \alpha \in B$
Thus $(F,A)^* \tilde{\subseteq} (G,B)^*$

Definition 3.15

Let (F, A), (G, B) $\in SMG(X)$. Then

i) (F, A) $\tilde{\vee}$ (G, B) = (H, A x B)

with $H(\alpha_1,\alpha_2) = F(\alpha_1,\alpha_2) \cup G(\alpha_1,\alpha_2)$ $\forall (\alpha_1,\alpha_2) \in A \times B$

ii) (F, A) $\tilde{\wedge}$ (G, B) = (K, A x B)

with $K(\alpha_1,\alpha_2) = F(\alpha_1,\alpha_2) \tilde{\cap} G(\alpha_1,\alpha_2)$ $\forall (\alpha_1,\alpha_2) \in A \times B$

Proposition 3.16 Let (F, A),(G,B) $\in SMG(X)$, then

(i) ((F, A) $\tilde{\vee}$ (G, B))*= $(F,A)^* \tilde{\vee}$ (G, B)*

(ii) ((F, A) $\tilde{\wedge}$ (G, B))*= $(F,A)^* \tilde{\wedge}$ (G, B)*

Proof:

(i) Let (F, A), (G,B) $\in SMG(X)$

(F, A) $\tilde{\vee}$ (G, B) = (H, A x B)

with $H(\alpha_1,\alpha_2) = F(\alpha_1,\alpha_2) \tilde{\cup} G(\alpha_1,\alpha_2)$ $\forall (\alpha_1,\alpha_2) \in A \times B$

$let\ x \in$ ((F, A) $\tilde{\vee}$ (G, B))*
$\Longrightarrow x \in (H(\alpha_1,\alpha_2))^* \forall (\alpha_1,\alpha_2) \in A \times B$
$\Longrightarrow x \in (F(\alpha_1,\alpha_2))^* \tilde{\cup} (G(\alpha_1,\alpha_2))^*$
i.e. $x \in (F(\alpha_1,\alpha_2))^* or\ x \in (G(\alpha_1,\alpha_2))^*$
$\Longrightarrow (F(\alpha_1,\alpha_2))^* \in (F,A)^*$ or $(G(\alpha_1,\alpha_2))^* \in (G,B)^* \forall (\alpha_1,\alpha_2) \in A \times B$
$\Longrightarrow (F(\alpha_1,\alpha_2))^* \tilde{\cup} (G(\alpha_1,\alpha_2))^* \in (F,A)^* \tilde{\cup} (G,B)^*$
i.e. $(F(\alpha_1,\alpha_2) \tilde{\cup} G(\alpha_1,\alpha_2))^* \in (F,A)^* \tilde{\cup} (G,B)^*$
that is, $(H(\alpha_1,\alpha_2))^* \in (F,A)^* \tilde{\vee} (G,B)^*$
((F, A) $\tilde{\vee}$ (G, B))$^* \subseteq (F,A)^* \tilde{\vee} (G,B)^*$………….. (1)
Now let $y \in (F,A)^* \tilde{\vee} (G,B)^*$
$\Longrightarrow y \in (F(\alpha_1,\alpha_2))^* \tilde{\cup} (G(\alpha_1,\alpha_2))^*$ $\forall (\alpha_1,\alpha_2) \in A \times B$
i.e. $y \in (F(\alpha_1,\alpha_2))^*$ or $y \in (G(\alpha_1,\alpha_2))^*$
since $(F(\alpha_1,\alpha_2))^* \in (F,A)^*$ and $(G(\alpha_1,\alpha_2))^* \in (G,B)^* \forall (\alpha_1,\alpha_2) \in A \times B$
$\Longrightarrow y \in ((F,A)\ \tilde{\cup}\ (G,B))^*$
That is, $(F,A)^* \tilde{\vee}$ (G, B)* $\subseteq$((F, A) $\tilde{\vee}$ (G, B))* …….. (2)
By (1) and (2) equality is established.

