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A Generalized Form of G^*-Open Sets

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Abstract— Generalization of sets in topological space is a very important concept in modern mathematics for structural oriented techniques. To enrich the structure and properties of generalized open sets, we define the notion of p^*-open sets by using topological space and generalized topological space both on the set X. The main aim of this paper is to define p^*-open sets and its generalized sets p^*-I-open sets, I_{p^*}-open sets and weakly I_{p^*}-open sets in a space (both topological space and generalized topological space) via ideal with illustrative examples. Also, we study the significant properties of newly defined open sets in the set X.

Keywords—Generalized topology, ideal topological spaces, $\boldsymbol{G}$-open sets, $\boldsymbol{p}^*$-open sets, $\boldsymbol{I_{p^*}}$-open sets.

I. Introduction

The notions of generalized form of open sets, such as semi-open sets [1], pre-open sets [2], α-open sets [3] and β-open sets [4] have been studied by several mathematician. The concept of generalized topology was introduced by Csaszar[5] in 2002 and he studied generalized form of open sets such as semi-open set, pre open set, α -open set and β -open set ([6], [7]) in generalized topological spaces. Significantly, author has been defined new form of basic operations on generalized topological space.

In accordance with the review of the field, it has been noticed that the large number of papers are based on the study of generalized open sets on topological space and developed the classes of these open sets with basic propeties more or less similar to previously defined open sets ([5], 6], [8]).

One of the famous and important branch of research in pure mathematics is ideal on topological space and this field of research has been studied by Kuratowski [9] and Vaidyanathaswamy [10]. After this study, many researchers applied extended this concept in different topological spaces ([10], [11],[12]).

Aim of this paper is that to use this concept of ideals in the generalized topological space. Roy and Sen [6] have considered two structures viz., topological structure and generalized topological structures on set X and defined a new concept called G^*-open set. We have also cosidered the above two structure on set X and defined the concept of p^*-open set. Further by taking an ideal I on X. We have introduced p^*-I-open sets, I_{p^*}-open sets and weakly I_{p^*}-open sets and studied them significantly.

The organization of the paper is as follows:
The paper opens with the introduction of the field. In section II mentions **Preliminaries**. Section III defines **$\mathbf{p}^*$-open sets** and section IV proposes **$\mathbf{p}^*$ - $\boldsymbol{I}$ -open sets**. In section V introduces **$\boldsymbol{I_{p^*}}$ -open sets** and section VI discusses **weakly $\mathbf{I_{p^*}}$-open sets**. The last section concludes the paper.

II. Priliminaries

In this section, we so through the basic definitions and results are required in studying the defined the concepts. We use notations G , (X,τ) and (X,τ,I) for generalized topology, topological space and ideal topological space respectively on the set X throughout this paper.

Definition 2.1: Let X be a non-empty set and let G be a family of subsets of X, then G is said to be a **generalized topology [5]** on X, if following two conditions are satisfied viz.

(i) $\phi \in \mathrm{G}$;

(ii) Arbitrary union of members of G is a member of G.

The pair (X,G) is called a **generalized topological space** [5]. The members of G are called G**-open sets** [5] and their complements are called G**-closed sets** [5]. The generalized topology G is said to be **strong** [5] if $\mathrm{X} \in \mathrm{G}$.

Definition 2.2: Let X be a non-empty set and I be a family of subsets of X, then I is said to be an **Ideal**[9] on X if it satisfies following two conditions viz,;

(i) If $A \in I$ and $B \subseteq A$ then $B \in I$. (Hereditary Property)

(ii) If $A, B \in I$ then $A \cup B \in I$. (Finite Additivity)

A topological spaces (X, τ) with ideal I is said to be **Ideal Topological Spaces** [9] and denoted by (X, τ, I).

Definition 2.3: Let G be a generalized topology on topological space X and A be a subset of X [5]. Then G – **interior** [5]of A is denoted by $i_G(A)$ and is defined to be the union of all G -open sets contained in A. The **G-closure** [5] of A is denoted by $c_G(A)$ and is defined to be the intersection of all G -closed sets containing A.

Remark 2.1[5]**:** Since arbitrary union of G-open sets is a G-open set and arbitrary intersection of G-closed sets is a G-closed set, it follows that $i_G(A)$is a G-open set and $c_G(A)$ is a G-closed set. Thus $i_G(A)$is the largest G-open set contained in A and $c_G(A)$ is the smallest G-closed set containing A.

