

CASP for Truncated Pareto Weibull Distribution Based on CUSUM Schemes and its Optimization

M. C. Chandra Prasad^{1*}, P. Mohammed Akhtar², G.Venkatesulu³

^{1,2,3}Dept. of Statistics, Sri Krishnadevaraya University, Ananthapuramu-515003

*Corresponding Author: prasu.here@gmail.com, Tel.: 9908102558

Available online at: www.isroset.org

Received: 03/Dec/2018, Accepted: 21/Dec/2018, Online: 31/Dec/2018

Abstract:- CUSUM charts are firstly introduced by E. S. Page, in 1954. They are used to identify a sudden and persistent change in the production average. Many researchers worked in this area and developed optimization techniques by updating the previous one. In this direction, all the optimization techniques are developed under the assumption that variable with regard to quality characteristic is distributed according to certain probability law. In the present study, we assume that variable under study follows the Truncated Pareto Weibull Distribution and its Optimization of CASP-CUSUM Schemes. Critical comparison is made based on the obtained numerical results.

Keywords— CASP – CUSUM Schemes, O C Curve, Optimal Truncated Point, Truncated Pareto Weibull Distribution

I. INTRODUCTION

In any organization, quality products help to maintain customer satisfaction, loyalty and reduce the risk of replacing the faulty goods. Companies can build a reputation and accreditation with recognized quality standard. Hence, the manufacturer shows keen interest in maintaining the quality of the product.

Most manufacturers find difficulty in maintaining and providing a good quality product to the customer. A major reason behind this is 'Variation.' There is a certain level of variability in every product. The 'variability' is termed as the level of difference from the desired level of output for a specified product or service. This means a product which shows more variability possess less quality. To maintain a good quality, one should make sure that products possess less variability during the manufacturing process till the end product. In short "*Quality Improvement is the reduction of the variability.*"

Further, the variation produced in any production process follows "Statistical Laws." This paves the way to apply sophisticated techniques to control the quality. These techniques are known as "Statistical Quality Control Techniques."

Sampling plans are one of the statistical tools in testing the quality of a product. It is the middle path between 100% inspection and no inspection. They are used to decide whether to accept or reject a lot of finished goods. These

techniques may not have direct impact in controlling the quality but it has indirect effect in improving the quality of the products. CUMMULATIVE ACCEPTING SAMPLING PLANS – CUSUM SCHEMES (CASP-CUSUM Schemes) are widely used in the industries in life testing of the products. It is powerful, versatile and diagnostic tool in Statistical Quality Control as it reduces time, cost, manpower and machinery.

Hawkins, D. M. [1] proposed a fast accurate approximation for ARL's of a CUSUM Control Charts. This approximation can be used to evaluate the ARL's for Specific parameter values and the out of control ARL's location and scale CUSUM Charts.

Kakoty. S., Chakraborty A.B. [2] determined CASP-CUSUM charts under the assumption that the variable under study follows a Truncated Normal Distribution. Generally, truncated distributions are employed in many practical phenomena where there is a constraint on the lower and upper limits of the variable under study. For example, in the production of engineering items, the sorting procedure eliminates items above or bellows designated tolerance limits. It is worthwhile to note that any continuous variable is first approximated as an exponential variable.

Vardeman.S, Di-ou Ray [3] introduced CUSUM control charts under the restriction that the values are regard to quality is exponentially distributed. Further, the phenomena under study are the occurrence of the rate of rare events and

the inter-arrival times for a homogenous poisson process are identically independently distributed exponential random variables.

Lonnie. C. Vance [4], considered Average Run Length of Cumulative Sum Control Charts for controlling normal means and to determine the parameters of a CUSUM Chart. To determine the parameters of CUSUM Chart the acceptable and rejectable quality levels along with the desired respective ARL's are considered.

Mohammed Akhtar. P and Sarma K.L.A.P [5] analyzed and Optimization of CASP-CUSUM Schemes based on truncated two parametric Gamma distributions and evaluates L (0), L (O) and a probability of Acceptance and also Optimized CASP-CUSUM Schemes based numerical results.

Narayana Murthy, B.R. and Mohammed Akhtar.P [6] proposed an Optimization of CASP CUSUM Schemes based on Truncated Log-logistic distribution and evaluate the probability of acceptance for different Parameter values.

Sainath.B and Mohammed Akhtar.P [7] studied an Optimization of CASP-CUSUM Schemes based on truncated Burr distribution and the results were analyzed at different values of the parameters.