(ii) Let (F, A), (G,B) $\in SMG(X)$

(F, A) $\tilde{\wedge}$ (G, B) = (H, A x B)

where $H(\alpha_1,\alpha_2) = F(\alpha_1,\alpha_2) \tilde{\cap} G(\alpha_1,\alpha_2)$ $\forall (\alpha_1,\alpha_2) \in A \times B$
and $(H(\alpha_1,\alpha_2))^* \in$ ((F, A) $\tilde{\wedge}$ (G, B))*
for any $x \in (H(\alpha_1,\alpha_2))^* \forall (\alpha_1,\alpha_2) \in A \times B$
we have $x \in (F(\alpha_1,\alpha_2))^*$ and $x \in (G(\alpha_1,\alpha_2))^*$
$\Longrightarrow (F(\alpha_1,\alpha_2))^* \in (F,A)^*$ and $(G(\alpha_1,\alpha_2))^* \in (G,B)^* \forall (\alpha_1,\alpha_2) \in A \times B$
$\Longrightarrow (F(\alpha_1,\alpha_2))^* \tilde{\wedge} (G(\alpha_1,\alpha_2))^* \in (F,A)^* \tilde{\cap} (G,B)^*$
That is
((F, A) $\tilde{\wedge}$ (G, B))$^* \subseteq (F,A)^* \tilde{\cap} (G,B)^*$ ……….. (1)
Now let $y \in (F(\alpha_1,\alpha_2))^*$ and $y \in (G(\alpha_1,\alpha_2))^*$
$\Longrightarrow y \in ((F,A)\ \tilde{\cap}\ (G,B))^*$
That is, $(F,A)^* \tilde{\wedge}$ (G, B)* $\subseteq$((F, A) $\tilde{\vee}$ (G, B))* ….. (2)
By (1) and (2) equality is established.

Definition 3.17 Let (F, A) $\in SMG(X)$. Then raising to arithmetic power denoted by:
$(F,A)^n$ is defined:
$(F,A)^n = (F^n, A)$where F^n: A$\longrightarrow P(X^n)$
Given by
$F^n(\alpha) = (F(\alpha))^n$ (n = 0, 1, 2, ….)

Proposition 3.18 Let (F, A) $\in SMG(X)$, then $(F,A)^n \in SMG(X)$.

Proof:
Let (F, A) $\in SMG(X)$
For every $x \in X$ and $\alpha \in A, n \in \mathbb{N}$
$C_{F^n(\alpha)}(x) = C_{(F(\alpha))^n}$
$= (C_{F(\alpha)}(x))^n$
Since $C_{F(\alpha)}(x)\ \forall \alpha \in A = (F,A) \in SMG(X)$
$(F,A)^n \in SMG(X)$.

Proposition 3.19 Let (F, A), (G,B) $\in SMG(X)$, then for $n \in \{0, 1, 2,…\}$

i. ((F, A) $\tilde{\cup}$ (G, B))^n= $(F,A)^n \tilde{\cup}$ (G, B)^n
ii. ((F, A) $\tilde{\cap}$(G, B))^n= $(F,A)^n \tilde{\cap}$ (G, B)^n

Proof:

i. Let (F, A), (G,B) $\in SMG(X)$, and $n \in \{0, 1, 2, …\}$ for any $x \in X$
We have $C_{((F\tilde{\cup}G)^n(\alpha)}(x)$
$= C_{((F\tilde{\cup}G)(\alpha))^n}(x)$
$= (C_{((F\tilde{\cup}G)(\alpha)}(x))^n$ for $\forall\ x \in X$ and $\alpha \in A \cup B$ (by definition)
But $C_{((F\tilde{\cup}G)(\alpha))}(x)$ = max $\{C_{F(\alpha)}(x), C_{G(\alpha)}(x)\}$
Then $(C_{((F\tilde{\cup}G)(\alpha)}(x))^n$
= (max $\{C_{F(\alpha)}(x), C_{G(\alpha)}(x)\})^n$
= max $\{(C_{F(\alpha)}(x))^n, (C_{G(\alpha)}(x))^n\}$
= max $\{(C_{F^n(\alpha)}(x), C_{G^n(\alpha)}(x)\}$
$= (F,A)^n \tilde{\cup} (G,B)^n$
Thus ((F, A) $\tilde{\cup}$ (G, B))^n= $(F,A)^n \tilde{\cup}$ (G, B)^n

