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Optimum Process Mean Setting for Product with Rework Process Under second order autocorrelation

Vispute S[1*].and Singh J. R.[2]

[1*,2] School of Studies in Statistics, Vikram University Ujjain, India 456010

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Abstract: In Paper the effects of second order autocorrelation on the determination of the optimum process mean in statistical process control. It directly affects the process defective rate, production cost, scrap cost, and rework cost. Lee et al.(2000) presented a filling problem for determining the optimum process mean and screening limits. They considered three grades of product, assumed a normal quality characteristic, and adopted the piecewise linear profit function for measuring the profit per item. However, they have not included the scrap cost and the perfect rework process in their model. In this chapter, we further propose a modified Lee et al.'s model with rework process for determining the optimum process mean under second order autocorrelation when the roots are (i) real and distinct (ii) real and equal and (iii) complex conjugate. Both perfect rework and imperfect rework processes for the product are considered in the model. Negative autocorrelation and positive autocorrelation are seriously affected on optimum mean and expected profit.

Keywords: Optimum Process Mean, Autocorrelation, Scrap Cost Nomenclature

Introduction: The optimum process mean setting has been a major topic in modern statistical process control. It may not be equal to the target value because the costs of below and above the specification limits are different. The determination of the optimum process mean should achieve the minimum expected cost per item or the maximum expected profit per item. There is considerable attention paid to the study of economic selection of the process mean. Recently, Li (1997, 2002), Li and Chirng (1999), Li and Cherng(2000), Li and Chou (2001), Li and Wu (2001, 2002), Wu and Tang (1998), Maghsoodloo and Li (2000), and Phillips and Cho (2000), have addressed different problems of unbalanced tolerance design with the asymmetric quadratic and linear quality loss functions. The piecewise linear profit function of the quality characteristic is usually applied in the filling/canning problem for determining the optimum manufacturing target and other important parameters, see for example, Springer (1951), Hunter and Kartha (1977), Carlsson (1984, 1989), Bisgaard et al. (1984), Golhar (1987, 1988), Golhar and Pollock (1988, 1992), Rahim and Banerjee (1988), Arcelus and Rahim (1990), Boucher and Jafari (1991), Al-Sultan (1997), Pulak and Al-Sultan (1996), Al-Sultan and Al-Fawzan (1997), Al-Sultan and Pulak (1997), Lee and Jang (1997), Misiorek and Barnett (2000), Lee and Elsayed (2002), Lee et al. (2000, 2001), and Duffuaa and Siddiqi (2002). In Misiorek and Barnett's (2000) model, the aim is to fix the filling mean of the process in order to maximize the expected profit per container. The profit for a container depends on the filling value of the material, i.e, whether or not it is over-filled, under-filled, or rejected. Misiorek and Barnett's (2000) model considered that the expense of recapturing over-filled material is a cost per unit, the expense of emptying out under-filled containers and putting the material back into the process is a constant cost, and that the containers from under-filled items are discarded. They also considered the expected profit per container for the following four special cases: (1) the containers from under-filled items are discarded, (2) the containers from under-filled items are re-used, (3) the containers from under-filled items are discarded and there is no overflow, and (4) all under-filled containers are topped-up and over-flowed material is captured.Lee et al. (2000, 2001) presented the problem of a joint determination of optimum process mean and screening limits. The quality characteristic of the performance variable or surrogate variable is considered as the screening variable. Their models involved selling and discounted prices as well as production, inspection, rework and penalty costs. The normal and bivariate normal distributions are assumed and used in Lee et al.'s (2000, 2001) models. The screening of a product with three grades, using single stage screening and two stage screening are considered. The objective of their models is to maximize the expected profit per item.

For the filling/canning industry, the product needs to be produced within the specification limits. The manufacturing cost per unit considers the fixed and variable production costs and the constant inspection cost. The variable production cost is proportional to the value of the quality characteristic. A product usually cannot be sold at a higher price for the constant label content. If a product is above the upper specification limit (USL), it will cause an increment in the manufacturing cost. This is not a desirable situation for the production department. If a product is below the lower specification limit (LSL), it will cause a loss of goodwill for the manufacturer. The company may face

Corresponding Author: Vispute. S.

customers' claims or penalty due to government's laws. This is not a desirable case for the marketing department. Thus, the canning manufacturing industry needs product conformance. The penalty cost due to loss of goodwill is usually higher than the finite manufacturing cost. Hence, a product is usually put to scrap when it is below LSL and put to rework when it is above the USL. Taguchi (1986) proposed the optimum tolerance design with 100% inspection under the assumptions of no scrap and perfect rework for product. However, it is hard to get an overall perfect product in the production process. Hence, in our paper, we consider the possibility of the filled product having rework, either perfect or imperfect, and scrap.

Lee et al. (2000) proposed the inspection of three grades of product and adopted the piecewise linear profit function for measuring the profit per item. However, they have not included the scrap cost and perfect reprocessing in their model. In this paper, we propose a modified Lee et al.'s (2000) model with rework process for determining the optimum process mean. The production cost, inspection cost, rework cost and scrap cost are included in the modified model. Both perfect rework and imperfect rework process for the product are considered. A numerical example and sensitivity analysis of parameters are provided for illustration. Previous researchers addressed a product scrap that is sold at a reduced price in the market. Our modified model addressed the case that a scrapped product cannot be sold in the market, instead it involves a scrap cost. These are the main differences between our model and the Lee et al.'s (2000) model.