Venkatesulu.G and Mohammed Akhtar.P [8] deduced Truncated Gompertz Distribution and its Optimization of CASP-CUSUM Schemes by changing different parameter values, and the obtained numerical results are critically compared.

Venkatesulu.G and Mohammed Akhtar.P [9] determined Truncated Lomax Distribution and its Optimization of CASP-CUSUM Schemes by changing the values of the parameters and finally, critical comparisons are drawn based on the obtained numerical results.

In the present section, the importance of the *Quality* and a brief literature review is presented on *Sampling plans* and *CUSUM Schemes*. In Section-2, we provide the definition of the Pareto Weibull distribution and its truncation. In Section-3, we provide the description of the Type-C OC Curve. The method for obtaining the different solutions is presented in the section-4. We developed a computer program for obtaining ARL's and P (A) for different parameters such that the variable under study follows Truncated Pareto Weibull distribution and the results obtained are presented in tabular form under section-5. Finally, In Section-6, we made conclusions based on the results obtained in the section-5.

II. PARETO WEIBULL DISTRIBUTION

It is well known that Weibull distribution is widely used for analyzing lifetime data in reliability engineering. It has wide applications in the field of Automotives, aerospace, Electronics etc. There are several generalization forms of Weibull distribution. Pareto Weibull distribution is a one of such generalization of Weibull distribution has some similar properties of another Weibull family of distributions. In the present study, we use Pareto Weibull Distribution given by M. H. Tahir et al. [10]

A continuous random variable X is said to follow Pareto Weibull distribution if it assumes non-negative values with parameters b, and θ and its probability density function is given by

$$f(x) = f(x; b, \alpha, \theta) = \frac{b\alpha}{\theta^\alpha} x^{\alpha-1} \left(\left(\frac{x}{\theta} \right)^\alpha - 1 \right)^{b-1} e^{-\left(\left(\frac{x}{\theta} \right)^\alpha - 1 \right)^b} \dots\dots (2.1)$$

Truncated Pareto Weibull distribution

It is defined as the ratio of the probability density function of Pareto Weibull distribution to its probability cumulative distribution function at the point B.

A random variable X is said to follow Truncated Pareto Weibull distribution if its probability density function is given by

$$f_B(X) = \frac{\frac{b\alpha}{\theta^\alpha} x^{\alpha-1} \left(\left(\frac{x}{\theta} \right)^\alpha - 1 \right)^{b-1} e^{-\left(\left(\frac{x}{\theta} \right)^\alpha - 1 \right)^b}}{1 - e^{-\left(\left(\frac{B}{\theta} \right)^\alpha - 1 \right)^b}} \dots\dots (2.2)$$

Where 'B' is the upper truncated point of Truncated Pareto Weibull distribution

III. DESCRIPTION OF THE TYPE-C OC CURVE

Beattie [11] has suggested the method for constructing the continuous acceptance sampling plans. The procedure, suggested by him consists of a chosen decision interval namely, "Return interval" with the length h', above the decision line is taken. We plot on the chart the sum $s_m = \sum(x_i - k)$, x_i 's (i = 1, 2, 3...) are distributed independently and k is the reference value. If the sum lies in the area of the normal chart, the product is accepted and if it lies on the return chart,

then the product is rejected, subject to the following assumptions.

1. When the recently plotted point on the chart touches the decision line, then the next point to be plotted at the maximum. i.e., $h=h'$.
2. When the decision line is reached or crossed from above, the next point on the chart is to plot at the baseline.

When the CUSUM falls in the return chart, network or a change of specification may be employed rather than outright rejection.

The procedure, in brief, is given below.

1. Start plotting the CUSUM at 0.
2. The product is accepted when $s_m < h$, when $s_m < 0$, return cumulative to 0
3. When $h < S_m < h+h'$ the product is rejected; when S_m crossed h , i.e., when $S_m > h+h'$ and continue rejecting Product until $S_m > h+h'$ return cumulative to $h+h'$

The Type-C OC function is defined as the probability of acceptance of an item as the function of the incoming quality when the sampling rate is same in acceptance and rejection regions. Then the probability of acceptance $P(A)$ is given by

$$P(A) = \frac{L(0)}{L(0) + L'(0)} \dots\dots 3.1$$

Where $L(0)$ = Average Run Length in acceptance zone and

$L'(0)$ = Average Run Length in rejection zone.

