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Statistical Model for Poverty Index

S. S. Singh[1*], S. S. Singh[2], S. P. Singh[3]

[1,2,3]Department of Statistics, Manipur University, Imphal-795003, India

Corresponding Author: salamchandra@gmail.com Tel.:91-8731920833

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Abstract - The incidence of poverty measured in Poverty Gap (PG) introduced by Foster-Greer-Thorbecke (FGT) is adopted as poverty indicator and the Lorenz curve and Gini-concentration co-efficient were used to explain the income inequality of households in the selected village. A village, called Thanga Karang comprising of 257 households, is considered as selected village. Household survey was conducted to collect the data related to poverty. A statistical analysis of PG is carried out and fitted to the beta distribution with parameters α and β by method of moments and the method of maximum likelihood. The statistically significant values of α and β are respectively $\hat{\alpha}$= 0.2989 and $\hat{\beta}$= 3.4722 by method of moments; $\hat{\alpha}$= 1.459 and $\hat{\beta}$= 3.001 by method of maximum likelihood. Therefore the PG, an index of poverty is best fit to the assumed model (i.e. Beta distribution). The Gini-concentration co-efficient is found to be 0.5253. It suggests that income inequality among the household is observed in 52.53 percentages.

***Keywords*:** Poverty, Head Count Ratio, Poverty Gap, Lorenz curve and Gini-concentration co-efficient.

I. INTRODUCTION

Poverty is regarded as a universal phenomenon because of its prevalence all over the world. It is said to be as old as the human civilization and exists in different forms. In recent times, poverty has gained an increasing importance and concern of the world development authorities and the United Nations. Certain poverty reduction programs are being taken up at present in accordance to the changing scenario of global poverty. As the world progresses and population increases, the concept and dimension of poverty itself has changed a lot. Also poverty differs from region to region and from one country to another depending on the available resources, the value of money, mode of income facilities and according to the standard of living. As a matter of fact, poverty is of different forms and is definitely multidimensional in nature. The World Bank Organization describes poverty as:

Poverty has many faces, changing from place to place and across time, and has been described in many ways. Most often, poverty is a situation people want to escape. So poverty is a call to action for the poor and wealthy alike, a call to change the world so that many more may have enough to eat, adequate shelter, access to education and health, protection from violence, and a voice in what happens in their communities (2008-2009).

Poverty is generally measured on the index of income and income related indices such as housing, education, consumption, health and social assets which function as primary variables. Poverty depends largely on the lack of income which can be either low income or inadequate income expenditures. The multidimensional approach to poverty is not a new way of studying poverty and it has been used, observed and studied by many economists and critics alike. In the study of multidimensional poverty, the concept of income and income inequality remains an important factor. The dimensions of income, living standard, education, health and social security are taken up as important indices of studying poverty which works on the FGT unidimensional measure of Alkire-Foster methodology [7]. Income and household assets are the traditional measures associated with poverty measurement and the measures of international poverty line, mortality rate and human development index (HDI) provides a more realistic picture of poverty assessment [1]. Coming to the Indian context, India being a land of diverse cultural and geographical variations, the occurrence of poverty differs from one state to another as is also the urban and rural poverty. Two alternative measures of poverty i.e. the Head–count ratio and Foster-Greer-Thorbeck (FGT) index measures the extent and depth of poverty and consumption inequalities among the SC’s and ST’s, the rural-urban differences and inter-state variations [3]. The rise and decline of Indian poverty is examined on the factors of headcount ratio and size of the poor population by using the measures of poverty gap index (PGI) and squared poverty gap index [6]. Looking in the problem of poverty in Manipur both in the hill and valley districts, it has been found that the factors of unemployment and lack of or low level of agricultural land holdings prove to be the key factors for the people to be in poverty [5]. Identifying the poor from the non-poor requires many factors and income remains one of the most important tools for the classification of poverty. Salam Sovachandra Singh made a study on the measurement of poverty which is an income level supposed to be just enough to avoid inadequate consumption. In simple words, it is an abstraction which is essential to measure the extent of poverty in a given country based on family size, location and characteristics of the head of the household [4].