Theorem 2.3 [5]: Let (X, G) be a generalized topological space and $\{A_\alpha\}_{\alpha\in\Lambda}$ be a family of subsets of X, then

(i) $\bigcup_{\alpha\in\Lambda} i_G(A_\alpha) \subseteq i_G(\bigcup_{\alpha\in\Lambda} A_\alpha)$,

(ii) $i_G(\bigcap_{\alpha\in\Lambda} A_\alpha) \subseteq \bigcap_{\alpha\in\Lambda} i_G(A_\alpha)$.

Theorem 2.4 [9]**:** Let (X, τ) be a topological space and $\{A_\alpha\}_{\alpha\in\Lambda}$be a family of subsets of X, then

(i) $\bigcup_{\alpha\in\Lambda} cl(A_\alpha) = cl(\bigcup_{\alpha\in\Lambda} A_\alpha)$,

(ii) $cl(\bigcap_{\alpha\in\Lambda} A_\alpha) = \bigcap_{\alpha\in\Lambda} cl(A_\alpha)$.

Definition 2.4 [13]**:** Let (X, τ, I) be an ideal topological space and $A \subset X$, then the set $A^\star = \{x \in X : A \cap U \notin I$ for each neighborhood U of $x\}$ is called the local function of A with respect to ideal I and topologyτ**.**

Theorem 2.5 [13]**:** Let (X, τ, I) be an Ideal topological space and $A \subset X$, then the map $Cl^\star: \wp(X) \to \wp(X)$, defined by $Cl^\star(A) = A \cup A^\star$, $A^\star$ denotes local function of A. Then $cl^*$is called kuratowski closure operator.

Definition 2.5: Let X be a non-empty set. Then a Kuratowski closure operator [13]$cl^*: \wp(X) \to \wp(X)$ satisfies following conditions:

(i) $cl^\star(\emptyset) = \emptyset$

(ii) $A \subset cl^\star(A), A \in \wp(X)$

(iii) $cl^\star(A \cup B) = cl^\star(A) \cup cl^\star(B),\ A, B \in \wp(X)$

(iv) $cl^\star(cl^\star(A)) = cl^\star(A)$.

Theorem 2.6 [13]**:** Let (X, τ, I) be ideal topological space and $\{A_\alpha\}_{\alpha\in\Lambda}$be a family of subsets of X, then

(i) $\bigcup_{\alpha\in\Lambda} cl^\star(A_\alpha) = cl^\star(\bigcup_{\alpha\in\Lambda} A_\alpha)$

(ii) $cl^\star(\bigcap_{\alpha\in\Lambda} A_\alpha) \subseteq \bigcap_{\alpha\in\Lambda} cl^\star(A_\alpha)$.

Definition 2.6: let G be a generalized topology on a topological space (X, τ), then a subset A of X is called $\mathbf{G^*}$**-open set** [14]if $A \subseteq cl(i_G(A))$.

Definition 2.5: Let G be a generalized topology on an ideal topological space (X, τ, I), then a subset A of X is called $\mathbf{I_G}$**-open set** [8] if there exists a G-open set U such that $U\setminus A \in I$ and $A\setminus cl(U) \in I$.

Definition 2.6: Let G be a generalized topology on an ideal topological space (X, τ, I), thena subset A of X is called **weakly I_G-open set** [8], if $A = \phi$ or if $A \neq \phi$, then there is a non-empty G-open set U such that $U\setminus A \in I$. The complement of a weakly I_G-open set is called weakly I_G-closed set [8].

III. p^*-OPEN SETS AND PROPERTIES OF p^*-OPEN SETS

Definition 3.1: Let A be a subset of X in generalized topology G on (X, τ). Then set A is a $\boldsymbol{p^*}$**-open** set in X, if $A \subset i_G(cl(A))$. The set $X\setminus A$ is $\boldsymbol{p^*}$**-closed set**.

Theorem 3.1: Every G-open set is a p^*-open set in generalized topology G on (X, τ).

Proof: Let A be any arbitrary G-open set in generalized topology G on (X, τ). Since A is G-open set, then $A = i_G(A)$. It is well known that $A \subseteq cl(A)$, by using this fact on set A, we have $A = i_G(A) \subseteq i_G(cl(A))$. It implise that A is a p^*-open set in X. Therefore, every G-open set is a p^*-open set.

Theorem 3.2: Every open set in generalized topology Gon $(X, \tau)$is p^*-open set provided G is finer than τ.

Proof: Let A be any arbitrary open set in generalized topology G on (X, τ).It is given that $G \supseteq \tau$, this implies that every open set is G-open.Since every G-open set is a p^*-open (cf. Theorem 3.1), then A is a p^*-open. Hence every open set is p^*-open in generalized topology Gon (X, τ).