Page E.S. [12] has introduced the formulae for $L(0)$ and $L'(0)$ as

$$L(0) = \frac{N(0)}{1 - P(0)} \dots\dots 3.2$$

$$L'(0) = \frac{N'(0)}{1 - P'(0)} \dots\dots 3.3$$

Where

$P(0)$ =Probability for the test starting from zero on the normal chart,

$N(0)$ = ASN for the test starting from zero on the normal chart,

$P'(0)$ = Probability for the test on the return chart and

$N'(0)$ = ASN for the test on the return chart

He further obtained integral equations for the quantities

$P(0)$, $N(0)$, $P'(0)$, $N'(0)$ as follows:

$$P(Z) = F(k - z) + \int_0^h P(y) f(y + k - z) dy \dots\dots (3.4)$$

$$N(Z) = 1 + \int_0^h N(y) f(y + k - z) dy \dots\dots(3.5)$$

$$P'(Z) = \int_{k+z}^h f(Y) dy + \int_0^h P'(y) f(-y + k + z) dy \dots\dots(3.6)$$

$$N'(Z) = 1 + \int_0^h N'(y) f(-y + k - z) dy \dots\dots(3.7)$$

$$F(x) = 1 + \int_1^h f(x) dx \dots\dots(3.8)$$

$$F(k - z) = 1 + \int_A^{k-z} f(y) dy \dots\dots(3.9)$$

and z is a distance of the starting of the test in the normal chart from zero.

IV. METHOD OF SOLUTION

Using the procedure suggested by Kakoty, S and Chakraborty, A.B.[2], we solve the above equations for Truncated Pareto Weibull distribution (2.1) and we obtained the solutions for $P(0)$, $N(0)$, $P'(0)$ and $N'(0)$. Using these values $L(0)$, $L'(0)$ and $P(A)$ are calculated for various parameters namely b , α , θ , h , h' and B are given in Tables 5.1 to 5.22.

V. COMPUTATION OF ARL'S AND P (A)

We developed computer program to solve the above equations and we get the following results given in the tables (5.1) to (5.22).

TABLE – 5.1
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 3$
 $k = 1$ $h = .1$ $h' = 0.1$

B	L(0)	L'(0)	P(A)
2.1	36.91	1.24	0.967504978
2	47.86	1.24	0.974690437
1.9	69.41	1.25	0.982366741
1.8	131.40	1.25	0.990581691
1.7	2050.19	1.25	0.999389291

TABLE – 5.2
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 3$
 $k = 1$ $h = .15$ $h' = 0.15$

B	L(0)	L'(0)	P(A)
2.7	42.17	1.39	0.968077898
2.6	52.13	1.40	0.973936141
2.5	69.15	1.40	0.980162203
2.4	104.91	1.40	0.986788511
2.3	227.86	1.41	0.993851364

TABLE – 5.3
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 3$
 $k = 1$ $h = .18$ $h' = 0.18$

B	L(0)	L'(0)	P(A)
3.1	52.30	1.50	0.972195268
3	65.00	1.50	0.977428377
2.9	86.98	1.51	0.982975781
2.8	134.27	1.51	0.988863766
2.7	309.67	1.52	0.995121419

TABLE – 5.4
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 3$
 $k = 1$ $h = .2$ $h' = 0.2$

B	L(0)	L'(0)	P(A)
3.4	64.10	1.57	0.976047635
3.3	81.01	1.58	0.98088485
3.2	111.63	1.58	0.986004055
3.1	183.97	1.59	0.99142772
3	564.99	1.60	0.997180998

TABLE -5. 5
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 2$ $\theta = 3$
 $k = 1$ $h = .1$ $h' = 0.1$

B	L(0)	L'(0)	P(A)
1.4	17.43	1.11	0.939924538
1.3	23.03	1.12	0.953822672
1.2	33.64	1.12	0.967884004
1.1	60.83	1.12	0.981967926
1	272.19	1.12	0.995909691

TABLE – 5.6
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 2$ $\theta = 3$
 $k = 1$ $h = .15$ 0.15

B	L(0)	L'(0)	P(A)
1.4	15.20	1.15	0.929548919
1.3	19.58	1.15	0.944389343
1.2	27.30	1.15	0.959458947
1.1	44.30	1.15	0.974607885
1	110.52	1.16	0.989657462

TABLE – 5.7
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 2$ $\theta = 3$
 $k = 1$ $h = .18$ $h' = 0.18$

