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Bioeconomic Modelling of a Low Growth Rate Fish Species Obeying Modified Logistic Growth Function: A Comparative Study with Schaefer Model

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***Abstract*:** This paper develops a mathematical model for growth and exploitation of a fish species obeying modified logistic growth function. A low growth rate species compared to the growth rate which is directly proportional to the population density is considered. Considering the harvesting rate as the catch-per-unit-effort (CPUE) hypothesis, conditions for existence of the biological equilibrium and the bioeconomic equilibrium are studied. Maximum sustainable yield is discussed with graphical representation. The nature of stability of the dynamical and the bionomic steady states are also examined. The results are compared with those of the Schaefer model. The results are illustrated and explained with the help of numerical example.

***Keywords*:** Modified logistic growth functions; maximum sustainable yield; bioeconomic equilibrium; catch-per-unit-effort hypothesis; Schaefer model; biotechnical productivity.

I. INTRODUCTION

One of the most commonly used commercial fishery models is the Schaefer model [1] developed by biologist M.B. Schaefer. Schaefer considered a fish population obeying the logistic law of growth function and adopted the catch-per-unit-effort (CPUE) hypothesis [2] to represent the catch rate function. The logistic growth function is $F(x) = rx\left(1 - \frac{x}{k}\right)$ where r is the intrinsic growth rate of the species and k is the environmental carrying capacity, which was first developed by P.F. Verhulst [3] in the year 1838 and first used in the model of human population. In spite of various limitations of the logistic growth function, it is still being used by most of the researchers for its simplicity. Researchers like Pradhan and Chaudhuri [4], Ganguly and Chaudhuri [5], Ray and Pradhan [6], Chaudhuri and Johnson [7] and many other researchers discussed several mathematical fishery models by using logistic growth function as the growth function of the species. Pradhan and Chaudhuri [8] discussed a bioeconomic modeling of selective harvesting in an inshore-offshore fishery using logistic growth function for the species of inshore area. Gompertz law of growth function [9] is also an important growth function. Pradhan and Chaudhuri [10] discussed a bioeconomic modeling of a single species fishery with Gompertz law of growth.

Many alternative terms for the growth function have been suggested by May [11]. Modified logistic growth function in the form of $rx^{\alpha}\left(1 - \frac{x}{k}\right), \alpha > 0$ is one of the important growth functions but it is being used very rare in the mathematical models for its complexity. It is very difficult to study the existence and the stability analysis of the non-trivial steady states of the species obeying the modified logistic growth function in the form of $rx^{\alpha}\left(1 - \frac{x}{k}\right)$ when $0 < \alpha < 1$ or $\alpha > 1$. For $\alpha = 1$ the modified growth function becomes the simple logistic growth function which is being used in maximum fishery models. For $0 < \alpha < 1$, the growth rate of the species is low and for $\alpha > 1$, the growth rate of the species is high compared to the growth rate which is directly proportional to the population density. Ray and Pradhan [12] discussed the dynamical behaviour of an exploited fish species obeying modified logistic growth function for both the cases $0 < \alpha < 1$ and $\alpha > 1$ with taxation as a control instrument.

In this paper it is assumed that the fish species obeys the modified logistic growth function in the form of $rx^{\frac{1}{2}}\left(1 - \frac{x}{k}\right)$ where the natural growth rate of the species is proportional to the square root of the population density and so the growth rate is very low compared to the growth rate which is directly proportional to the population density. Considering the harvesting rate as the catch-per-unit-effort hypothesis (CPUE) [2], steady state analysis is discussed in section-IV. It is seen that, the mathematical model of the low growth rate species requires no restriction on the parameters for existence of the non-trivial steady state whereas for the Schaefer model the non-trivial steady state exists when the harvesting effort is less than the BTP (biotechnical productivity) [2] of the species. Stability of non-trivial steady states for exploited system of the model is examined in section-V. In section-VI, it is seen that sustainable yield increases with increase of the effort in certain range and then decreases with increase of the effort up to infinity. Maximum sustainable yield occurs in this model which is less than the maximum sustainable yield for the Schaefer model. In section-VII, it is proved that bioeconomic equilibrium exists when the fishing cost per unit effort is less than a certain amount which depends on market price of the harvested fish, environmental carrying capacity and catchability coefficient of the species or when the market price is greater than a certain level which depends on fishing cost per unit effort, environmental carrying capacity and catchability coefficient of the species. Different results are explained in numerically and graphically for the set of hypothetical parameter values for both the cases in section-IX.