Remark 3.1: p^*-open set is neither open nor G-open set in generalized topology G on (X, τ). For example, let us consider $X = \{b_1, b_2, b_3, b_4\}$ and topology $\tau = \{\phi, \{b_1\}, \{b_1, b_2\}, \{b_1, b_3\}, \{b_1, b_2, b_3\}, X\}$ with generalized topology $G = \{\phi, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, X\}$ on X. If we consider $A = \{b_1, b_4\}$, then clearly set A is a p^*-open set in generalized topology G on (X, τ) but it is neither open nor G-open set.

Proposition 3.1: Let A be a non-empty p^*-open set in generalized topology G on (X, τ), then $i_G(cl(A))$ is non-empty.

Proof: Obviously.

Theorem 3.3: Let A be a non-empty set in generalized topology G on (X,τ). Then A is p^*-open iff $\exists$ a G-open set W in X such that $cl(A) \supset W \supset A$.

Proof: Let A be a p^*-open set in G on (X,τ) .Then $A \subset i_G(cl(A)) = W$ (say). This shows that $A \subset W$, W is G-open set in X. Since $W = i_G\big(cl(A)\big) \subset cl(A)$, then we obtain $U \subset cl(A)$. Finally, we conclude that there must exist a G-open set W such that $cl(A) \supset W \supset A$.

Conversely, suppose $A \subset X$ and W is a G-open set in X such that $cl(A) \supset W \supset A$. Now, it is given that $cl(A) \supset W$, this implies that $i_G\big(cl(A)\big) \supset i_G(W) = W$, hence $i_G\big(cl(A)\big) \supset W$. Also $A \subset W$, then we get $A \subset i_G\big(cl(A)\big)$. Thus, A is a p^*-open set in G on (X,τ).

Theorem 3.4: Any arbitrary union of p^*-open sets in generalized topology G on (X,τ) is a p^*-open set.

Proof: Suppose $\{A_k\}_{k\in J}$ is a family of p^*-open sets in generalized topology Gon (X,τ) where J is an index set. By the definition of p^*-open sets, $A_k \subset i_G\big(\mathrm{cl}\,(A_k)\big), \forall\, k \in J$. Let us suppose that $\cup_{k\in J} A_k = A$. Then we have, $i_G\big(cl(A)\big) = i_G\Big(cl\big(\cup_{k\in J} A_k\big)\Big) \supset i_G\Big(\cup_{k\in J} cl(A_k)\Big) \supset \cup_{k\in J} i_G\big(cl(A_k)\big) \supset \cup_{k\in J} A_k = A$.This concludes that $A \subset i_G(Cl(A))$. Hence, A is a p^*-open set in generalized topology G on (X,τ).

Remark 3.2: The intersection of two p^*-open sets may or may not be p^*-open. For example, let us suppose that $X = \{b_1, b_2, b_3, b_4\}$ and topology $\tau = \{\phi, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, X\}$ and generalized topology $G = \{\phi, \{b_1, b_3\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, \{b_2, b_3, b_4\}, X\}$ on X. Take $A = \{a, c, d\}$ and $B = \{b, c, d\}$. We can easily verify that A and B both are p*-open set in X but $A \cap B = \{c, d\}$ is not a p^*-open set in G on (X,τ).

Proposition 3.1: The collection of p^*-open sets in G on (X,τ) forms a generalized topology on X.

Proof: Obviously.

Theorem 3.5: Let A be a p^*-open set in G on (X,τ) with property that $B \subseteq A \subseteq cl(B)$. Then B is a p^*-open set.

Proof: Suppose A is any arbitrary p^*-open set on X, then there exists a G-open set W such that $cl(A) \supset W \supset A$, then $B \subseteq W$ and also $cl(A) \subseteq cl(B)$.It implies that $W \subseteq cl(B)$. Hence $B \subseteq W \subseteq cl(B)$. By using Theorem 3.3, B is p^*-open set in G on (X,τ).

Proposition 3.3: Any subset A of X is a p*-open in generalized topology G on (X,τ) if $\mathrm{cl}(A) = i_G\big(\mathrm{cl}(A)\big)$.

Proof: Let A be any arbitrary set in G on (X,τ). We have $\mathrm{cl}(A) = i_G\big(\mathrm{cl}(A)\big)$. Since $A \subseteq \mathrm{cl}(A)$, then $A \subseteq i_G\big(\mathrm{cl}(A)\big)$. By using Definition 3.1, we can conclude that A is p*-open set.