B	L(0)	L'(0)	P(A)
1.3	17.49	1.17	0.937369943
1.2	23.65	1.17	0.952896774
1.1	36.00	1.17	0.968530655
1	72.36	1.17	0.98408705
0.9	1802.62	1.17	0.999351084

TABLE – 5.8
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 2$ $\theta = 3$
 $k = 1$ $h = .2$ $h' = 0.2$

B	L(0)	L'(0)	P(A)
1.3	16.13	1.18	0.932033718
1.2	21.36	1.18	0.947793722
1.1	31.23	1.18	0.963676214
1	56.23	1.18	0.979493201
0.9	235.51	1.18	0.995025635

TABLE – 5.9

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 3$
 $k = 1$ $h = .1$ $h' = 0.1$

B	L(0)	L'(0)	P(A)
1.5	35.80	1.37	0.96318984
1.4	49.97	1.38	0.973210394
1.3	77.53	1.38	0.982480228
1.2	151.21	1.39	0.990900457
1.1	868.41	1.39	0.998397231

TABLE – 5.14

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 4$
 $k = 1$ $h = .15$ $h' = 0.15$

B	L(0)	L'(0)	P(A)
2.4	42.64	1.30	0.970379412
2.3	54.87	1.30	0.9767735
2.2	78.09	1.31	0.983522892
2.1	139.08	1.31	0.990656197
2	731.58	1.32	0.998204887

TABLE – 5.10

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 3$
 $k = 1$ $h = .15$ $h' = 0.15$

B	L(0)	L'(0)	P(A)
1.7	30.80	1.70	0.94760406
1.6	40.77	1.73	0.95939517
1.5	57.96	1.75	0.970742106
1.4	93.72	1.77	0.981494844
1.3	208.67	1.79	0.991513908

TABLE – 5.15

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 2$ $\theta = 4$
 $k = 1$ $h = .18$ $h' = 0.18$

B	L(0)	L'(0)	P(A)
1.3	28.73	1.15	0.961388111
1.2	39.19	1.15	0.971384704
1.1	59.93	1.15	0.98109442
1	119.46	1.16	0.990422904
0.9	1586.23	1.16	0.999272048

TABLE -5. 11

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 3$
 $k = 1$ $h = .18$ $h' = 0.18$

B	L(0)	L'(0)	P(A)
1.8	32.22	2.00	0.941628754
1.7	42.87	2.04	0.954631627
1.6	61.75	2.08	0.967447937
1.5	103.25	2.12	0.979908168
1.4	261.84	2.15	0.991838455

TABLE -5. 16

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 2$ $\theta = 4$
 $k = 1$ $h = .2$ $h' = 0.2$

B	L(0)	L'(0)	P(A)
1.3	26.59	1.16	0.958111882
1.2	35.53	1.16	0.968302429
1.1	52.22	1.16	0.978206515
1	93.65	1.16	0.987727225
0.9	358.44	1.16	0.996763587

TABLE – 5.12

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 3$
 $k = 1$ $h = .2$ $h' = 0.2$

B	L(0)	L'(0)	P(A)
1.9	31.59	2.23	0.934011221
1.8	41.80	2.29	0.948025286
1.7	59.83	2.35	0.962161124
1.6	99.26	2.41	0.976254523
1.5	248.07	2.48	0.990120828

TABLE -5. 17

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 4$
 $k = 1$ $h = .1$ $h' = 0.1$

B	L(0)	L'(0)	P(A)
1.5	71.11	1.27	0.982398272
1.4	100.98	1.28	0.987517834
1.3	160.33	1.28	0.992089987
1.2	328.08	1.28	0.996113479
1.1	3174.99	1.28	0.999596536

TABLE – 5.13

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 4$
 $k = 1$ $h = .1$ $h' = 0.1$

B	L(0)	L'(0)	P(A)
2	36.50	1.19	0.968492866
1.9	47.33	1.19	0.975481808
1.8	68.40	1.19	0.98287493
1.7	127.29	1.19	0.990705967
1.6	1210.77	1.20	0.999012649

TABLE – 5.18

Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 4$
 $k = 1$ $h = .15$ $h' = 0.15$

B	L(0)	L'(0)	P(A)
1.7	55.45	1.49	0.973768413
1.6	74.35	1.50	0.980227172
1.5	107.27	1.51	0.986159563
1.4	177.06	1.51	0.991540849
1.3	414.30	1.52	0.99635613

TABLE -5. 19
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 4$
 $k = 1$ $h = .18$ $h' = 0.18$