Fish is not only an important source of income but it also has great significances in the social and cultural life of the people of Manipur. The island village of Thanga Karang is basically backward in certain sectors of health, transport, communication, education and fishing remains as the main occupation and source of income for the villagers. With the deteriorating eco-system of the Loktak Lake, the decreasing fish population due to the construction of Ithai Barrage and the increasing number of fishing hands, the socio-economic condition of the people of Thanga Karang is still very poor. Majority of the fish productions of the state are supplied by the fishing folks who inhabit in and around the Loktak Lake, which is the largest fresh water lake in North East India. Since there is no road connectivity to Thanga Karang, it remains an isolated island and slow in its development. Waterways are the only mode of transportation currently available in this area. Taking into consideration of the role of fishing of this selected village, the present work is an attempt to study the poverty of the fishing folks of Thanga Karang based on their income. The objective of the study is to determine the probability distribution of poverty indicator and to evaluate the income inequality among households. The analysis of poverty is based only on the income and financial level of the people of Thanga Karang.

II. DATA

Data were collected from the households located in Thanga Karang island. The study is mainly based on the primary sources of data which were collected through questionnaire interviews. The total 257 household was taken up with census survey method.

III. METHOD

The incidence of poverty or HCR (Head Count Ratio) and the PG (Poverty Gap) can be obtained by the generalized measures of poverty introduced by Foster-Greer-Thorbecke (FGT) (1984) [2] Poverty measure are defined as $F(\alpha, t) = \frac{1}{N}\sum_{j=1}^{N}\left(\frac{t-Y_j}{t}\right)^{\alpha} I(Y_j \le t)$. Here, Y_j is a measure of income for individual/household j, N is the no. of individuals/households and α is sensitively parameter. Setting $\alpha = 0$ defines the HCR, F(0,t) whereas setting $\alpha = 1$ defines PG, F(1,t).

Poverty gap (PG) as a measure of poverty is treated as incidence (index) of poverty. Since its value ranges from 0 to 1, the suitable probability distribution of PG is Beta-distribution with parameters (α, β). The probability model of PG is given as follows:

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1;\ \alpha, \beta > 0 \quad \text{---------------- 1)}$$

By method of moments, the estimates of α and β for the beta distribution are given by $\hat{\beta} = \frac{\alpha(1-\bar{x})}{\bar{x}}$ --------- (2)

And $\hat{\alpha} = \bar{x}\left[\frac{\bar{x}(1-\bar{x})}{s^2} - 1\right]$ ---------------- (3)

Where the $\bar{x} = Sample\ mean$ and $s^2 = Sample\ variance$

Maximum Likelihood Method:

Let $x_1, x_2, x_3, x_4, \ldots\ldots.. x_n$ be a random sample from Beta distribution B(α, β).

$$\therefore f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$$

The likelihood function of the sample observations is

$$L(\alpha,\beta) = \frac{\lceil(\alpha+\beta)^n}{(\lceil\alpha)^n(\lceil\beta)^n}\prod_{i=1}^{n} x_i^{\alpha-1}\prod_{i=1}^{n}(1-x_i)^{\beta-1}$$

$$\therefore \log L(\alpha,\beta) = n\lceil(\alpha) - n\log\lceil\alpha+\beta - n\log\lceil(\alpha) - n\log\lceil(\beta) + (\alpha-1)\sum_{i=1}^{n}\log x_i + (\beta-1)\sum_{i=1}^{n}\log(1-x_i) \ldots\ldots\ldots\ldots (1)$$

The Likelihood equations are $\frac{\partial}{\partial\alpha}\log(\alpha,\beta) = 0$

$\Longrightarrow \frac{n}{\lceil(\alpha+\beta)}\frac{\partial}{\partial\alpha}\lceil(\alpha+\beta) - \frac{n}{\lceil(\alpha)}\frac{\partial}{\partial\alpha}\lceil\alpha + \sum_{i=1}^{n}\log x_i = 0 \ldots\ldots\ldots\ldots\ldots (2)$

And $\frac{\partial}{\partial\beta}\log L(\alpha,\beta) = 0$

$\Longrightarrow \frac{n}{\lceil(\alpha+\beta)}\frac{\partial}{\partial\beta}\lceil(\alpha+\beta) - \frac{n}{\lceil(\beta)}\frac{\partial}{\partial\beta}\lceil(\beta) + \sum_{i=1}^{n}\log(1-x_i) = 0 \ldots\ldots\ldots\ldots. (3)$