ii. Let (F, A), (G,B) $\in SMG(X)$, and $n \in \{0, 1, 2, …\}$

We have $C_{((F\tilde{\cap}G)^n(\alpha)}(x)$

$C_{((F\tilde{\cap}G)(\alpha))^n}(x) = (C_{((F\tilde{\cap}G)(\alpha)}(x))^n$

$\forall x \in X$ and $\alpha \in A \tilde{\cap} B$ (by definition.)

But $C_{((F\tilde{\cap}G)(\alpha))}(x)$ = min{$C_{F(\alpha)}(x)$, $C_{G(\alpha)}(x)$}

Then $(C_{((F\tilde{\cap}G)(\alpha)}(x))^n$

= (min $\{C_{F(\alpha)}(x), C_{G(\alpha)}(x)\})^n$

= min $\{(C_{F(\alpha)}(x))^n, (C_{G(\alpha)}(x))^n\}$

= min $\{(C_{(F(\alpha))^n}(x), (C_{(G(\alpha))^n}(x)\}$

= min $\{(C_{F^n(\alpha)}(x), (C_{G^n(\alpha)}(x)\}$

= $(F, A)^n \tilde{\cap} (G, B)^n$

Thus ((F, A) $\tilde{\cap}$(G, B)$)^n$ = $(F, A)^n \tilde{\cap}$ (G, B$)^n$

Proposition 3.20 Let (F, A) ∈ $SMG(X)$. Then for $n \in \{0, 1, 2, \ldots\}$

(F, A) $\tilde{\subseteq} (G, B) \Longrightarrow (F, A)^n \tilde{\subseteq} ((G, B)^n$.

Proof:

Let (F, A), $(G, B) \in SMG(X)$

Such that (F, A) $\tilde{\subseteq} (G, B)$

This imply for any $\alpha \in A \Longrightarrow \alpha \in B$ and F(α) $\leq G(\alpha)$

Now for any $x \in X$,

$C_{F^n(\alpha)}(x) = (C_{F(\alpha)}(x))^n$

and $C_{G^n(\alpha)}(x) = (C_{G(\alpha)}(x))^n$

Since F(α) $\leq G(\alpha)\ \forall\ \alpha \in A$,

we have $C_{F^n(\alpha)}(x) \leq C_{G^n(\alpha)}(x)$

i.e., $(C_{F(\alpha)}(x))^n$

$\leq (C_{G(\alpha)}(x))^n\ \forall\ x \in X \text{ and } \alpha \in B$.

hence $(F, A)^n \tilde{\subseteq} ((G, B)^n$.

Definition 3.21 Let $(F, A), (G, B) \in SMG(X)$.

Then an arithmetic multiplication denoted by $(F, A) \tilde{\odot} (G, B)$ is given by

(F, A) $\tilde{\odot}$ (G, B) = (H, C) where C = A∩B and $H(\alpha) = F(\alpha) \odot G(\alpha) \alpha \in C$ where $\odot$ is the mset arithmetic multiplication.

Proposition 3.22 Let $(F, A), (G, B) \in SMG(X)$. Then (F, A) $\tilde{\odot}$ $(G, B) \in SMG(X)$.

Proof:

Let $(F, A), (G, B) \in SMG(X)$.