II. STATEMENT OF THE PROBLEM

The population dynamics of the fishery resource is given by the equation

$$\frac{dx}{dt} = F(x) - h(t) \qquad (1)$$

where $x(t)$ is the population density, $F(x)$ is the growth rate function and $h(t)$ is the harvesting rate at time t.

Regarding the rate of harvesting, biologist M.B. Schaefer [1] made the catch-per-unit effort hypothesis [2] and took $h(t) = qEx$, where E denotes the harvesting effort and q denotes the catchability coefficient. This particular form of $h(t)$ implies random and independent search for harvesting as well as equal availability of one individuals to the harvesting set-up.

It is assumed that the growth rate function is the modified logistic growth function i.e. $F(x) = rx^{\alpha}\left(1 - \frac{x}{k}\right)$, where r is the intrinsic growth rate of the species, k is the environmental carrying capacity and α is a positive constant. Considering the growth rate of the species as the modified logistic growth function and the harvesting rate as the catch-per-unit-effort hypothesis (CPUE), the equation (1) becomes

$$\frac{dx}{dt} = rx^{\alpha}\left(1 - \frac{x}{k}\right) - qEx,\ \alpha > 0 \qquad (2)$$

Now we find the steady states of the equation (2) and the corresponding sustainable yield and hence the maximum sustainable yield of the fish species for $\alpha = \frac{1}{2}$.

Definition: Maximum Sustainable Yield.

The maximum sustainable yield (briefly denoted by MSY) of a biological resource population is the maximum rate at which it can be harvested even after maintaining the population at a constant level.

III. THE MATHEMATICAL MODEL FOR $\alpha = \frac{1}{2}$

In this model it is assumed that the growth rate of the species is proportional to the square root of the population density x under ideal conditions where the availability of space and other resources do not inhibit the growth.

For $\alpha = \frac{1}{2}$, the growth function of the species is $F(x) = rx^{\frac{1}{2}}\left(1 - \frac{x}{k}\right)$ (3)

Here the growth rate of the species is low compared to the growth rate which is simply proportional to the population density. This growth function is very useful to determine the dynamical behaviour of a low growth rate species.

Now, $F^{/}(x) = r\left(\frac{1}{2\sqrt{x}} - \frac{3}{2k}\sqrt{x}\right)$ and $F^{/}(x) = 0$ implies $x = \frac{k}{3}$.

Therefore, $F^{/}(x) > 0$ for $0 < x < \frac{k}{3}$ and $F^{/}(x) < 0$ for $\frac{k}{3} < x < k$.

Thus the growth function of the species is increasing in the interval $\left(0, \frac{k}{3}\right)$ and decreasing in the interval $\left(\frac{k}{3}, k\right)$. The growth of the species is maximum at the population level $\frac{k}{3}$. In the Schaefer model [1], the growth of the species is maximum at the population level $\frac{k}{2}$.

Thus it is seen that for the unexploited fishery the growth of the population of a low growth rate species is faster than the growth of the species obeying simple logistic growth law.

For this growth function, equation (2) becomes $\frac{dx}{dt} = rx^{\frac{1}{2}}\left(1 - \frac{x}{k}\right) - qEx$ (4)

IV. STEADY STATES

Equation (4) has the trivial steady state at $x = 0$. The non-trivial steady state of (4) is given by $ky - \frac{1}{y} = \frac{qkE}{r}$ where $y = \frac{1}{\sqrt{x}} > 0$. (5)

$\Rightarrow y = \frac{\frac{qkE}{r} + \sqrt{\left(\frac{qkE}{r}\right)^2 + 4k}}{2k}$, since $y > 0$.