In chapter the effects of second order autocorrelation on the determination of the optimum process mean in statistical process control. It directly affects the process defective rate, production cost, scrap cost, and rework cost. Lee et al.(2000) presented a filling problem for determining the optimum process mean and screening limits. They considered three grades of product, assumed a normal quality characteristic, and adopted the piecewise linear profit function for measuring the profit per item. However, they have not included the scrap cost and the perfect rework process in their model. In this chapter, we further propose a modified Lee et al.'s model with rework process for determining the optimum process mean under second order autocorrelation when the roots are (i) real and distinct (ii) real and equal and (iii) complex conjugate. Both perfect rework and imperfect rework processes for the product are considered in the model. Negative autocorrelation and positive autocorrelation are seriously affected on optimum mean and expected profit.

Modified Lee et al.'s Model with Scrap Cost

Nomenclature

- a — the selling price in the modified Lee et al.'s model
- a_1 — the selling price for the primary market in the Lee et al.'s model
- a_2 — the selling price for the secondary market in the Lee et al.'s model
- b — the fixed production cost per item
- c — the variable production cost per item
- n is the sample size
- $E(TP_1)$ — the expected profit per item for the perfect rework model
- $E(TP_2)$ — the expected profit per item for the imperfect rework model
- f (y) — the normal probability density function

$$f(y)=\frac{1}{\sqrt{2\pi\sigma}}e^{\frac{1}{2}(\frac{y-\mu}{\sigma^2})^2},-\infty<y<\infty$$

- *i* — the inspection cost per item
- L_1 — the pre-specified specification limit for the item with grade A in the Lee et al.'s model
- M_1 — the upper specification limit in the modified Lee et al.'s model
- L_2 — the pre-specified specification limit for the item with grade B in the Lee et al.'s model
- M_2 — the lower specification limit in the modified Lee et al.'s model
- $P(y)$ — the profit per item for the Lee et al.'s model
- $P(yr)$ — the profit for a rework item for the Lee et al.'s model
- *r* — the rework cost for the item with grade C in the Lee et al.'s model and the rework cost in the modified Lee et al.'s model
- *s* — the scrap cost in the modified Lee et al.'s model
- TP_1 — the profit per item for the perfect rework
- TP_2 — the profit per item for the imperfect rework model
- *y* — the quality characteristic of the performance variable
- *yr* — the quality characteristic of a reworked item, Where it is assumed that *y* and *yr*are independent and identically distributed
- μ — the unknown process mean
- σ — the known process standard deviation
- $\Phi(z)$ — The cumulative probability of a standard normal random variable with a probability density function

In Lee et al.'s (2000) model, which considers performance as variable, the objective is to maximize the expected profit per item and obtain the optimum process mean. The profit for an item depends on the value of a normal quality characteristic, Y. Each item is classified into three grades A, B, and C. Grade A items are sold to primary market and grade B items are sold to secondary market. Grade C items are reworked and the rework process is the same as the original production process. Let L_1 be the pre-specified specification limit for grade A and L_2 be the pre-specified specification limit for grade B, where $L_2< L_1$. If an item has $y > L_1$, it is sold at a fixed price a1 to the primary market. If an item has $L_2 \leq y \geq L_1$, it is sold at a fixed price a_2 ($< a_1$) to the secondary market. If an item has $y < L_2$, it is reworked by the same production process at a rework cost r ($< a_2 < a_1$).

It is assumed that the quality characteristic y is normally distributed with an unknown process mean μ and a known standard deviation σ. Let the production cost per item be b + cy, where b is the fixed production cost and c is the variable production cost per item. Let i be the inspection cost per item and yr be the quality characteristic of a reworked item. It is assumed that y and y are identically and independently distributed.

From Lee et al. (2000), we have the profit per item as follows:

$$p(Y)=\begin{cases} a_1-b-cy-i, & y>L_1 \\ a_2-b-cy-i, & L_2\le y\le L_1 \\ P(y_r)-r-i, & y<L_2 \end{cases} \qquad (1)$$

Assume that the reworked item has the same profit as the non-reworked item, i.e., $P(y) = P(y_r)$. Hence, the expected profit per item is

$$E[p(y)]=\int_{L_1}^{\infty}(a_1-b-cy-i)f(y)dy+\int_{L_2}^{L_1}(a_2-b-cy-i)f(y)dy +\int_{L_2}^{L_1}(E[P(y)]-r-i)f(y)dy \qquad (2)$$

where$f(y)$ is the normal probability density function of Y.
From Lee et al. The above Eq. (2) can be rewritten as

$$E[P(y)]=\{a_1\Phi\left(\frac{\mu-L_1}{\sigma/\sqrt{n}}\right)+a_2\left[\Phi\left(\frac{L_1-\mu}{\sigma/\sqrt{n}}\right)-\Phi\left(\frac{L_2-\mu}{\sigma/\sqrt{n}}\right)\right]-(b-c\mu)\Phi\left(\frac{\mu-L_2}{\sigma/\sqrt{n}}\right)-r\Phi\left(\frac{L_2-\mu}{\sigma/\sqrt{n}}\right) -c\sigma/\sqrt{n}\,\Phi\left(\frac{\mu-L_2}{\sigma/\sqrt{n}}\right)-i\}/\Phi\left(\frac{\mu-L_2}{\sigma/\sqrt{n}}\right) \qquad (3)$$

the first derivative of Eq. (3) with respect to μ, set it equal to zero, and adopted the bisection method for finding the optimal μ that maximizes the expected profit per item and multiple in equation (3) we obtain expected profit for n items .