$(F, A) \tilde{\odot} (G, B)$ = (H, A∩B) (from definition)

such that H$(\alpha) = F(\alpha) \odot G(\alpha)$

for any $\alpha \in$ A∩B

Clearly $F(\alpha)$ and $G(\alpha)$ are msubgroups of M(by definition)

And $F(\alpha) \odot G(\alpha)$ is a msubgroup of M

thus H(α) is a msubgroup of M.

in particular, (H, A∩ B) is a soft mgroup.

Thus (F, $A) \tilde{\odot} (G, B)$ is a soft mgroup.

Definition 3.23

Let $(F, A) \in SMG(X)$ and $k \in \mathbb{N}$ then the scalar multiplication denoted $k(F, A)$ is given by

$k(F, A)$ = (H, A) where H(α) = $kF(\alpha)$.

Proposition 3.24

Let $(F, A) \in SMG(X)$ then $k(F, A) \in SMG(X)$.

Proof:

Let $(F, A) \in SMG(X)$

We need to show that $k(F, A) \in SMG(X)$ for $k \in \mathbb{N}$

For any $\alpha \in A$ and $x \in X$ with k = {0, 1, 2, …}

We have $C_{kF(\alpha)}(x)$ = k. $C_{F(\alpha)}(x)$

k. $C_{F(\alpha)}(x) = kF(\alpha)$

= H(α) (but H(α) is a msubgroup of M)

Hence $k(F, A) \in SMG(X)$.

Definition 3.25

Comparable soft mgroups

Let (F_1, A_1), (F_2, A_2) ∈ SMG(X), then (F_1, A_1), (F_2, A_2) are said to be comparable if

$(F_1, A_1) \subseteq (F_2, A_2)$ or $(F_2, A_2) \subseteq (F_1, A_1)\ \forall x \in X$ and $\alpha \in E$.

Definition 3.26

Regular soft mgroup

A soft mgroup (F, A) is said to be regular if

$C_{F(\alpha)}(x) = C_{F(\alpha)}(y)\ \forall$ x, y ∈ X, $\alpha \in A$.

Proposition 3.27 Let (F, A), (G, B)∈ $SMG(X)$ such that (F, A), (G, B) are regular. Then;

(i) (F, A) $\tilde{\cup}$ (G, B)

(ii) (F, A) $\tilde{\cap}$ (G, B)

are regular.

Proof:

(i) Let (F, A), (G, B) ∈ $SMG(X)$ and (F, A), (G, B) be regular.

Now (F, A) $\tilde{\cup}$ (G, B) = (H, C) where H(α) = F(α) $\tilde{\cup}$ $G(\alpha)$ and C = $A \cup B$

For any $\alpha \in A \cup B$, clearly $F(\alpha) \text{ and } G(\alpha)$ are regular msubgroups (by definition)

thus $H(\alpha)$ is a regular msubgroup of M.

in particular, (H, A $\cup$ B) is a regular soft mgroup.

Thus $(F, A) \tilde{\cup} (G, B)$ is regular.

(ii) Let (F, A), (G, B) ∈ $SMG(X)$ and (F, A), (G, B) be regular.

Now (F, A)$\tilde{\cap}$(G, B) = (H, C) where

$H(\alpha)$ = F(α) $\tilde{\cap}$ $G(\alpha)$ and C = $A \cap B$

For any $\alpha \in A \cap B$, clearly $F(\alpha) \text{ and } G(\alpha)$ are regular msubgroups (by definition)

thus $H(\alpha)$ is a regular msubgroup of M.

in particular, (H, A∩ B) is a regular soft mgroup.

Thus $(F, A) \tilde{\cap} (G, B)$ is regular.

Proposition 3.28

Let (F, A) ∈ $SMG(X)$ such that (F, A) is regular. Then for $n \in \mathbb{N}$, and a scalar k,

(i) (F, A$)^n$

(ii) $k(F, A)$

are regular.