Therefore, the non-trivial steady state of the exploited fish species is

$$x_1 = \frac{4k^2}{\left\{\frac{qkE}{r} + \sqrt{\left(\frac{qkE}{r}\right)^2 + 4k}\right\}^2}. \quad (6)$$

This model requires no restriction of the parameters for existence of the non-trivial steady state but in the Schaefer model non-trivial steady state exists only when the harvesting effort is less than the BTP of the species i.e. $E < \frac{r}{q}$. From (6), it is clear that the steady state monotonically decreases with the increase of effort and $x \to k$, the environmental carrying capacity as $E \to 0$. This result confirms that the steady state is equal to the environmental carrying capacity for the unexploited fishery.

V. STABILITY ANALYSIS OF NON-TRIVIAL STEADY STATE

$$\frac{d}{dx}\left(\frac{dx}{dt}\right) = \frac{r}{2\sqrt{x}} - \frac{3r\sqrt{x}}{2k} - qE \text{ by (4)}$$
$$= \frac{r}{2k\sqrt{x}}(k - 3x) - qE.$$

Therefore, $\frac{d}{dx}\left(\frac{dx}{dt}\right)_{at\ x=x_1} = \frac{r}{2k\sqrt{x_1}}\{k - 3x_1 - 2(k - x_1)\} = -\frac{r}{2k\sqrt{x_1}}(k + x_1) < 0$ using (5).

The non-trivial steady state $x_1 = \frac{4k^2}{\left\{\frac{qkE}{r} + \sqrt{\left(\frac{qkE}{r}\right)^2 + 4k}\right\}^2}$ of the equation (4) is always stable.

The important characteristic of this type of model is that the non-trivial steady state of the growth equation always exists and it is stable for every parameter values. So the modified logistic growth function $F(x) = rx^{\frac{1}{2}}\left(1 - \frac{x}{k}\right)$ has an important role for existence and stability of the non-trivial steady state for exploited fishery.

VI. MAXIMUM SUSTAINABLE YIELD

Corresponding to the effort E, the sustainable yield $Y(E)$ is given by

$$Y(E) = qEx_1 = \frac{4qEk^2}{\left\{\frac{qkE}{r} + \sqrt{\left(\frac{qkE}{r}\right)^2 + 4k}\right\}^2}, \text{ by (6)} \quad (7)$$

Therefore, $\frac{dY(E)}{dE} = Y\left\{\frac{1}{E} - \frac{2kq}{r\sqrt{\left(\frac{qkE}{r}\right)^2 + 4k}}\right\}$ (8)

Now, $\frac{dY(E)}{dE} = 0 \Rightarrow E = \frac{r\sqrt{\left(\frac{qkE}{r}\right)^2 + 4k}}{2kq}$, since $Y \neq 0$.

$$\Rightarrow \frac{4k^2q^2E^2}{r^2} = \frac{k^2q^2E^2}{r^2} + 4k \Rightarrow E = \frac{2}{\sqrt{3k}}\left(\frac{r}{q}\right).$$

Therefore, the sustainable yield attains its extreme value at an effort level $\frac{2}{\sqrt{3k}}\left(\frac{r}{q}\right)$.

From (8), we have

$$\frac{1}{Y}\frac{d^2Y}{dE^2} - \frac{1}{Y^2}\frac{dY}{dE} = -\frac{1}{E^2} + \frac{k^2q^2}{r^2\left\{\left(\frac{kqE}{r}\right)^2 + 4k\right\}}$$

Therefore, $\left(\frac{d^2Y}{dE^2}\right)_{at\ E=\frac{2}{\sqrt{3k}}\left(\frac{r}{q}\right)} = -\frac{45}{16}\frac{kq^2}{r^2}Y^* < 0$ by (8)

where $Y^* = Y\left(\frac{2r}{q\sqrt{3k}}\right) = \frac{2r}{\sqrt{3k}}x_1 > 0$ by (7).

Therefore, $Y(E)$ is maximum when the effort level $E_{MSY} = \frac{2}{\sqrt{3k}}\left(\frac{r}{q}\right)$ and maximum sustainable yield $MSY = qE_{MSY}x_1 = \frac{4qEk^2}{\left(\frac{qkE}{r}+\frac{2kqE}{r}\right)^2} = \frac{2r\sqrt{k}}{3\sqrt{3}}$.