All items are inspected prior to shipment to the customers. If an item has $Y>M_1$, it is reworked by the same production process at a rework cost r. Items with $Y<M_2$ are scrapped at a scrap cost s. Items with $M_2\le Y\ge M_1$ are shipped to the market at a price a. The production cost per item is linear in Y, that is $b + cy$, where b is the fixed production cost and c is the variable production cost. Let $i$be the inspection cost per item. Assume that the rework process is perfect. The perfect rework model, which include production, inspection, rework and scrap costs, is as follows:

$$TP_1=\begin{cases} a-b-cy-i-r, & y>M_1 \\ a-b-cy-i, & M_2\le y\le M_1 \\ 0-b-cy-i-s, & y<M_2 \end{cases} \qquad (4)$$

From Eq. (4), the expected profit per item is

$$E(TP_1)=\int_{M_1}^{\infty}(a-b-cy-i-r)f(y)dy+\int_{M_2}^{M_1}(a-b-cy-i)f(y)dy+\int_{-\infty}^{M_2}(0-b-cy-i-s)f(y)dy$$

$$=a-b-cy-i-r\left[1-\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)\right]-(a+s)\Phi\left(\frac{M_2-\mu}{\sigma/\sqrt{n}}\right) \qquad (5)$$

multiple in equation (5) we obtain expected profit for n items

To find the optimum process mean μ*, Eq. (5) is differentiated with respect to μ and set equal to 0, giving:

$$E'(TP_1)=-c+\frac{r}{\sigma/\sqrt{n}}\phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)+\frac{a+s}{\sigma/\sqrt{n}}\phi\left(\frac{M_2-\mu}{\sigma/\sqrt{n}}\right) \qquad (6)$$

If the second derivative of Eq. (5) is negative, that is,

$$\frac{a+s}{(\sigma/\sqrt{n})^3}(L_2-\mu)\phi\left(\frac{M_2-\mu}{\sigma/\sqrt{n}}\right)+\frac{r(M_1-\mu)}{(\sigma/\sqrt{n})^3}\phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)<0,$$

Then μ* is optimal. We use MSEXCEL for obtaining the optimum process mean.

Consider an imperfect process. The reworked product may be scrapped, perfect, or reworked again. The rework process can be continued. It is assumed that the quality characteristic of any reworked product is the same as that of the original process. The imperfect rework model, which include production cost, inspection cost, rework cost, and scrap cost is as follows:

$$TP_2=\begin{cases}E(TP_2)-b-cy-i-r, & y>M_1\\ a-b-cy-i, & M_2\le y\le M_1\\ 0-b-cy-i-s, & y<M_2\end{cases} \qquad (7)$$

From Eq. (7), the expected total cost per item is

$$E(TP_2)=\int_{M_1}^{\infty}[E(TP_2)-b-cy-i-r]f(y)+\int_{M_2}^{M_1}(a-b-cy-i)f(y)dy+\int_{\infty}^{M_2}(0-b-cy-i-s)f(y)$$

$$=E(TP_2)[1-\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)]-b-c\mu-i-r[1-\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)+a\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)-(a-s)\Phi\left(\frac{M_2-\mu}{\sigma/\sqrt{n}}\right)$$

Expected total cost for n items is given by

$$E(TP_2)=r+a-\frac{b+c\mu+i+r}{\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)}-\frac{(a+s)\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)}{\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)} \qquad (8)$$

To find the optimum process mean μ*, is differentiated with respect to μ and set equal to 0, giving:

$$E'(TP_2)=-\frac{[c-\frac{a+s}{\sigma/\sqrt{n}}\phi\left(\frac{M_2-\mu}{\sigma/\sqrt{n}}\right)]}{\Phi\left(\frac{M_1-\mu}{\sigma/\sqrt{n}}\right)}-\frac{[b+c\mu+i+r+(a+s)\Phi(\frac{M_2-\mu}{\sigma/\sqrt{n}})]\phi(\frac{M_1-\mu}{\sigma/\sqrt{n}})}{\sigma/\sqrt{n}[\Phi(\frac{M_1-\mu}{\sigma/\sqrt{n}})]^2} \qquad (9)$$

The second derivative also negative then μ* is optimum.

Modified Lee et al.'s Model with Scrap Cost under second order autocorrelation: Consider a manufacturing process where a quality characteristic is measured at equidistance time points 1, 2, 3, … n. This situation may occur in a discrete manufacturing process which produces discrete time 1, 2, 3 ... n, with one quality characteristic of interest. It may also occur in a continuous manufacturing process where the quality characteristic of interest is measured at discrete equidistant time points. We denote the behavior of the quality characteristic as x_1, x_2,....x_n. It will assumed that on EPC control action can be represented by some controllable variable or factor x_t, such that

$$x_t = \mu + \xi_t, \qquad (10)$$

Where μ is a constant, and ξ_t is a stationary time series with zero mean and standard deviation σ. A Durbin and Watson (1950) "d" statistic can be used to detect the presence or absence of serial correlation. The problem, however, is that to do once the suspicion of dependence via the serial correlation test is confirmed. If serial correlation exists we use identification techniques to define the nature of ξ_t. When identification is complete, the likelihood function can provide maximum likelihood estimate of the parameters of the identified model.