B	L(0)	L'(0)	P(A)
1.8	53.87	1.67	0.9699561
1.7	72.10	1.68	0.977241218
1.6	104.06	1.69	0.984028697
1.5	172.97	1.70	0.99027878
1.4	420.76	1.71	0.995961547

TABLE – 5.20
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 3$ $\theta = 4$
 $k = 1$ $h = .2$ $h' = 0.2$

B	L(0)	L'(0)	P(A)
1.9	49.69	1.80	0.964952171
1.8	65.62	1.82	0.973017931
1.7	92.88	1.83	0.980635285
1.6	148.96	1.85	0.98775053
1.5	325.45	1.86	0.994318485

TABLE – 5.21
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 3$
 $k = 3$ $h = .18$ $h' = 0.18$

B	L(0)	L'(0)	P(A)
6.9	205.96	1.41	0.99321413
6.8	257.20	1.41	0.994554937
6.7	346.48	1.41	0.995949388
6.6	541.01	1.41	0.997400165
6.5	1293.02	1.41	0.998909712

TABLE -5. 22
 Values of ARL's and Type C-OC Curves when
 $b = 1$ $\alpha = 1$ $\theta = 3$
 $k = 3$ $h = .2$ $h' = 0.2$

B	L(0)	L'(0)	P(A)
8	303.34	1.47	0.99518317
7.9	377.19	1.47	0.996120453
7.8	504.25	1.47	0.997093558
7.7	774.22	1.47	0.998103976
7.6	1737.03	1.47	0.999153495

VI. CONCLUSIONS

At the hypothetical values of the parameters b, α, θ, k, h and h' are given at the top of each table, we determine optimum truncated point B at which P (A) the probability of accepting an item is maximum and also obtained ARL's values which represents the acceptance zone L(0) and rejection zone L (0) values. The values of truncated point B of random variable

X, L (0), L (0) and the values for Type-C Curve, i.e. P (A) are given in columns I, II, III, and IV respectively.

From the above tables 5.1 to 5.22 we made the following conclusions:

1. From the tables 5.1 to 5.22 it is observed that the values of P (A) are increased as the value of truncated point decreases. Thus, the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
2. From the tables 5.1 to 5.22 it is observed that the value of L (0) and P (A) is increased as the value of truncated point decreases thus the truncated point of the random variable and the various parameters for CASP-CUSUM are related.
3. From the tables 5.1 to 5.22 it is observed that the minimum and maximum truncated points are 0.9 and 7.6 corresponding to highest probabilities for the various parameters for CASP-CUSUMs.
4. The various relations exhibited among the ARL's and Type-C OC curves with the parameters of the CASP-CUSUM based on the above tables 5.1 to 5.22 are observed from the following Table 6.2.

Table – 6.2

b	α	θ	$h=h'$	K	B	L(0)	L'(0)	P(A)
1	1	3	0.1	1	1.7	2050.190920	1.252885	0.999389291
1	1	3	0.15	1	2.3	227.856380	1.409673	0.993851364
1	1	3	0.18	1	2.7	309.667390	1.518143	0.995121419
1	1	3	0.2	1	3.0	564.986880	1.597215	0.997180998
1	2	3	0.1	1	1.0	272.186160	1.117899	0.995909691
1	2	3	0.15	1	1.0	110.521390	1.155018	0.989657462
1	2	3	0.18	1	0.9	1802.624150	1.170459	0.999351084
1	2	3	0.2	1	0.9	235.505570	1.177348	0.995025635
1	3	3	0.1	1	1.1	868.405030	1.394088	0.998397231
1	3	3	0.15	1	1.3	208.667360	1.785931	0.991513908
1	3	3	0.18	1	1.4	261.839510	2.154607	0.991838455
1	3	3	0.2	1	1.5	248.066280	2.475144	0.990120828
1	1	4	0.1	1	1.6	1210.771850	1.196605	0.999012649
1	1	4	0.15	1	2.0	731.584410	1.315632	0.998204887
1	2	4	0.18	1	0.9	1586.228030	1.155515	0.999272048
1	2	4	0.2	1	0.9	358.442260	1.163844	0.996763587
1	3	4	0.1	1	1.1	3174.985110	1.281589	0.999596536
1	3	4	0.15	1	1.3	414.297790	1.515163	0.99635613
1	3	4	0.18	1	1.4	420.757020	1.706099	0.995961547
1	3	4	0.2	1	1.5	325.449070	1.859601	0.994318485
1	1	3	0.18	3	6.5	1293.018680	1.411304	0.998909712
1	1	3	0.2	3	7.6	1737.034550	1.471616	0.999153495