The corresponding population level $x_{MSY} = \frac{4k^2}{\left(\frac{qkE}{r}+\frac{2kqE}{r}\right)^2} = \frac{k}{3}$.

It is noted that, in the case of Schaefer model [1] the MSY population level is $\frac{k}{2}$. Thus the present model prescribes a lower MSY population level than the Schaefer model.

In fishery literature the ratio of the intrinsic growth rate and the environmental carrying capacity $\left(\frac{r}{q}\right)$ is called the Biotechnical Productivity (BTP) [2].

Therefore, the effort for which the maximum sustainable yield occurs is $\frac{2}{\sqrt{3k}}$ times of the BTP of the fish species and MSY is independent of the catchability coefficient of the species for this modified logistic growth function.

From (7), we see that $Y(E) = 0$ for $E = 0$ and $Y(E) \to 0$ as $E \to \infty$.

Therefore, the sustainable yield is an increasing function of effort in the interval $\left(0, \frac{2r}{\sqrt{3k}q}\right)$ and a decreasing function of $\left(\frac{2r}{\sqrt{3k}\,q}, \infty\right)$.

VII. BIONOMIC EQUILIBRIUM

Let *TR* be the total revenue earned by selling the harvested biomass and *TC* be the total cost of effort for harvesting the species.

The *economic equilibrium* is said to be achieved when $TR = TC$.

Let c be the fishing cost per unit effort and p be the market price per unit harvested biomass.

Therefore, $TR = pqEx$ and $TC = cE$.

The *economic equilibrium* is the solution of the equation $TR - TC = 0 \Rightarrow pqEx - cE = 0$.

When the *biological equilibrium* and the *economic equilibrium* hold simultaneously for a species, then it is called *bionomic equilibrium*.

Thus the *bionomic equilibrium* (x_∞, E_∞) of an exploited species is the solution of the following system of equations:

$$\left.\begin{aligned} F(x,E) &= rx^{\frac{1}{2}}\left(1-\frac{x}{k}\right) - qEx = 0 \\ \Pi(x,E) &= pqEx - cE = 0 \end{aligned}\right\} \qquad (9)$$

From (9) we have $x_\infty = \frac{c}{pq}$ and $E_\infty = \frac{r}{q\sqrt{x_\infty}}\left(1-\frac{x_\infty}{k}\right) = \frac{r\sqrt{pq}}{q\sqrt{c}}\left(1-\frac{c}{pqk}\right)$.

Now, $E_\infty > 0 \Rightarrow c < pqk$ or $p > \frac{c}{qk}$.

Thus the bionomic equilibrium exists if $c < pqk = c_{max}$ or $p > \frac{c}{qk} = p_{min}$. In other word we can say that the bionomic equilibrium exists if the cost price ratio $\frac{c}{p}$ is less than kq.

Therefore, the upper limit of the fishing cost per unit effort is $c_{max} = pqk$ for existence of the bionomic equilibrium when the market price p per unit harvested biomass is assumed to be constant and the lower limit of the market price per unit harvested biomass is $p_{min} = \frac{c}{qk}$ for existence of bionomic equilibrium when the fishing cost *c* per unit effort is assumed to be constant.

Thus the necessary and sufficient condition for existence of bionomic equilibrium is one of the following conditions.

(i) For the fixed market price, the fishing cost per unit effort is $c < pqk = c_{max}$

(ii) For the fixed fishing cost, the market price per unit harvested biomass is $p > \frac{c}{qk} = p_{min}$.