Suppose that a correlation test revealed the presence of data dependence and identification technique suggested autoregressive model of order two AR (2) say, then we can express ξ_t of equation (10) as

$$\xi_t = \alpha_1 \, \xi_{t-1} + \alpha_2 \, \xi_{t-2} + \in_k , \; t = 1,2, ... n \tag{11}$$

Where

(i) $\in_t \sim N \left(0, \; \sigma_{\in}^{\;2}\right)$

(ii) $\text{cov}\left(\in_t , \in_\gamma\right) = \begin{cases} \sigma_{\in}^{\;2} & t = \gamma \\ 0 & t \neq \gamma \end{cases}$

The Class of stationary models that assume the process to remain in equilibrium about a constant mean level μ. The variance of AR (2) process is given by:

$$\sigma^2 = \left(\frac{1-\alpha_2}{1+\alpha_2}\right) \frac{\sigma_{\in}^{\;2}}{\left[(1-\alpha_2)^2 - \alpha_1^{\;2}\right]}. \tag{12}$$

Following Kendall and Stuart (1976) it can be shown that for stationary, the roots of the characteristic equation of the process in equation (11)

$$\phi\,(B) = 1 - \alpha_1 \; B - \alpha_2 B^2 \tag{13}$$

must lies outside the unit circle, which implies that the parameters α_1 and α_2 must satisfy the following conditions :

$$\begin{aligned} \alpha_2 + \alpha_1 &< 1 \\ \alpha_2 - \alpha_1 &< 1 \\ -1 < \alpha_2 &< 1 \end{aligned} \tag{14}$$

Now If $G_1^{\;-1}$ and $G_2^{\;-1}$ are the roots of the characteristic equation of the process given by equation (13) then

$$G_1 = \frac{\alpha_1 + \sqrt{\alpha_1^{\;2} + 4\alpha_2}}{2} \tag{15}$$

$$G_2 = \frac{\alpha_1 - \sqrt{\alpha_1^{\;2} + 4\alpha_2}}{2} \tag{16}$$

For stationary we require that $\mid G_i \mid < 1, \;\; i \; = 1, \; 2.$ Thus, three situations can theoretically arise:

(i) Roots G_1 and G_2 are real and distinct $\left(i.e., \; \alpha_1^{\;2} - 4\alpha_2 > \; 0\right)$

(ii) Roots G_1 and G_2 are real and equal $\left(i.e., \; \alpha_1^{\;2} - 4\alpha_2 = \; 0\right)$

(iii) Roots G_1 and G_2 are complex conjugate $\left(i.e., \; \alpha_1^{\;2} - 4\alpha_2 < \; 0\right)$.

When the serial correlation is present in the data, we have for the distribution of the sample mean $\bar{x}$, its mean and variance is given by,

$$E\,(\bar{x}) = \mu$$

$$\text{Var}\left(\bar{x}\right) = \frac{\sigma^2}{n} \; \lambda_{ap}\left(\alpha_1, \alpha_2, \; n\right), \tag{17}$$

Where $\lambda_{ap}\left(\alpha_1, \alpha_2, n\right)$ depends on the nature of the roots G_1 and G_2 , and for different situations is given as follows :

(i) If G_1 and G_2 are real and distinct,

$$\begin{aligned} \lambda_{ap}\left(\alpha_1, \alpha_2, n\right) &= \left[\frac{G_1\left(1-G_2^2\right)}{\left(G_1-G_2\right)\left(1+G_1G_2\right)}\,\lambda\left(G_1, n\right) - \frac{G_2\left(1-G_1^{\;2}\right)}{\left(G_1-G_2\right)\left(1+G_1G_2\right)}\,\lambda\left(G_2, n\right)\right] \\ &= \lambda_{rd}\left(\alpha_1, \alpha_2, n\right), \end{aligned} \tag{18}$$

Where, $\lambda(G, n) = \left[\frac{1+G}{1-G} - \frac{2G}{n}\frac{(1-G^n)}{(1-G)^2}\right]$

(ii) If G_1 and G_2 are real and equal

$$\lambda_{ap}(\alpha_1, \alpha_2, n) = \left(\frac{1+G}{1-G}\right) - \frac{2G(1-G^n)}{n(1-G)^2}\left[1 + \frac{(1+G)^2(1-G^n) - n(1-G^2)(1+G^n)}{(1+G^2)(1-G^n)}\right]$$

$$= \lambda_{re}(\alpha_1, \alpha_2, n) \quad (19)$$

(iii) If G_1 and G_2 and complex conjugate

$$\lambda_{ap}(\alpha_1, \alpha_2, n) = \left[\gamma(d, u) + \frac{2d}{n}(W(d,u,n) + z(d,u,n))\right] \quad (20)$$

$$= \lambda_{cc}(\alpha_1, \alpha_2, n)$$

Where $\gamma(d, u) = \frac{1-d^4+2d(1-d^2)\cos u}{(1+d^2)(1+d^2-2d\cos u)}$,

$$W(d,u,n) = \frac{2d(1+d^2)\sin u - (1+d^4)\sin 2u - d^{n+4}\sin((n-2)u)}{(1+d^2)(1+d^2-2d\cos u)^2 \sin u},$$

$$Z(d,u,n) = \frac{2d^{n+3}\sin(n-1)u - 2d^{n+1}\sin(n+1)u + d^n\sin((n+2)u)}{(1+d^2)(1+d^2-2d\cos u)^2\sin u},$$