Proof:

(i) Let (F, A) $\in SMG(X)$ such that (F, A) is regular

For any $x \in X$ and $\alpha \in A, n \in \mathbb{N}$

$C_{F^n(\alpha)}(x)$

$= C_{(F(\alpha))^n}(x)$

$= (C_{F(\alpha)}(x))^n$

Since $\forall x \in X$ and $\alpha \in A$,

$C_{F(\alpha)}(x) = (F, A) \in SMG(X)$ is regular

$(C_{F(\alpha)}(x))^n = (F, A)^n \in SMG(X)$ is regular.

(i) Let (F, A) $\in SMG(X)$ such that (F, A) is regular

For any $x \in X$ and $\alpha \in A, k \in \mathbb{N}$

$C_{kF(\alpha)}(x) = k.C_{F(\alpha)}(x)$

But $C_{F(\alpha)}(x)$ = (F, A) is regular.

Thus k. $C_{F(\alpha)}(x) = k$(F, A) is regular.

Example 3.29

Let X = {e, a, b, c} be the Klein's 4-group and M = [e, a, b, c$]_{2,2,2,2}$ be a mgroup over X. Suppose (F, A) is a soft mgroup, with E= $\{\alpha_1, \alpha_2, \alpha_3\}$ a set of parameters such that $A = \{\alpha_1, \alpha_2\}$ with F(α_1) = [e, a, b, c$]_{1,1,1,1,1}$ and F(α_2) = [e, a, b, c$]_{1,1,1,1,1}$. Then (F, A) is a regular soft mgroup.

Definition 3.30 Count of an element in a soft mgroup

Let $SMG(X)$ be the set of all soft mgroups of a group X. The count of an element in

(F,A) $\in SMG(X)$ is the number of occurrences of its object. The order of (F, A) is the sum of the count of each of its elements and is given by:

|(F, A)| = $\sum C_{F(\alpha)}(x) \forall x \in X$ and $\forall \alpha \in A$.

Definition 3. 31

Absolute soft mgroup

Let (F, A) be a soft mgroup over M, the soft mgroup (F, A) is called absolute over M if

F(α) = M $\forall \alpha \in A$.

Definition 3.32

Commutator in a soft mgroup

Let (F_1, A_1),$(F_2, A_2) \in$ SMG(X), the commutator soft mgroup of (F_1, A_1) and (F_2, A_2) is the soft mgroup generated by the commutator

$[(F_1, A_1), (F_2, A_2)] = \{F_1(\alpha), F_2(\alpha) \forall \alpha \in E\}$

$= [x, y] \forall x \in F_1(\alpha)$ and $y \in F_2(\alpha) \forall \alpha \in E\}$ with $[x, y] = x^{-1}y^{-1}xy$.

Proposition 3.33 Suppose $(F, A) \in SMG(X)$ then $C_{F(\alpha)}([x,y]) = C_{F(\alpha)}(e)$ iff (F, A) is Abelian.

Proof:

Assume $C_{F(\alpha)}([x,y]) = C_{F(\alpha)}(e)$ where e is the identity element in (F, A)

Then $C_{F(\alpha)}(x^{-1}y^{-1}xy) = C_{F(\alpha)}(e) \forall \alpha \in A$

$\Rightarrow C_{F(\alpha)}((yx)^{-1}xy) = C_{F(\alpha)}(e) \forall \alpha \in A$

$\Rightarrow C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx) \forall \alpha \in A$

Therefore, (F, A) is abelian.