VIII. STABILITY ANALYSIS OF BIONOMIC EQUILIBRIUM

The variational matrix of the system of equations (9) is

$$V(x,E) = \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial E} \\ \frac{\partial \Pi}{\partial x} & \frac{\partial \Pi}{\partial E} \end{pmatrix} = \begin{pmatrix} \frac{r}{2\sqrt{x}}\left(1-\frac{x}{k}\right) - \frac{r\sqrt{x}}{k} - qE & -qx \\ pqE & pqx - c \end{pmatrix}.$$

Therefore, $V(x_\infty, E_\infty) = \begin{pmatrix} -\left(\frac{r}{2\sqrt{x_\infty}} + \frac{r\sqrt{x_\infty}}{2k}\right) & -qx_\infty \\ pqE_\infty & 0 \end{pmatrix}$.

Now, $Trace$ of $V(x_\infty, E_\infty) = -\left(\frac{r}{2\sqrt{x_\infty}} + \frac{r\sqrt{x_\infty}}{2k}\right) < 0$ and $detV(x_\infty, E_\infty) = pq^2 x_\infty E_\infty > 0$.

Therefore, the eigen values of the variational matrix $V(x_\infty, E_\infty)$ are both real negative or complex conjugate with negative real parts. So the bionomic equilibrium (x_∞, E_∞) of the system of equations (9) is always stable, if it exists. This equilibrium is either stable node or stable focus according as the eigen values are real negative or complex conjugate with negative real parts. The discriminant of the characteristic equation of the variational matrix $V(x_\infty, E_\infty)$ is

$$\Delta = \{traceV(x_\infty, E_\infty)\}^2 - 4detV(x_\infty, E_\infty) = \left(\frac{r}{2\sqrt{x_\infty}} + \frac{r\sqrt{x_\infty}}{2k}\right)^2 - 4pq^2 x_\infty E_\infty \qquad (10)$$

If $\Delta \geq 0$, then the bionomic equilibrium is a stable node and if $\Delta < 0$, then the bionomic equilibrium is a stable focus.

IX. EXAMPLE

Let us consider a set of hypothetical data as $r = 5, k = 100, q = 1\%$ and $E = 25$ in appropriate units.
The nontrivial steady state of the fish population satisfying the growth equation (4) is $x_1 = 63.37$ and it is a stable steady state.

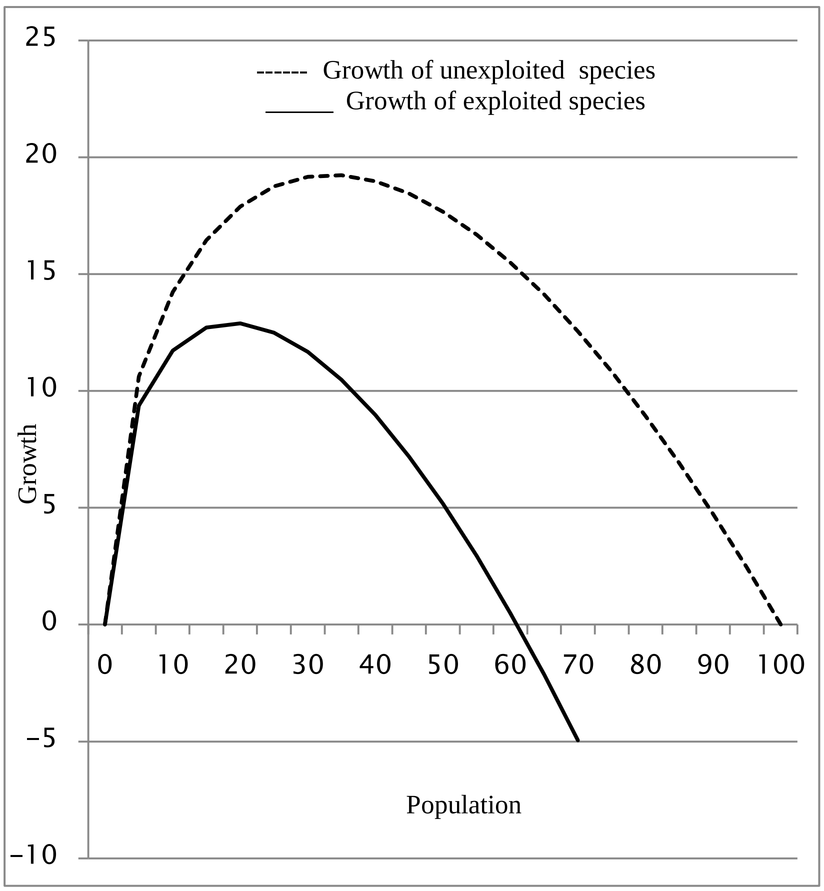

Fig.1 Growth curves for the unexploited and exploited fishery for the model $\alpha = \frac{1}{2}$