$d^2 = -\alpha_2$,

And $u = \cos^{-1}\left(\frac{\alpha_1}{2d}\right)$.

The x_t denote the change in the level of the compensating variable model at the time t, i.e., the adjustment made at the time point t. The ε_t is Gaussian white noise with variance $\sigma_\in^2$. throughout, we suppose that the noise variance is known. In practice, this is justified if reliable estimates of $\sigma_\in^2$ are available from the evaluation of a large number of previous values of the process, e.g., during the setup phase. The real - valued parameters α_1 and α_2 (the autoregressive parameters) determines the influence of the preceding time point (t - 1) and (t - 2) on the present time point t. We assume an in-control value $\alpha_1 = \alpha_2 = O$ for the auto regression parameters. It is possible that the auto regression parameters may shift to an out-of-control value $(\alpha_1,\alpha_2) \neq 0$.

Further, the distribution of the sample average will have mean μ and standard deviation $\frac{\sigma^2}{n}\lambda(\alpha_1,\alpha_2,n)$

For optimum value of mean and expected profit for n items for perfect rework process under dependency we put the value of equation (17) in equation (5) and (16) then

$$E(TP_1) = a - b - cy - i - r\left[1 - \Phi\left(\frac{M_1-\mu}{\sigma\lambda(\alpha_1,\alpha_2,n)/\sqrt{n}}\right)\right] - (a+s)\Phi\left(\frac{M_2-\mu}{\sigma\lambda(\alpha_1,\alpha_2,n)/\sqrt{n}}\right) \quad (21)$$

$$E(TP_1) = -c + \frac{r}{\sigma\lambda(\alpha_1,\alpha_2,n)/\sqrt{n}}\phi\left(\frac{M_1-\mu}{\sigma\lambda(\alpha_1,\alpha_2,n)/\sqrt{n}}\right) + \frac{a+s}{\sigma\lambda(\alpha_1,\alpha_2,n)/\sqrt{n}}\phi\left(\frac{M_2-\mu}{\sigma\lambda(\alpha_1,\alpha_2,n)/\sqrt{n}}\right)$$

(22)

For optimum value of mean and expected profit for n items for imperfect rework process under dependency we put the value of equation (17) in equation (8) and (9) then

$$E'(TP_2) = r + a - \frac{b + c\mu + i + r}{\Phi\left(\frac{M_1 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}}\right)} - \frac{(a+s)\Phi\left(\frac{M_1 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}}\right)}{\Phi\left(\frac{M_1 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}}\right)} \quad (23)$$

$$E'(TP_2) = -\frac{[c - \frac{a+s}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}}\phi\left(\frac{M_2 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}}\right)]}{\Phi\left(\frac{M_1 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}}\right)}$$

$$-\frac{[b + c\mu + i + r + (a+s)\Phi(\frac{M_2 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}})]\phi(\frac{M_1 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}})}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}\,[\Phi(\frac{M_1 - \mu}{\sigma\lambda(\alpha_1, \alpha_2, n)/\sqrt{n}})]^2} \quad (24)$$

Numerical Illustration and conclusion:

Consider the packing plant of a tea drink. The plant consists of two processes: an inspection process and a filling process. Inspection is performed by measuring the ingredients of the tea drink. Assume that the ingredients of the tea drink is above the USL which increases the manufacturing cost and that the tea drink cannot be sold at a higher price. Hence, the producer adopts a rework for it. If the ingredients of the tea drink is below the LSL, a penalty cost due to government's law may occur. Hence, the producer adopts a scrap for it. For the rework of a product, there exists the perfect and imperfect rework cases. Conforming ingredients of the tea drink is canned by a filling machine and moved to the dispatching stages on a conveyor belt. From theoretical considerations and past experience, it is known that the ingredients of the tea drink *Y* is normally distributed with a known standard deviation $\sigma = 0.25$ and an unknown mean μ. Let the target value of the mean be 40.75. Assume that the cost components and the specification limits for *Y* are $a = 5$, $s = 0.3$, $r = 0.1$, $b = 0.1$, $c = 0.06$, $i= 0.04$, $M_1 = 41.5$ and $M_2 = 40$ n = 15, 20. The producer would like to determine the optimum process mean for maximizing the expected profit per item and n items under the dependency.