Theorem 3.8: Any arbitrary subset $A$of X is a p*-open set in generalized topology G on (X,τ) if and only if for every $x \in A$, then there is a p*-open set W in X such that $x \in W \subseteq A$.

Proof: Suppose A isany arbitrary p*-open set in generalized topology G on (X,τ). For any arbitrary $x \in A$ there exist a p*-open set in X. Since A itself a p^*-open set then condition satisfies trivially for $x \in A$. It is clear that $x \in A$ is an arbitrary element then this condition is true for every $x \in A$.

Conversely, suppose $A \subseteq X$ having the property that for each $x \in A$ there exists a p*-open set W_x in X such that $x \in W_x \subseteq A$. Then, clearly we have $A = \cup_{x\in A} W_x$. Since, arbitary union of p*-open set is p*-open (see theorem 3.4), therefore A is a p*-open set in X.

Remark 3.3: The concept of G^*-open sets and p^*-open sets are independent to each other in generalized topology G on (X,τ). The counter examples is given as follows:

Case I. A subset A of X is G*-open set but not p^*-open set.

Suppose $X = \{b_1, b_2, b_3, b_4\}$ and let us consider topology $\tau = \{\phi, \{b_4\}, X\}$ and generalized topology $G = \{\phi, \{b_2\}, \{b_2, b_3\}, X\}$ on X. If we consider $A = \{b_1, b_2\}$, then clearly A is a G*-open set in X but not a p*-open set.

Case II. A subset A of X is neither G^*-open set nor p^*-open set.

Suppose $X = \{b_1, b_2, b_3, b_4\}$ and topology $\tau = \{\phi, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, X\}$, generalized topology

$G = \{\phi, \{b_1, b_4\}, \{b_3, b_4\}, \{b_1, b_3, b_4\}, X\}$

on X. If we consider $A = \{b_1, b_2\}$, then A is neither G*-open set nor p^*-open set

Case III. A subset A of X is G^*-open set but not p^*-open set.

Suppose $X = \{b_1, b_2, b_3, b_4\}$ and topology $\tau = \{\phi, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, X\}$, generalized topology $G = \{\phi, \{b_1, b_4\}, \{b_3, b_4\}, \{b_1, b_3, b_4\}, X\}$ on X. Consider $A = \{b_2, b_4\}$, Then clearly A is a p^*-open set in X but not G*-open set.

With reference to above stated examples, it is clear that the concept of p^*-open sets and G*-open sets are independent to each other.

IV. p*-*I*-OPEN SETS AND THEIR PROPERTIES

Definition 4.1: Let A be a subset of X in generalized topology G on (X,τ,I) is a ***p**-I-open** set, if $A \subset i_G(cl^\star(A))$. The set $X \backslash A$ is a ***p**-I-closed** set.

Theorem 4.1: Every G-open set is a p*-I-open set in generalized topology G on (X,τ,I).

Proof: Suppose A is any arbitrary G-open set in X. By using definition of G-open set, $A = i_G(A)$ and we know that

$A \subset cl^{\star}(A)$, then we have $i_G(A) \subset i_G(Cl^{\star}(A))$. Hence $A \subset i_G(cl^{\star}(A))$. This shows that A is a p^*-I-open set in G on (X, τ, I).

Theorem 4.2: Every open set in generalized topology G on $(\mathrm{X}, \tau, \mathrm{I})$is p*-I-open set provided G is finer than τ.

Proof: Let A be any arbitrary open set in generalized topology G on (X, τ, I). It is given that $\mathrm{G} \supseteq \tau$, this implies that every open set is G-open. Since every G-open set is a p*-I-open (cf. Theorem 4.1), then A is a p*-I-open. Hence every open set is p*-I-open in generalized topology Gon $(\mathrm{X}, \tau, \mathrm{I})$.

Remark 4.1: A p*-I-open set is neither open nor G-open set in generalized topology G on $(\mathrm{X}, \tau, \mathrm{I})$. For example, Let us consider $\mathrm{X} = \{ \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4 \}$ and topology $\tau = \{\phi, \{\mathrm{b}_1\}, \{\mathrm{b}_1, \mathrm{b}_2\}, \{\mathrm{b}_1, \mathrm{b}_4\}, \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_4\}, \mathrm{X}\}$ with generalized topology $\mathrm{G} = \{\phi, \{\mathrm{b}_2, \mathrm{b}_3\}, \{\mathrm{b}_3, \mathrm{b}_4\}, \{\mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4\}, \mathrm{X}\}$ on X. If we consider $\mathrm{A} = \{\mathrm{b}_2\}$, then clearly set A is a p^*-I-open set in G on (X, τ) but it is neither open nor G-open set.