From the above table 6.2, we conclude that the optimum CASP-CUSUM schemes for which the values of ARL and P (A) reach their maximum i.e., 3174.99, **0.999596536** respectively, is

$$\left[\begin{array}{l} B = 1.1 \\ b = 1 \\ \alpha = 3 \\ \theta = 4 \\ k = 1 \\ h = 0.1 \\ h' = 0.1 \end{array} \right]$$

REFERENCES

- [1]. Hawkins, D.M. (1992). "A Fast Accurate Approximation for Average Lengths of CUSUM Control Charts". *Journal on Quality Technology*, Vol. 24(No.1): 37-43.
- [2]. Kakoty, S and Chakravaborthy, A.B., (1990), "A Continuous Acceptance Sampling Plan for Truncated Normal distribution based on Cumulative Sums", *Journal of National Institution for Quality and Reliability*, Vol.2 (No.1): 15-18.
- [3]. Vardeman, S. And Di-ou Ray. (1985). "Average Run Length for CUSUM schemes where observations are Exponentially Distributed", *Technometrics*, vol. 27 (No.2):145-150.
- [4]. Lonnie, C. Vance. (1986). "Average Run Length of CUSUM Charts for Controlling Normal means". *Journal of Quality Technology*", Vol.18:189-193.
- [5]. Akhtar, P. Md. and Sarma. K.L.A.P.(2004). "Opimization of CASP-CUSUM Schemes based on Truncated Gamma Distribution". *Bulletin of Pure and applied sciences*, Vol-23E (No.2):215-223.
- [6]. Narayana Muthy, B. R, Akhtar, P. Md and Venkataramudu, B.(2012) "Optimization of CASP- CUSUM Schemes based on Truncated Log-Logistic Distribution". *Bulletin of Pure and Applied Sciences*, Vol-31E (Math&Stat.): Issue (No.2) pp243-255.
- [7]. B.Sainath, P.Mohammed Akhtar, G.Venkatesulu, and Narayana Muthy, B. R, (2016) "CASP CUSUM Schemes based on Truncated Burr Distribution using Lobatto Integration Method". *IOSR Journal of Mathematics (IOSR-JM)*, Vol-12, Issue 2, pp54-63.
- [8]. Venkatesulu.G, Akhtar.P.Md, Sainath.B, Narayana Murthy B.R.(2017), "Truncated Gompertz Distribution and its Optimization of CASP-CUSUM Schemes" *Journal of Research in applied Mathematics*, Vol-3, Issue7, pp:19-28.
- [9]. Venkatesulu.G, Akhtar.P.Md, Sainath.B, Narayana Murthy B.R.(2018), "Continuous Acceptance sampling plans for Truncated Lomax Distribution Based on CUSUM schemes" *International Journal Mathematics Trends and Technology (IJMTT) – Vol -55 , No-3*.
- [10]. M. H. Tahir, Gauss M. Cordeiro, Ayman Alzaatreh, M. Mansoor and M. Zubair. "A New Weibull-Pareto Distribution: Properties and Applications". Vol-45, 2016.
- [11]. Beattie, B.W.(1962). "A Continuous Acceptance Sampling procedure based upon a cumulative Sums Chart for a number of defective". *Applied Statistics*, Vol. 11 (No.2): pp: 137-147.
- [12]. Page, E.S.,(1954) "Continuous Inspection Schemes", *Biometrika*, Vol. XLI, pp. 104- 114.

AUTHORS PROFILE

Mr. M. C. Chandra Prasad is pursuing Ph. D. in Statistics from Sri Krishnadevaraya University, Anantapuramu, Andhra Pradesh, India. His research topic is "CASP – CUSUM Schemes based on Truncated Mixed Weibull Distributions". He has 5 years of experience in teaching Statistics for Graduates and CA-CPT students. He has published a paper in reputed international journal.

Prof. P. Md. Akhtar received Ph. D. degree in Statistics in 1993 from Sri Krishnadevaraya University, Anantapuramu, Andhra Pradesh, India. He taught the subjects Statistical Quality Control, Time Series and Forecasting, Theory of Estimation, Computer Programming etc, to the students of M.Sc., Statistics. He produced four Ph. D., degrees in Statistics and one Ph. D., degree in Computer Science and Technology. He published more than 35 research papers in various reputed journals.