By solving Eq. (22), the optimum process mean for the perfect rework and By solving Eq. (23), the optimum process mean for the imperfect rework are process is obtain for independent case and different situation of autocorrelation. Sensitivity Analysis also performed for a, b, c, i , r , s under the independent case and different situation of autocorrelation. In tables 1.1 to 1.4

Table:1.1 Optimum process mean and expected profit of the perfect and imperfect rework model under independent case with respect to different values of model parameter

a	Perfect		Imperfect		b	Perfect		Imperfect		c	Perfect		Imperfect	
	μ_1	Expected	μ_2	Final value		μ_1	Final	μ_2	Final value		μ_1	Final	μ_2	Final value
0.5	40.6186	-2.0824755	40.6189	-2.082476	0.5	40.78999	2.008194	40.7904	2.00819132	0.5	40.601	-15.4835	40.59464	-15.48339
1	40.6399	-1.5852346	40.6666	-1.585022	1	40.78999	1.508194	40.7904	1.50819132	1	40.519	-35.7594	40.51661	-35.7594
1.5	40.641	-1.0878021	40.6969	-1.086659	1.5	40.78999	1.008194	40.7904	1.00819132	1.5	40.461	-56.0042	40.46497	-56.00415
2	40.6421	-0.5903055	40.719	-0.587861	2	40.78999	0.508194	40.7904	0.50819132	2	40.43	-76.2264	40.42455	-76.2262
2.5	40.6499	-0.0920942	40.7363	-0.088809	2.5	40.78999	0.008194	40.7904	0.00819132	2.5	40.395	-96.4299	40.39032	-96.42971
3	40.67	0.4076073	40.7503	0.4104093	3	40.78999	-0.49181	40.7904	-0.4918087	3	40.36	-116.617	40.35994	-116.6171
3.5	40.681	0.906833	40.7624	0.9097435	3.5	40.78999	-0.99181	40.7904	-0.9918087	3.5	40.333	-136.79	40.3321	-136.7901
4	40.783	1.4090806	40.773	1.4091636	4	40.78999	-1.49181	40.7904	-1.4918087	4	40.31	-156.95	40.30593	-156.9495
4.5	40.79	1.908588	40.782	1.9086521	4.5	40.78999	-1.99181	40.7904	-1.9918087	4.5	40.283	-177.096	40.28084	-177.0962
5	40.831	2.407416	40.7903	2.4081919	5	40.78999	-2.49181	40.7904	-2.4918087	5	40.26	-197.231	40.25633	-197.2305
5.5	40.89	2.9047902	40.7978	2.9077746	5.5	40.78999	-2.99181	40.7904	-2.9918087	5.5	40.236	-217.353	40.23193	-217.3525
6	40.999	3.3976036	40.8045	3.4073936	6	40.78999	-3.49181	40.7904	-3.4918087	6	40.21	-237.462	40.20715	-237.4623
6.5	41.11	3.8874314	40.8109	3.9070408	6.5	40.78999	-3.99181	40.7904	-3.9918087	6.5	40.186	-257.56	40.1814	-257.5595
7	41.219	4.3738054	40.8165	4.4067158	7	40.78999	-4.49181	40.7904	-4.4918087	7	40.155	-277.643	40.15377	-277.6434
7.5	41.398	4.8419562	40.822	4.9064104	7.5	40.78999	-4.99181	40.7904	-4.9918087	7.5	40.124	-297.713	40.12256	-297.7127
8	41.412	5.339038	40.827	5.4061253	8	40.78999	-5.49181	40.7904	-5.4918087	8	40.09	-317.765	40.08339	-317.7647
8.5	41.459	5.8289734	40.8316	5.9058579	8.5	40.78999	-5.99181	40.7904	-5.9918087	8.5	40.01	-337.79	40	-337.79
9	41.499	6.3202196	40.836	6.4056047	9	40.78999	-6.49181	40.7904	-6.4918087	9	39.999	-357.789	40.009	-357.7949
9.5	41.501	6.8197804	40.8404	6.9053624	9.5	40.78999	-6.99181	40.7904	-6.9918087	9.5	39.989	-377.779	40.00009	-377.7901
10	41.699	7.2793616	40.8442	7.4051357	10	40.78999	-7.49181	40.7904	-7.4918087	10	39.979	-397.757	40.00099	-397.7915

Table:1.2 Optimum process mean and expected profit of the perfect and imperfect rework model under independent case with respect to different values of model parameter