Theorem 4.3: Let A be a non-empty set in generalized topology G on (X, τ, I). Then A is p^*-I-open iff $\exists$ a G-open set W in X such that $cl^{\star}(A) \supset W \supset A$.

Proof: Let A be a p^*-I-open set in G on (X, τ, I) .Then $A \subset i_G(cl^{\star}(A)) = W$ (say). This shows that $A \subset W$,W is G-open set in X. Since $W = i_G(cl^{\star}(A)) \subset cl^{\star}(A)$, then we obtain $W \subset cl^{\star}(A)$ .Finally, we conclude that there must exist a G-open set W such that $cl^{\star}(A) \supset W \supset A$.

Conversely, suppose $A \subset X$ and W is a G-open set in X such that $cl^{\star}(A) \supset W \supset A$. Now, it is given that $cl^{\star}(A) \supset W$, this implies that $i_G(cl^{\star}(A)) \supset i_G(W) = W$, hence $i_G(cl^{\star}(A)) \supset W$. Also $A \subset W$, then we get $A \subset i_G(cl^{\star}(A))$. Thus, A is a p^*-I-open set in G on (X, τ, I).

Proposition 4.1: Let A be a non-empty p^*-I-open set in generalized topology G on (X, τ, I), then $i_G(Cl^{\star}(A))$ is non-empty.

Proof: Obviously.

Theorem 4.4: An arbitrary union of p^*-I-open sets is p^*-I-open in generalized topology on ideal topological space.

Proof: Suppose $\{A_\beta\}_{\beta \in J}$ is a family of p^*-I-open sets in X, where $\beta$is an index set. Then by using definition of p^*-I-open set, $A_\beta \subseteq i_G(Cl^{\star}(A_\beta))$, $\forall\ \beta \in J$. Let us suppose that $\cup_{\beta \in J} A_\beta = A$ (say) is a arbitrary union of A_β's. Then

$$i_G(cl^{\star}(A)) = i_G\left(cl^{\star}\left(\cup_{\beta \in J} A_\beta\right)\right) \supset i_G\left(\cup_{\beta \in J} cl^{\star}(A_\beta)\right) \supset \cup_{\beta \in J} i_G\left(cl^{\star}(A_\beta)\right) \supset \cup_{\beta \in J} A_\beta = A.$$

This concludes that $\mathrm{A} \subset i_G(cl^{\star}(A))$. Hence A is p^*-I-open set in G on (X, G).

Remark 4.1: Intersection of two p^*-I-open sets may or may not be p^*-I-open. For example, let us consider $\mathrm{X} = \{ \mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4\}$ and consider a topology $\tau = \{\phi, \{\mathrm{b}_1\}, \{\mathrm{b}_1, \mathrm{b}_2\}, \{\mathrm{b}_1, \mathrm{b}_3\}, \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3\}, \mathrm{X}\}$ along with generalized topology $\mathrm{G} = \{\phi, \{\mathrm{b}_1, \mathrm{b}_2\}, \{\mathrm{b}_1, \mathrm{b}_3\}, \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3\}, \{\mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4\}, \mathrm{X}\}$ and ideal $I = \{\phi, \{b_2\}, \{b_3\}, \{b_2, b_3\}\}$ on X. Suppose $\mathrm{A} = \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3\}$ and $B = \{b_2, b_3, b_4\}$,then we can easily verify that A and B both are p*-I-open set in X but $A \cap B = \{b_2, b_3\}$ is not a p^*-I-open set in G on $(\mathrm{X}, \tau, \mathrm{I})$.

Proposition 4.2: The collection of all p^*-I-open sets in generalized topology G on $(\mathrm{X}, \tau, \mathrm{I})$ forms a generalized topology on X.

Theorem 4.5: Let A be any p^*-I-open set in generalized topology G on (X, τ, I) with properties that $B \subseteq A \subseteq cl^{\star}(B)$. Then B is p^*-I-open set.

Proof: Suppose A is a p^*-I-open set in X, then there exists a G-open set W such that $A \subseteq W \subseteq cl^{\star}(A)$. Since, it is given that $B \subseteq A$, this implies that$B \subseteq W$. Also, $cl^{\star}(W) \subseteq cl^{\star}(B)$, it show that $W \subseteq cl^{\star}(B)$. Then, we have $B \subseteq W \subseteq cl^{\star}(B)$. By using the Theorem 4.3, B is a p^*-I-open set in G on (X, τ, I).