For these parameter values corresponding $MSY = 19.25, E_{MSY} = 57.74$ and $x_{MSY} = 33.33$.
Corresponding to the above parameters the growth curves for the unexploited and exploited fishery are shown in Figure-1 and the yield effort curve is shown in Figure-2.

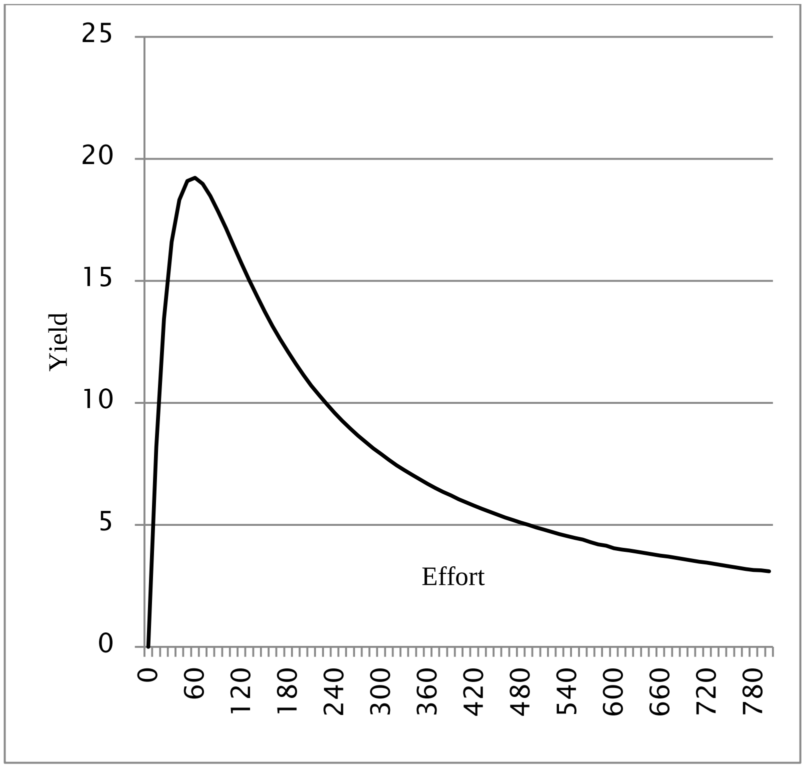

Fig.2. The yield effort curve for the model $\alpha = \frac{1}{2}$

Let the market price per unit harvested biomass be $p = 15$ which is assumed to be fixed.
Therefore, for existence of non-trivial bionomic equilibrium, the fishing cost per unit effort must be less than $c_{max} = pqk = 15$.
For a fixed market price $p = 15$, we have the following different bionomic equilibrium and their corresponding nature of stability for different fishing cost per unit effort.

Table-1: Bioeconomic equilibrium and their stability for different fishing costs

Fishing cost per unit effort (c)	Equilibrium level of population (x_∞)	Equilibrium level of effort (E_∞)	Sign of Δ	Nature of stability
1	6.67	180.69	Negative	Stable focus
2	13.33	118.67	Negative	Stable focus
3	20	89.44	Negative	Stable focus
4	26.67	71	Negative	Stable focus
5	$33.33 = x_{MSY}$	$57.74 = E_{MSY}$	Negative	Stable focus
6	40	47.43	Negative	Stable focus
7	46.67	39.03	Negative	Stable focus
8	53.33	31.95	Negative	Stable focus
9	60	25.82	Negative	Stable focus
10	66.67	20.47	Negative	Stable focus
11	73.33	15.57	Negative	Stable focus
12	80	11.18	Negative	Stable focus
13	86.67	7.16	Negative	Stable focus
14	93.33	3.45	Negative	Stable focus
15	**100**	**0**	**Positive**	**Stable Node**

The Fig.3 shows the bionomic equilibrium levels corresponding to progressively increasing of fishing cost per unit effort.