But $\lceil(\alpha) = (\alpha-1)! \cong (\alpha-1)^{\alpha-1}e^{-(\alpha-1)}\sqrt{2\pi(\alpha-1)}$ $\quad [\because n! \cong n^n e^{-n}\sqrt{2\pi n}\,]$

$$= \sqrt{2\pi}\,(\alpha-1)^{\alpha-1}\left[e^{-(\alpha-1)}(\alpha-1)^{\frac{1}{2}}\right]$$

$$\therefore \frac{\partial}{\partial\alpha}\lceil\alpha \cong \sqrt{2\pi}\left[(\alpha-1)^{\alpha-1}\left\{-(\alpha-1)e^{-(\alpha-1)}(\alpha-1)^{\frac{1}{2}} + e^{-(\alpha-1)}\frac{1}{2}(\alpha-1)^{-\frac{1}{2}}\right\} + e^{-(\alpha-1)}(\alpha-1)^{\frac{1}{2}}(\alpha-1)^{\alpha-1}\{\log(\alpha-1)+1\}\right]$$

$$= \sqrt{2\pi}(\alpha-1)^{\alpha-1}e^{-(\alpha-1)}(\alpha-1)^{\frac{1}{2}}\left[-(\alpha-1) + \frac{1}{2}(\alpha-1)^{-1} + \log(\alpha-1) + 1\right]$$

$$\therefore \frac{1}{\lceil\alpha}\frac{\partial}{\partial\alpha}\lceil\alpha = \frac{\sqrt{2\pi}(\alpha-1)^{\alpha-1}e^{-(\alpha-1)}(\alpha-1)^{\frac{1}{2}}\left[-(\alpha-1) + \frac{1}{2(\alpha-1)} + \log(\alpha-1) + 1\right]}{\sqrt{2\pi}(\alpha-1)^{\alpha-1}e^{-(\alpha-1)}(\alpha-1)^{\frac{1}{2}}}$$

$$\Longrightarrow \frac{1}{\lceil\alpha}\frac{\partial}{\partial\alpha}\Big\lceil\alpha = \log(\alpha-1) + 1 + \frac{1}{2(\alpha-1)} - (\alpha-1)$$

Similarly = $\frac{1}{\beta}\frac{\partial}{\partial\beta}\lceil\beta = \log(\beta-1) + 1 + \frac{1}{2(\beta-1)} - (\beta-1)$

And $= \frac{1}{\lceil\alpha+\beta}\frac{\partial}{\partial\alpha}\Big\lceil(\alpha+\beta) = \log(\alpha+\beta-1) + 1 + \frac{1}{2(\alpha+\beta-1)} - (\alpha+\beta-1)$

$$= \frac{1}{\lceil\alpha+\beta}\frac{\partial}{\partial\beta}\Big\lceil(\alpha+\beta) = \log(\alpha+\beta-1) + 1 + \frac{1}{2(\alpha+\beta-1)} - (\alpha+\beta-1)$$

Equation (2) becomes

$$\Longrightarrow n\left[\log(\alpha+\beta-1) + 1 + \frac{1}{2(\alpha+\beta-1)}(\alpha+\beta-1)\right] - n\left[\log(\alpha-1) + 1 + \frac{1}{2(\alpha-1)} - (\alpha-1)\right] + \sum_{i=1}^{n}\log x_i = 0 \ldots\ldots\ldots\ldots\ldots. (4)$$

$$\Rightarrow n\left[\log(\alpha+\beta-1)+1+\frac{1}{2(\alpha+\beta-1)}(\alpha+\beta-1)\right]-n\left[\log(\beta-1)+1+\frac{1}{2(\beta-1)}-(\beta-1)\right]$$
$$+\sum_{i=1}^{n}\log(1-x_i)=0\ldots\ldots\ldots\ldots(5)$$

Subtracting (4) from (5), we get

$$\Rightarrow n\left[\log(\alpha-1)+1+\frac{1}{2(\alpha-1)}-(\alpha-1)\right]-n\left[\log(\beta-1)+1+\frac{1}{2(\beta-1)}-(\beta-1)\right]+\sum_{i=1}^{n}\log(1-x_i)-\sum_{i=1}^{n}\log x_i=0$$

$$\Rightarrow \left[\log(\alpha-1)+1+\frac{1}{2(\alpha-1)}-(\alpha-1)\right]-\left[\log(\beta-1)+1+\frac{1}{2(\beta-1)}-(\beta-1)\right]+\frac{1}{n}\sum_{i=1}^{n}\log(1-x_i)-\frac{1}{n}\sum\log x_i=0$$

$$\Rightarrow \left[\log(\alpha-1)+1+\frac{1}{2(\alpha-1)}-(\alpha-1)\right]-\left[\log(\beta-1)+1+\frac{1}{2(\beta-1)}-(\beta-1)\right]-0.09309+0.79513$$
$$=0\ldots\ldots\ldots\ldots(6)$$

Gini Coefficient and Lorenz Curve:

Gini coefficient is defined as the ratio of the inequality area and the area under the curve of complete equal distribution.