Proposition 4.3: Any subset A of X is a p*-open in generalized topology G on (X, τ) if $\mathrm{cl}^{\star}(\mathrm{A}) = \mathrm{i}_{\mathrm{G}}(\mathrm{cl}^{\star}(\mathrm{A}))$.

Proof: Let A be any arbitrary set in G on (X, τ). We have $\mathrm{cl}^{\star}(\mathrm{A}) = \mathrm{i}_{\mathrm{G}}(\mathrm{cl}^{\star}(\mathrm{A}))$. Since $\mathrm{A} \subseteq \mathrm{cl}^{\star}(\mathrm{A})$ then $\mathrm{A} \subseteq \mathrm{i}_{\mathrm{G}}(\mathrm{cl}^{\star}(\mathrm{A}))$. By using Definition 4.1, we can conclude that A is p*-I-open set.

V. I_{p^*}-OPEN SETS AND THEIR PROPERTIES

Definition 5.1: Let A be a subset of X in generalized topology G on (X, τ, I) is a $\boldsymbol{I_{p^*}}$**-open set** if there exists a p*-open set W with properties that$\mathrm{W} \backslash \mathrm{A} \in \mathrm{I}$ and $\mathrm{A} \backslash \mathrm{cl}(\mathrm{W}) \in \mathrm{I}$.

Theorem 5.1: Every p*-open set is $\boldsymbol{I_{p^*}}$-open set in generalized topology Gon $(\mathrm{X}, \tau, \mathrm{I})$.

Proof: Suppose A is any arbitrary p^*-open set in X. Then we find always a p^*-open set W for which $W \backslash A = \phi$ and $A \subseteq cl(W)$. This implies $W \backslash A \in I$ and $A \backslash cl(W) \in I$. Thus A is I_{p^*}-open set in G on (X, τ, I).

Theorem 5.2: Every I_{G}-open set is $\boldsymbol{I_{p^*}}$-open set in generalized topology G on (X, τ, I).

Proof: Suppose A is any arbitrary I_G-open set in X. Then we find always a G-open set $W$with properties that $W \backslash A = \phi$ and $A \subseteq cl(W)$. Since every G-open set is p^*-open, then U is p^*-open for which $U \backslash A = \phi$ and $A \subseteq cl(U)$. This implies $U \backslash A \in I$ and $A \backslash cl(U) \in I$. Thus A is I_{p^*}-open set in G on (X, τ, I).

Remark 5.1: A I_{p^*}-open set is may or may not be I_G-open set in generalized topology G on ideal topological space (X, τ, I). For example, let us consider a set $X = \{b_1, b_2, b_3, b_4\}$ and consider the topology $\tau = \{\phi, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, X\}$ along with generalized topology $G = \{\phi, \{b_1, b_4\}, \{b_3, b_4\}, \{b_1, b_3, b_4\}, X\}$ and ideal $I = \{\phi, \{b_1\}\}$ on X. Suppose $A = \{b_2, b_4\}$. Then we can easily verify that A is a I_{p^*}-open set in X but is not I_G-open set in G on (X, τ, I).

Theorem 5.3: Let G be a generalized topology on an ideal topological space (X, τ, I). Then each G-open is $\boldsymbol{I_{p^*}}$-open set for every ideal I.

Proof: Suppose G is a generalized topology on ideal topological space (X, τ, I) and $A \subseteq X$ is G-open set in X. By the theorem 3.1 and theorem 5.1, A is p^*-I-open set in G on (X, τ, I).

Theorem 5.4: The union of finite number of I_{p^*} -open sets is an I_{p^*}-open set in generalized topology Gon (X, τ, I).

Proof: Let A and B be two I_{p^*}-open sets. Then there exists two p^*-open sets $V$and W such that $V\backslash A \in I$, $A\backslash cl(V) \in I$ and $W\backslash B \in I, B\backslash cl(W) \in I$. Let $G = V \cup W$ is p^*-open set and we have $G\backslash(A \cup B) \subseteq ((V\backslash A)\backslash B) \cup ((W\backslash B)\backslash A) \in I$. Also $A \cup B\backslash cl(G) = \Big(\big(A\backslash cl(V)\big)\backslash cl(W)\Big) \cup \Big(\big(B\backslash cl(W)\big)\backslash cl(V)\Big) \in I$. Thus $A \cup B$ is I_{p^*}-open.

Proposition 5.1: Let G be a generalized topology on an ideal topological space (X, τ, I) and A be a subset of X. If every non-empty p^*-open subset is dense in (X, τ). Then following conditions are satisfied if $A \subseteq X$ is I_{p^*}-open with $A \notin I$.