Conversely, let (F, A) be abelian

$\Rightarrow \forall x, y \in X, \alpha \in A$

we have $C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx)$

$\Rightarrow C_{F(\alpha)}(y^{-1}xy) = C_{F(\alpha)}(y^{-1}yx)$

$\Rightarrow C_{F(\alpha)}(y^{-1}xy) = C_{F(\alpha)}(x)$

$\Rightarrow C_{F(\alpha)}(x^{-1}y^{-1}xy) = C_{F(\alpha)}(x^{-1}x)$

$\Rightarrow C_{F(\alpha)}(x^{-1}y^{-1}xy) = C_{F(\alpha)}(e)$

$\Rightarrow C_{F(\alpha)}([x,y]) = C_{F(\alpha)}(e)$

Proposition 3.34 Let$(F, A) \in SMG(X)$ be abelian. Then $C_{F(\alpha)}[y,x]^{-1}) = C_{F(\alpha)}[y,x]$ $\forall x, y \in X, \alpha \in A$

Proof.

Suppose (F, A) is abelian, then we get

$C_{F(\alpha)}([x,y]^{-1}) \geq C_{F(\alpha)}([x,y])$

$= C_{F(\alpha)}(x^{-1}y^{-1}xy)$

$= C_{F(\alpha)}(y^{-1}x^{-1}yx)$

$= C_{F(\alpha)}([y,x])] \forall x, y \in X$ and $\alpha \in A$

$\Longrightarrow C_{F(\alpha)}([x,y]^{-1}) \geq C_{F(\alpha)}([y,x])$ …………….. (1)

Similarly, $C_{F(\alpha)}([y,x]) = C_{F(\alpha)}([y,x]^{-1})^{-1}$

$\geq C_{F(\alpha)}([y,x]^{-1}$

$= C_{F(\alpha)}((y^{-1}x^{-1}yx)^{-1})$

$= C_{F(\alpha)}((x^{-1}y^{-1}xy)^{-1})$

$= C_{F(\alpha)}([x,y]^{-1}$

$\Longrightarrow C_{F(\alpha)}([y,x]) \geq C_{F(\alpha)}([x,y]^{-1})$ …………….. (2)

(1) And (2) implies

$C_{F(\alpha)}([y,x]) = C_{F(\alpha)}([x,y]^{-1})$.

Definition 3.35

Normal soft mgroup

Let (F_1, A_1) be a soft mgroup over M, and (F_2, A_2) be a soft submgroup of (F_1, A_1), then we say that (F_2, A_2) is a normal soft submgroup of (F_1, A_1) written as

$(F_2, A_2) \tilde{\trianglelefteq} (F_1, A_1)$.

If $C_{F_2(\alpha)}(xyx^{-1})$

$\geq C_{F_2(\alpha)}(y) \forall x, y \in X$ and $\alpha \in E$.

Proposition 3.36 Let (F_1, A_1) and (F_2, A_2) be soft mgroups such that (F_1, A_1) is a soft submgroup of (F_2, A_2). Then (F_1, A_1) is a normal soft submgroup of (F_2, A_2) iff

(i) $C_{F_1(\alpha)}([x,y]) \geq C_{F_1(\alpha)}(x) \forall x, y \in X$ and $\alpha \in A_1 \cap A_2$.

(ii) $C_{F_1(\alpha)}([x,y]) = C_{F_1(\alpha)}(e) \forall x, y \in X$ and $\alpha \in A_1 \cap A_2$, where e is the identity in X

Proof

(i) ($\Rightarrow$)Suppose (F_1, A_1) is a normal submgroup of (F_2, A_2).