Gini Coefficient $=\frac{area\ between\ the\ curve\ of\ complete\ equal\ distribution\ and\ lorenz\ curve}{Area\ under\ the\ curve\ of\ complete\ equal\ distribution}$

It can be computed graphically by applying following formula,

Gini Coefficient $=\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|Y_i-Y_j|}{2n^2\mu}$

Where, $Y_i-Y_j = difference\ between\ income\ of\ every\ possible\ pair\ of\ individuals, n = number\ of\ poor\ and\ \mu = mean\ size\ income$

The Lorenz curve is a graphical representation of the degree of equality/inequality in the distribution of income.

IV. ANALYSIS & FINDINGS

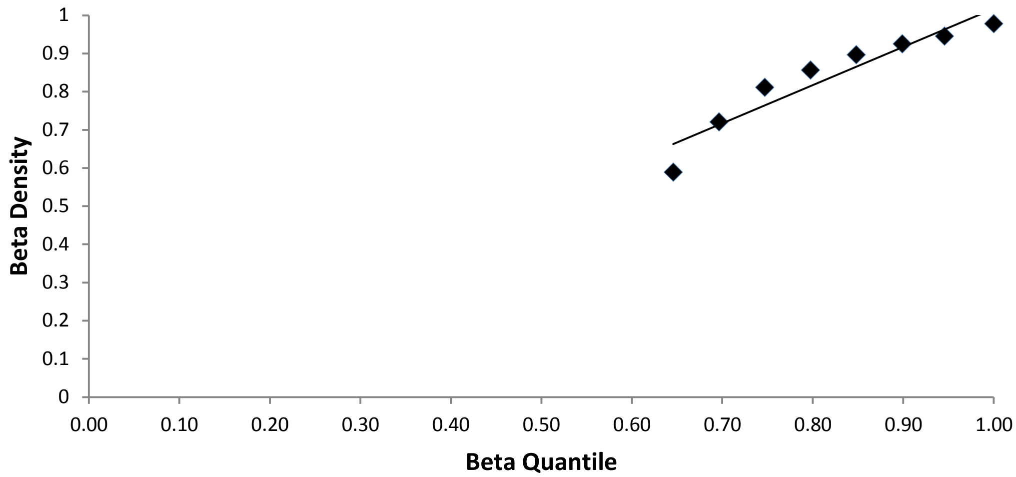
Now we have, $\bar{x}=0.079270046$, $s^2=0.015297356$

By method of moments, the estimates of α and β for the beta distribution using (2) and (3) are respectively $\hat{\alpha}=0.298941$ and $\hat{\beta}=3.47223$.

Substituting the estimated values of $\hat{\alpha}$ and $\hat{\beta}$ in the model (1), we have

$$f(x)=\frac{\lceil(0.298941+3.47223)}{\lceil(0.298941)\lceil(3.47223)}x^{0.298941-1}(1-x)^{3.47223-1}$$

$$=\frac{\lceil(3.7712)}{\lceil(0.298941,\lceil(3.47223)}x^{-0.7011}(1-x)^{2.47223}$$