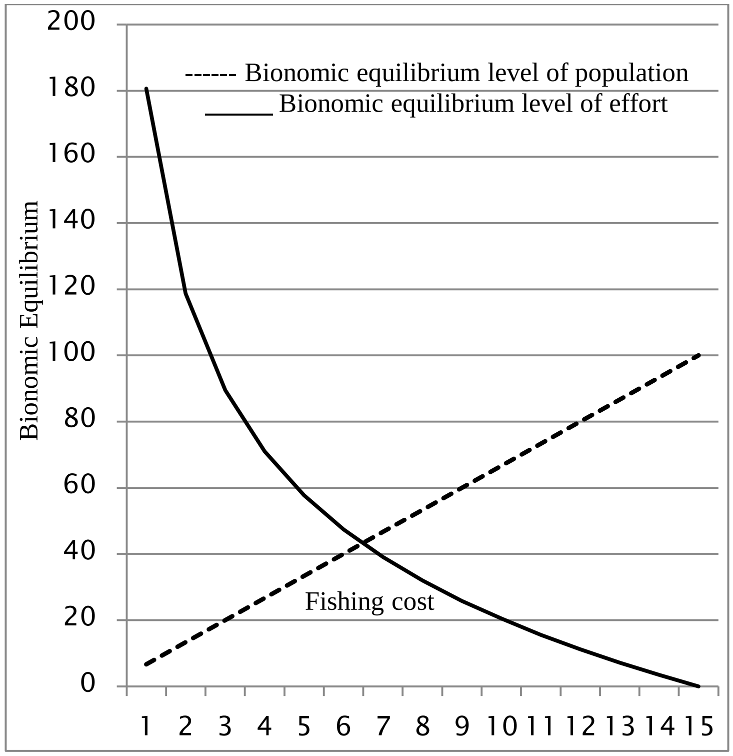

Fig.3. Bionomic equilibrium levels of population and effort corresponding to the increase of fishing cost per unit effort.

The dotted line is the line of bionomic equilibrium levels of population and the solid line is the graph of bionomic equilibrium levels of effort.

From the Fig.-3 and also from the table-1 it is clear that the bionomic equilibrium levels of population gradually increase and the bionomic equilibrium levels of effort gradually decrease corresponding to increase of fishing cost per unit effort. In reality it is true that when the cost of fishing increases the fishermen will be less interested to fishing and as a result the equilibrium level of effort decreases and the equilibrium level of population increases.
Again let the fishing cost per unit effort be $c = 10$ which is assumed to be fixed.
Therefore, for existence of non-trivial bionomic equilibrium, the market price per unit of harvested biomass must be greater than $p_{min} = \frac{c}{qk} = 10$.

For the fixed fishing cost per unit effort $c = 10$, we have the following different bionomic equilibrium and their corresponding nature of stability for different market price per unit of harvested biomass.

Table-2: Bioeconomic equilibrium and their stability for different market prices

Market price per unit biomass (p)	Equilibrium level of population (x_∞)	Equilibrium level of effort (E_∞)	Sign of Δ	Nature of stability
11	90.91	4.77	Negative	Stable focus
15	66.67	20.41	Negative	Stable focus
20	50	35.36	Negative	Stable focus
25	40	47.43	Negative	Stable focus
30	$33.33 = x_{MSY}$	$57.74 = E_{MSY}$	Negative	Stable focus
35	28.57	66.82	Negative	Stable focus
40	25	75	Negative	Stable focus

45	22.22	82.50	Negative	Stable focus
50	20	89.44	Negative	Stable focus
55	18.18	95.95	Negative	Stable focus
60	16.67	102.05	Negative	Stable focus
65	15.38	107.89	Negative	Stable focus
70	14.29	113.37	Negative	Stable focus
75	13.33	118.69	Negative	Stable focus
80	12.30	125.03	Negative	Stable focus
85	11.76	128.66	Negative	Stable focus
90	11.11	133.34	Negative	Stable focus
95	10.53	137.86	Negative	Stable focus
100	10	142.30	Negative	Stable focus
105	9.52	161.92	Negative	Stable focus