Let $x, y \in X$ and $\alpha \in A_1$, then

$C_{F_1(\alpha)}(x^{-1}y^{-1}xy) = C_{F_1(\alpha)}(x^{-1})(y^{-1}xy)$
$\geq C_{F_1(\alpha)}(x^{-1}) \wedge C_{F_1(\alpha)}(y^{-1}xy)$
$= C_{F_1(\alpha)}(x) \wedge C_{F_1(\alpha)}(x) = C_{F_1(\alpha)}(x)$
Hence $C_{F_1(\alpha)}(x^{-1}y^{-1}xy) \geq C_{F_1(\alpha)}(x)$
That is $C_{F_1(\alpha)}([x,y]) \geq C_{F_1(\alpha)}(x)$ ………………. (1)
(⇐) Suppose $C_{F_1(\alpha)}(x^{-1}y^{-1}xy) \geq C_{F_1(\alpha)}(x)$
then $\forall x, y \in X$ $and\ \alpha \in A_1$, we have $C_{F_1(\alpha)}(x^{-1}yx)$
$= C_{F_1(\alpha)}(yy^{-1}x^{-1}yx)$
$= C_{F_1(\alpha)}((y)y^{-1}x^{-1}yx)$
$\geq C_{F_1(\alpha)}(y) \wedge C_{F_1(\alpha)}(y^{-1}x^{-1}yx)$
$= C_{F_1(\alpha)}(y) \wedge C_{F_1(\alpha)}[(y,x]) = C_{F_1(\alpha)}(y)$
Thus $C_{F_1(\alpha)}(x^{-1}yx) \geq C_{F_1(\alpha)}(y)$
$\forall x, y \in X$ $and\ \alpha \in A_1$
Hence (F_1, A_1) is a normal soft submgroup of (F_2, A_2).

(ii) Suppose $x, y \in X$, $\alpha \in A_1$, and (F_1, A_1) is a normal soft submgroup of (F_2, A_2) .

Now we know that (F_1, A_1) is a normal soft submgroup of (F_2, A_2)
$\Leftrightarrow C_{F_1(\alpha)}(xy) = C_{F_1(\alpha)}(yx)\ \forall\ x, y \in X, \alpha \in A_1$
$\Leftrightarrow C_{F_1(\alpha)}(x^{-1}y^{-1}x) = C_{F_1(\alpha)}(y^{-1})$
$\forall\ x, y \in X, \alpha \in A_1$
$\Leftrightarrow C_{F_1(\alpha)}(x^{-1}y^{-1}xyy^{-1}) = C_{F_1(\alpha)}(y^{-1})$
$\forall\ x, y \in X, \alpha \in A_1$
$\Leftrightarrow C_{F_1(\alpha)}([x,y]y^{-1}) = C_{F_1(\alpha)}(y^{-1})$
$\forall\ x, y \in X, \alpha \in A_1$
Thus $C_{F_1(\alpha)}([x,y]) = C_{F_1(\alpha)}(e)$
$\forall\ x, y \in X, \alpha \in A_1$ where e is the identity in X.

Definition 3.37
Conjugate element in a soft mgroup and Conjugate soft mgroup
Let $(F, A) \in$ SMG(X) and $x, y \in X$, then x and y are called conjugate elements in (F, A) if for some $z \in X$ and $\alpha \in E$, $C_{F(\alpha)}(x) = C_{F(\alpha)}(zyz^{-1})$.
Two soft mgroups (F_1, A_1) ,(F_2, A_2) $\in$ SMG(X) are conjugate to each other if
$\forall\ x, y \in X$ and $\alpha \in E$,
$C_{F_1(\alpha)}(y) = C_{F_2(\alpha)}(xyx^{-1})$
and $(C_{F_1(\alpha)}(x) = C_{F_2(\alpha)}(yxy^{-1})$.

Definition 3.38
Soft Abelian mgroup [8]
A soft mgroup (F, A) over a mgroup M of a group X is called soft abelian mgroup if F(α)is an abelian submgroup of M, i.e. $C_{F(\alpha)}(xy) = C_{F(\alpha)}(yx) \forall\ x, y \in X$ *and* $\alpha \in A$

Remark 3.39 Let (F_1, A_1) be a normal soft mgroup over M. Then (F_1, A_1) is an abelian soft mgroup.