Goodness of Fit Plot For Beta Distribution

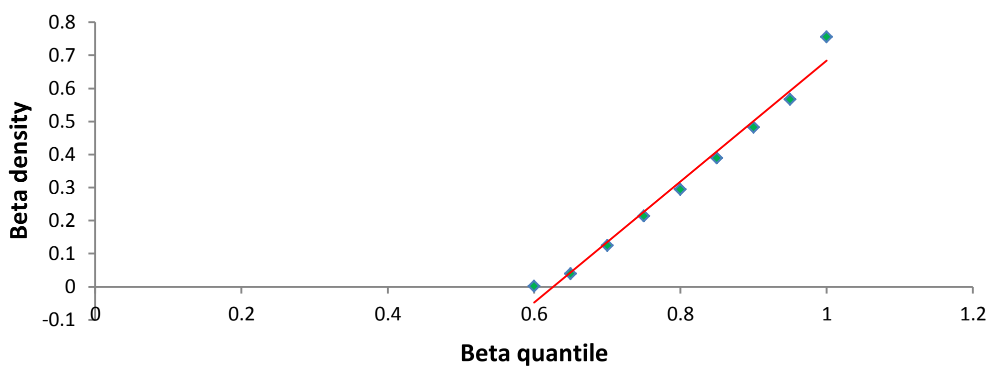
Goodness of fit for fitted beta distribution on poverty gap (PG) is constructed by plotting the cumulative probability curve with respect to quantiles of beta distribution. From the chart, it is observed that almost all points are nearer to the straight line drawn of quantiles of the distribution as shown in figure. Therefore the poverty gap, an index of poverty, is best fit to the Beta distribution with parameters $\hat{\alpha} = 0.298941$ and $\hat{\beta} = 3.47223$.

By method of maximum likelihood using (5) and (6), the estimates of α and β are obtained as $\hat{\alpha} = 1.459$ *and* $\hat{\beta} = 3.001$ respectively. The fitted model of poverty gap is given below:

$$\therefore f(x) = \frac{\lceil 4.460}{\lceil 1.459 \lceil 3.001} x^{1.459-1}(1-x)^{3.001-1}$$

$$= \frac{\lceil 4.460}{\lceil 1.459 \lceil 3.001} x^{0.459}(1-x)^{2.001}$$

Goodness of Fit:



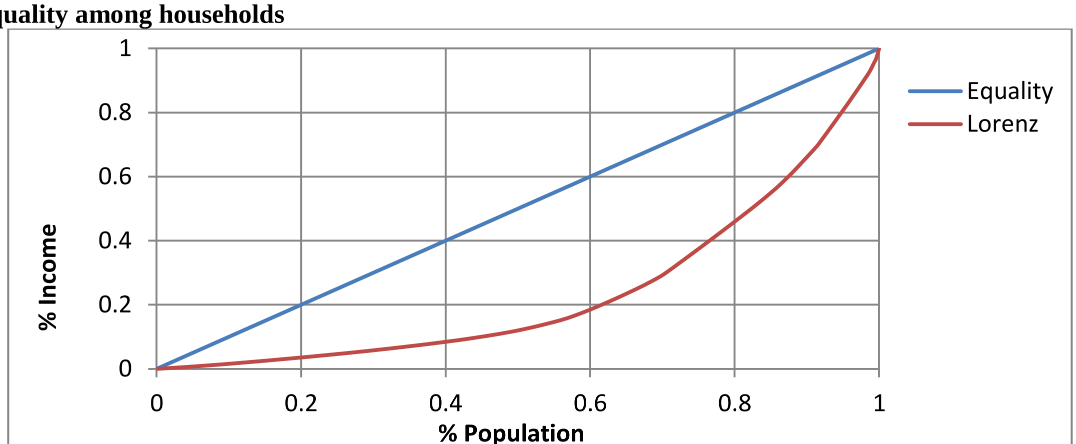
Goodness of Fit Plot for Beta Distribution

From the chart, it is observed that almost all points are nearer to the straight line drawn of quantiles of the distribution as shown in figure. Therefore the poverty gap, an index of poverty, is best fit to the Beta distribution with parameters $\hat{\alpha} = 1.459$ and $\hat{\beta} = 3.001$.

Incidence of poverty

In the present study, the HCR is found to be 0.4047 for households and for individuals is 0.4701. On the other hand, the Poverty Gap (PG) for households is 0.0793 and it is 0.0997 for individuals.

Income Inequality among households



Lorenz Curve for Income Inequality

Gini Coefficient = 0.525332

The Gini-concentration co-efficient is found to be 0.5253. It suggests that income inequality among the household is observed in 52.53percentage.

V. CONCLUSION

The probability distribution of poverty gap is best fit to the beta distribution and parameters of the distribution are estimated by methods of moments and maximum likelihood. The Gini-concentration coefficient and Lorenz's curve give clear picture of income inequality among the households of the study area. Income inequality is observed in above 50 percent of households in the village.

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