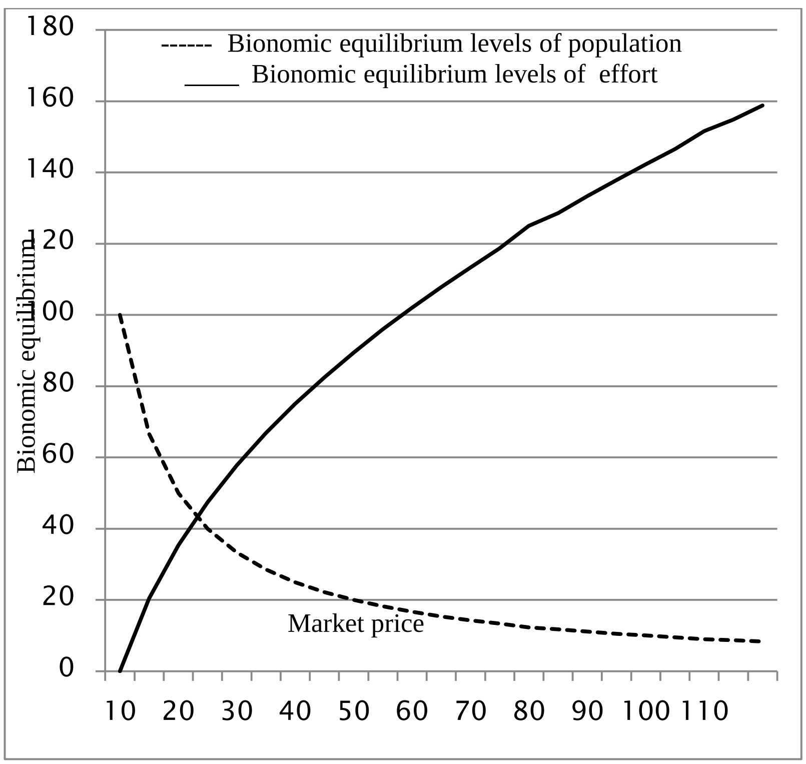

Fig.4. Bionomic equilibrium levels of population and effort corresponding to progressive increase of market price of fish.

In Fig.4, the dotted line is the line of bionomic equilibrium levels of population and the solid line is the line of bionomic equilibrium levels of effort.

From the Fig.4 and also from the table-2 it is clear that the bionomic equilibrium levels of population gradually decrease and the bionomic equilibrium levels of effort gradually increase corresponding to increase of market price of the fish. In reality, when the market price of fish increases the fishermen will become interested in fishing and give more effort to catch fish. As a result the equilibrium level of efforts increase and the equilibrium level of population decrease.

X. CONCLUSION

The present model is suitable for the fish species whose growth rate is very low compared to the growth rate which is directly proportional to the population density in the ideal conditions where the availability of space and other resources do not inhibit the growth. Though it is very difficult to study the dynamical behaviour of the fishery models with the modified logistic growth function in the form of $F(x) = rx^{\alpha}\left(1 - \frac{x}{k}\right)$, for all $\alpha > 0, \alpha \neq 1$, the results are more realistic compared to the various results of the models where the species obeys the simple logistic growth function in the form of $F(x) = rx\left(1 - \frac{x}{k}\right)$. It is seen that for the unexploited fishery the growth of the population of a low growth rate species is faster than the growth of the species obeying simple logistic growth law. In case of exploited fishery if the harvesting rate is considered as the catch-per-unit-effort (CPUE) hypothesis then the non-trivial steady state of the low growth rate species $\left(\alpha = \frac{1}{2}\right)$ always exists without any restriction of the parameters but in the Schaefer model non-trivial steady state exists only when the harvesting effort is less than the BTP of the species i.e. $E < \frac{r}{q}$. It is also seen that the MSY population level of the Schaefer model is greater than the MSY population level of the model for $\alpha = \frac{1}{2}$.

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