Proposition 3.40 Every soft submgroup (F_2, A_2) of an abelian soft mgroup (F_1, A_1) over M is normal.
Proof
Since (F_1, A_1)is soft abelian mgroup, F_1(α) for all α $\in A_1$ is an abelian submgroup of M and each F_2 (α) is a subgroup of F_1(α) for all α $\in A_1$.
It is well known that a subgroup of an abelian group is a normal subgroup which can be extended since every mgroup is a group, even though the converse is not true. This implies that (F_2, A_2) is a normal soft mgroup.

Definition 3.41
Soft mgroupoid
Definition: Let X be a group and M a mgroup over X. Let A$\subseteq E$ be a set of parameters. A soft mset (F, A) over M is called a soft mgroupoid over M iff ∀ $x, y \in$ M,
$C_{F(\alpha)}(xy) \geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)$ $\forall \alpha \in$ A.
Proposition 3.42
A soft mgroupoid (F, A) over a mgroup M, is a soft mgroup iff $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x)$
$\forall x, y \in$ M and α $\in A$.
Proof:
(⇒)
Assume (F, A) is a soft mgroup, it follows that, $C_{F(\alpha)}(xy)$
$\geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)$
and $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x)$) $\forall x, y \in$ X and α $\in A$.
(⇐) Suppose $C_{F(\alpha)}(x^{-1}) = C_{F(\alpha)}(x)$ $\forall x, y \in$ X and α $\in A$
It remains to show that $C_{F(\alpha)}(xy) \geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)$
Now $C_{F(\alpha)}(xy) = C_{F(\alpha)}[x(y^{-1})^{-1}]$
$\geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}((y^{-1})^{-1})$
$= C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y^{-1})$
$= C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)$
Hence $C_{F(\alpha)}(xy) \geq C_{F(\alpha)}(x) \wedge C_{F(\alpha)}(y)$
Therefore, (F, A) is a soft mgroup over M.

Definition 3.43
Soft semimgroup
A soft mset (F, A) over M is called a soft semimgroup over M iff $C_{F(\alpha)}(xyz) = C_{F(\alpha)}(yxz)$ ∀ x, y, z ∈ M and $\alpha \in A$.

Definition 3.44
Soft multimonoid (mmonoid)
A soft mset (F, A) over M is a soft multimonoid (mmonoid) over M, if it is a soft semimgroup of M and it satisfies $C_{F(\alpha)}(e) \geq C_{F(\alpha)}(x)$, ∀x ∈ M, and $\alpha \in A$ where e is the identity element in M.

4. Conclusion and Future Scope

In this paper, we have initiated some new results on the structures and Operations on Soft Multigroups. Some of the algebraic properties of soft mgroups were explicated. To extend the study of soft mgroups which was initiated by Nazmul and Samanta [8], we have introduced novel concepts such as commutators in soft mgroups, soft m-monoid, soft semi mgroup, soft mgroupoid, conjugate soft mgroups, and others and exemplified some important properties and operations in soft mgroups. We obtained many results, established some relationships, and extended some properties in soft mgroup theory. The foundations which we have established here can be used to gain insight into the higher-order structures of group theory. One can also further the study on soft mgroups by taking on: The cosets in soft mgroups, Product and the restricted product of soft mgroups, or considering several hybrid structures like Soft fuzzy mgroups and soft vague mgroups among others.

The theory of soft mgroups can be very useful in many areas such as structural Analysis, risk management, decision making, and cryptography.

Data Availability
Available on request.

Conflict of Interest
The authors declare that there is no conflict of interests regarding the publication of this paper

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Authors' Contributions
Author -1 researched the literature, conceived the study, developed most of the propositions, and wrote the future scope. Author-2 was the lead supervisor who made corrections and put forward suggestions in most of the proofs, and the structure of the literature. Author-3 suggested the propositions and supervised the proofs on the algebraic properties of soft mgroups, he also suggested the conclusion. All authors reviewed and approved the final version of the manuscript.

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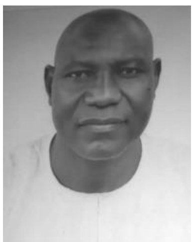

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