(i) If $A \subseteq B$ then B is I_{p^*}-open. (ii) $A \cup B$ is I_{p^*}-open for any subset B of X.

Proof: (i) Suppose that A is I_{p^*}-open, then there is p^*-open set U such that $U\backslash A \in I$ and $A\backslash cl(U) \in I$. Since $A \notin I$, this implies that $cl(U) \neq \phi$. Hence $U \neq \phi$ is non-empty set. It is given that $A \subseteq B$, we have $U\backslash B \subseteq G\backslash A \in I$ and $B\backslash cl(G) = B\backslash X = \phi \in I$. Therefore B is I_{p^*}-open set in G on (X, τ, I).

(ii) Since $A \cup B$ is subset of A and A is an I_{p^*}-open set. Then by condition (i), $A \cup B$ is I_{p^*}-open set in G on (X, τ, I).

Proposition 5.2: Let A be any subset of X and Wbe a p^*-open set in generalized topology G on (X, τ, I) has the property that $W\backslash A \in I$. If $A \subseteq cl(W)$, then A is I_{p^*}-open set.

Proof: Suppose G is a generalized topology on ideal topological space X with topology τ and ideal I. It is given that W is a p^*-open set in G with the property that $W\backslash A \in I$. If $A \subseteq cl(W)$, it clear that $A\backslash cl(W) = \phi$ and $\phi \in I$. Therefore, by the definition of I_{p^*}-open set, A is I_{p^*}-open set in G on (X, τ, I).

Proposition 5.3: Let C be the collection of all possible subsets of X. If any subset $A \in C$ is p^*-open and $A \in I$ which is dense in X, then every possible subset of X is I_{p^*}-open set.

Proof: Let B be any arbitrary subset of X and $A \in I$, then $A\backslash B \in I$. Now $B\backslash cl(A) = B\backslash X = \phi \in I$. If$A$ is a p^*-open set, and take $U = A$,then $U\backslash B = A\backslash B \in I$ and $B\backslash cl(U) = B\backslash cl(A) \in I$. By the definition 5.1, B is p^*-open set.

VI. WEAKLY $\mathbf{I_{p^*}}$-OPEN SET

Definition 6.1: Let A be a subset of X in generalized topology G on (X, τ, I) is a **weakly $\boldsymbol{I_{p^*}}$-open set**, if $A = \phi$ or if $A \neq \phi$, then there is a non-empty p^*-open set W with properties that $W\backslash A \in I$. The set $X\backslash A$ is weakly I_{p^*}-closed set.

Theorem 6.1: Every p^*-open set is weakly I_{p^*}-open set in generalized topology G on (X, τ, I).

Proof: Suppose A is any arbitrary p^*-open set in generalized topology G on (X, τ, I). Then there exists a p^*-open set W such that $W \setminus A \in I = \{\phi\}$. Hence A is a weakly I_{p^*}-open set in X.

Proposition 6.1: Every G-open set is a weakly I_G-open set in generalized topology G on (X, τ, I).

Proof: Suppose A is any arbitrary G-open set in X. It is well known that every G-open set is p^*-open set. Then by using Theorem 6.1, A is a weakly I_{p^*}-open setin G on (X, τ, I).

Theorem 6.2: Every weakly I_G-open set is weakly $\boldsymbol{I_{p^*}}$-open set in generalized topology G on (X, τ, I).

Proof: Suppose $A$is any arbitrary weakly I_G-open set in X. Then we find always a non-empty G-open set W such that $W\backslash A \in I$. Since every G-open set is p^*-open, then W is p^*-open for which $W\backslash A \in I$. Thus A is weakly I_{p^*}-open set in generalized topology G on (X, τ, I).

Remark 6.1: A weakly I_{p^*}-open set is may or may not be weakly I_G-open set in generalized topology G on (X, τ, I). For example, let us consider $X = \{b_1, b_2, b_3, b_4\}$ and topology $\tau = \{\phi, \{b_2\}, \{b_1, b_2\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}, X\}$ and generalized topology $G = \{\phi, \{b_1, b_4\}, \{b_3, b_4\}, \{b_1, b_3, b_4\}, X\}$ with ideal $I = \{\phi, \{b_1\}\}$ on X. Suppose $A = \{b_1\}$. Then we can easily verify that $A = \{b_1\}$ is a weakly I_{p^*}-open set but is not weakly I_G-open set in G on (X, τ, I).