	Perfect		Imperfect			Perfect		Imperfect			Perfect		Imperfect	
I	μ_1	Final value	μ_2	Final value	r	μ_1	Final value	μ_2	Final value	s	μ_1	Final value	μ_2	Final value
0.5	40.7904	1.9481913	40.742	1.9474153	0.5	40.816	2.408021	40.742	2.40692943	0.5	40.7933	-2.08248	40.742	2.4071156
1	40.7904	1.4481913	40.741	1.4473729	1	40.819	2.407618	40.742	2.40632208	1	40.8006	-1.58523	40.741	2.4063101
1.5	40.7904	0.9481913	40.742	0.9474153	1.5	40.819	2.40725	40.742	2.40571473	1.5	40.807	-1.0878	40.742	2.4056168
2	40.7904	0.4481913	40.742	0.4474153	2	40.82	2.406908	40.742	2.40510738	2	40.8132	-0.59031	40.742	2.4048675
2.5	40.7904	-0.0518087	40.742	-0.052585	2.5	40.7852	2.406591	40.742	2.40450004	2.5	40.8188	-0.09209	40.742	2.4041181
3	40.7904	-0.5518087	40.742	-0.552585	3	40.783	2.406295	40.742	2.40389269	3	40.8239	0.407607	40.742	2.4033688
3.5	40.7904	-1.0518087	40.742	-1.052585	3.5	40.782	2.406016	40.742	2.40328534	3.5	40.8289	0.906833	40.746	2.4030204
4	40.7904	-1.5518087	40.742	-1.552585	4	40.7632	2.405755	40.742	2.40267799	4	40.8334	1.409081	40.79	2.4052748
4.5	40.7904	-2.0518087	40.742	-2.052585	4.5	40.7579	2.405505	40.742	2.40207064	4.5	40.8379	1.908588	40.81	2.4054333
5	40.7904	-2.5518087	40.742	-2.552585	5	40.7531	2.40527	40.742	2.4014633	5	40.842	2.407416	40.83	2.405331
5.5	40.7904	-3.0518087	40.742	-3.052585	5.5	40.748	2.405048	40.742	2.40085595	5.5	40.8457	2.90479	40.84	2.4050935
6	40.7904	-3.5518087	40.742	-3.552585	6	40.7442	2.404834	40.742	2.4002486	6	40.8494	3.397604	40.8494	2.4048342
6.5	40.7904	-4.0518087	40.742	-4.052585	6.5	40.74	2.40463	40.742	2.39964125	6.5	40.8529	3.887431	40.853	2.4046293
7	40.7904	-4.5518087	40.742	-4.552585	7	40.75	2.404433	40.742	2.3990339	7	40.8564	4.373805	40.856	2.4044378
7.5	40.7904	-5.0518087	40.742	-5.052585	7.5	40.73	2.404246	40.742	2.39842656	7.5	40.8596	4.841956	40.859	2.4042528
8	40.7904	-5.5518087	40.742	-5.552585	8	40.72	2.404065	40.742	2.39781921	8	40.8627	5.339038	40.861	2.4040849
8.5	40.7904	-6.0518087	40.742	-6.052585	8.5	40.783	2.403892	40.742	2.39721186	8.5	40.8656	5.828973	40.863	2.4039223
9	40.7904	-6.5518087	40.742	-6.552585	9	40.783	2.403724	40.742	2.39660451	9	40.8685	6.32022	40.865	2.4037645
9.5	40.7904	-7.0518087	40.742	-7.052585	9.5	40.783	2.403561	40.742	2.39599716	9.5	40.8713	6.81978	40.866	2.4036209
10	40.7904	-7.5518087	40.742	-7.552585	10	40.783	2.403404	40.742	2.39538982	10	40.874	7.279362	40.87	2.4034526

Table:1.3 Optimum process mean and expected profit of the perfect and imperfect rework model under second order autocorrelation respect to different values of model parameter 'a'

	Roots are real & equal $\lambda(\alpha1=0.8,\alpha2=0.16,n=15)=3.544$				Roots are real & distinct $\lambda(\alpha1=0.3,\alpha2=0.6,n=15)=9.91$				Roots are complex $\lambda(\alpha1=0.8,\alpha2=-0.6,n=15)=1.06$			
	Perfect		Imperfect		Perfect		Imperfect		Perfect		Imperfect	
a	μ_1	Final value	μ_2	Final value	μ_1	Final value	μ_2	Final value	μ_1	Final value	μ_2	Final value
0.5	42.071	-2.2460623	39.93185	-2.582551	43.469	-2.491383	33.999	-2.4775618	39.351	-2.7953308	39.351	-2.7953308
1	42.198	-1.75886	38.1171	-2.70546	44.165	-1.936076	35	-2.5274567	39.3004	-2.7926351	39.3004	-2.7926351
1.5	42.281	-1.2669922	37.9826	-2.698602	44.58	-1.462169	36.00999	-2.5595591	39.2686	-2.7909139	39.2686	-2.7909139
2	42.343	-0.7729208	37.888	-2.693681	44.87	-0.980247	36.4	-2.5477526	39.2454	-2.7896574	39.2454	-2.7896574
2.5	42.392	-0.2775319	37.8157	-2.689864	45.095	-0.493989	36.74	-2.5209268	39.2272	-2.7886719	39.2273	-2.7886719
3	42.434	0.21863189	37.757	-2.686755	45.278	-0.004998	35.71	-2.4994964	39.21228	-2.7878634	39.2123	-2.7878634
3.5	42.469	0.71544714	37.7085	-2.684139	45.43	0.4858751	35.344	-2.4857037	39.2	-2.7871793	39.2	-2.7871793
4	42.5	1.21268043	37.6665	-2.681885	45.56	0.978101	35.09	-2.4750824	39.1893	-2.7865872	39.1893	-2.7865872
4.5	42.527	1.71027782	37.6291	-2.679908	45.675	1.4713322	34.895	-2.4663896	39.1796	-2.786066	39.1798	-2.786066
5	42.552	2.20810798	37.596	-2.678148	45.775	1.9653722	34.738	-2.4590188	39.1713	-2.7856008	39.1713	-2.7856008
5.5	42.574	2.70618818	37.567	-2.676565	45.865	2.4600421	34.6	-2.4526185	39.1637	-2.7851811	39.1637	-2.7851811
6	42.594	3.20445159	37.541	-2.675127	45.95	2.9552063	34.48	-2.446963	39.1568	-2.784799	39.1568	-2.784799
6.5	42.613	3.70284006	37.516	-2.673811	46.025	3.4508199	34.379	-2.4418986	39.1504	-2.7844484	39.1505	-2.7844484
7	42.63	4.20138194	37.494	-2.672597	46.094	3.9467958	34.285	-2.4373151	39.1445	-2.7841247	39.1446	-2.7841247
7.5	42.647	4.69998675	37.473	-2.671472	46.158	4.4430804	34.2	-2.4331304	39.139	-2.7838243	39.139	-2.7838243
8	42.662	5.19872503	37.454	-2.670424	46.216	4.9396403	34.124	-2.4292821	39.1342	-2.7835439	39.134	-2.7835439
8.5	42.676	5.69754976	37.436	-2.669444	46.272	5.436423	34.05	-2.4257213	39.1293	-2.7832813	39.1294	-2.7832813
9	42.69	6.19641819	37.419	-2.668523	46.323	5.9334181	33.986	-2.4224091	39.125	-2.7830343	39.125	-2.7830343
9.5	42.703	6.69536279	37.403	-2.667655	46.375	6.4305753	33.926	-2.4193138	39.1208	-2.7828012	39.1208	-2.7828012
10	42.715	7.19437859	37.388	-2.666835	46.419	6.9279187	33.869	-2.4164096	39.1168	-2.7825807	39.1168	-2.7825807