Theorem 6.3: Let G be a generalized topology on $(X, \tau, I)$and W be a singleton subset of X. If $W \in I \cap G$, then every non-empty subset of X is a weakly I_G-open set.

Proof: Suppose W is any arbitrary singleton subset of X and $W \in I \cap G$. If A is a non empty subset of X, then there exists a G-open set W such that $W \setminus A$ is either ϕ or W which is in I. Since every G-open set is p^*-open, therefore by using definition 6.1, A is a weakly $\mathrm{I}_{\mathrm{p}^*}$-open set in X.

Remark 6.2: The concept of $\mathrm{I}_{\mathrm{p}^*}$-open sets and weakly $\mathrm{I}_{\mathrm{p}^*}$-open sets are independent to each other in generalized topology G on (X, τ, I). The counter example is given as follows:

Case I. A subset A of X is $\mathrm{I}_{\mathrm{p}^*}$-open but not weakly $\mathrm{I}_{\mathrm{p}^*}$-open set.

Suppose $\mathrm{X} = \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3\}$ with topology $\tau = \{\phi, \{\mathrm{b}_1\}, \{\mathrm{b}_1, \mathrm{b}_2\}, \{\mathrm{b}_1, \mathrm{b}_3\}, \mathrm{X}\}$ and generalized topology $\mathrm{G} = \{\phi, \{\mathrm{b}_1, \mathrm{b}_3\}, \{\mathrm{b}_2, \mathrm{b}_3\}, \mathrm{X}\}$ along with ideal $\mathrm{I} = \{\phi, \{\mathrm{b}_3\}\}$ on X. Let us considerA $= \{\mathrm{a}_3\}$, then clearly A is a $\mathrm{I}_{\mathrm{p}^*}$-open set but not a weakly $\mathrm{I}_{\mathrm{p}^*}$-open set in generalized topology G on (X, τ, I).

Case II. A subset A of X is neither I_G-open set nor weakly I_G-open set.

Suppose $\mathrm{X} = \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4\}$ and let us consider topology $\tau = \{\phi, \{\mathrm{b}_1\}, \{\mathrm{b}_1, \mathrm{b}_2\}, \mathrm{X}\}$ and generalized topology $\mathrm{G} = \{\phi, \{\mathrm{b}_2, \mathrm{b}_4\}, \{\mathrm{b}_3, \mathrm{b}_4\}, \mathrm{X}\}$with ideal $\mathrm{I} = \{\phi, \{\mathrm{b}_4\}\}$ on X. If we consider $\mathrm{A} = \{\mathrm{b}_1, \mathrm{b}_2\}$, then A is neither I_G-open set nor weakly I_G-open set in generalized topology G on (X, τ, I).

Case III. A subset A of X is weakly $\mathrm{I}_{\mathrm{p}^*}$-open set but not $\mathrm{I}_{\mathrm{p}^*}$-open set in ideal topological spaces.

Suppose $\mathrm{X} = \{\mathrm{b}_1, \mathrm{b}_2, \mathrm{b}_3, \mathrm{b}_4\}$ and let us consider topology $\tau = \{\phi, \{\mathrm{b}_1\}, \{\mathrm{b}_1, \mathrm{b}_2\}, \mathrm{X}\}$ and generalized topology $\mathrm{G} = \{\phi, \{\mathrm{b}_2, \mathrm{b}_4\}, \{\mathrm{b}_3, \mathrm{b}_4\}, \mathrm{X}\}$ with ideal $\mathrm{I} = \{\phi, \{\mathrm{b}_4\}\}$ on X. Consider a set $\mathrm{A} = \{\mathrm{b}_2, \mathrm{b}_3\}$, Then clearly A is a weakly $\mathrm{I}_{\mathrm{p}^*}$-open set but not $\mathrm{I}_{\mathrm{p}^*}$-open set in generalized topology G on (X, τ, I).

With reference to above stated examples, it is clear thatthe concept of $\mathrm{I}_{\mathrm{p}^*}$-open sets and weakly $\mathrm{I}_{\mathrm{p}^*}$-open sets are independent to each other.

VII. Conclusion

Generalization of topological space is an important concept in modern mathematics and many mathematicians are giving different –different new generalized family of open sets. This concept has the broad applications in computer science, digital topology and computational topology for geometric and molecular design, data mining, information systems, quantum physics, particle physics, high energy physics and superstring theory. To study the complicated structures of these applicational areas, we need the rich properties and structure of generalized open sets in topological spaces. Therefore, we have defined such open sets which can be appropriate to fulfil the need of modern techniques. These open sets may enhance to study the complicated structure of various fields of research and development of the subjects.

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