Table:1.4 Optimum process mean and expected profit of the perfect and imperfect rework model under second order autocorrelation respect to different values of model parameter 'b'

	Roots are real & equal , λ(α1=0.8,α2=0.16,n=15)=3.544				Roots are real & distinct , λ(α1=0.3,α2=0.6,n=15)=9.91				Roots are complex , λ(α1=0.8,α2=-0.6,n=15)=1.06			
	Perfect		Imperfect		Perfect		Imperfect		Perfect		Imperfect	
b	μ_1	Final value	μ_2	Final value	μ_1	Final value	μ_2	Final value	μ_1	Final value	μ_2	Final value
0.5	42.552	1.80810798	37.5969	-3.078151	45.774999	1.5653722	38.9144	-2.5585125	39.17136	-3.1856008	39.1712	-3.1856008
1	42.551	1.30815249	37.5966	-3.578153	45.774999	1.0653722	38.593	-3.1358066	39.17136	-3.6856008	39.1712	-3.6856008
1.5	42.552	0.80810798	37.5963	-4.078156	45.774999	0.5653722	38.295	-3.6968344	39.17136	-4.1856008	39.1712	-4.1856008
2	42.552	0.30810798	37.5962	-4.578158	45.774999	0.0653722	38.0109	-4.2453474	39.17136	-4.6856008	39.1712	-4.6856008
2.5	42.552	-0.191892	37.5971	-5.078161	45.774999	-0.434628	37.739	-4.7840211	39.17136	-5.1856008	39.1712	-5.1856008
3	42.552	-0.691892	37.5966	-5.578164	45.774999	-0.934628	37.47	-5.314815	39.17136	-5.6856008	39.1712	-5.6856008
3.5	42.552	-1.191892	37.597	-6.078166	45.774999	-1.434628	37.2	-5.8391872	39.17136	-6.1856008	39.1712	-6.1856008
4	42.551	-1.6918475	37.597	-6.578169	45.774999	-1.934628	36.92	-6.3582202	39.17136	-6.6856008	39.1712	-6.6856008
4.5	42.552	-2.191892	37.597	-7.078172	45.774999	-2.434628	36.606	-6.8726737	39.17136	-7.1856008	39.1712	-7.1856008
5	42.551	-2.6918475	37.597	-7.578174	45.774999	-2.934628	36.13	-7.3828404	39.17136	-7.6856008	39.1712	-7.6856008
5.5	42.552	-3.191892	37.597	-8.078177	45.774999	-3.434628	35.9	-7.8890121	39.17136	-8.1856008	39.1712	-8.1856008
6	42.551	-3.6918475	37.597	-8.57818	45.774999	-3.934628	35.7	-8.3927512	39.17136	-8.6856008	39.1712	-8.6856008
6.5	42.552	-4.191892	37.597	-9.078182	45.774999	-4.434628	35.57	-8.8955018	39.17136	-9.1856008	39.1712	-9.1856008
7	42.551	-4.6918475	37.597	-9.578185	45.774999	-4.934628	38.91	-10.18688	39.17136	-9.6856008	39.1712	-9.6856008
7.5	42.552	-5.191892	37.5971	-10.07819	45.774999	-5.434628	38.91	-10.773677	39.17136	-10.185601	39.1712	-10.185601
8	42.552	-5.691892	37.5971	-10.57819	45.774999	-5.934628	38.91	-11.360474	39.17136	-10.685601	39.1712	-10.685601
8.5	42.551	-6.1918475	37.5972	-11.07819	45.774999	-6.434628	38.91	-11.947272	39.17136	-11.185601	39.1712	-11.185601
9	42.552	-6.691892	37.5973	-11.5782	45.774999	-6.934628	38.91	-12.534069	39.17136	-11.685601	39.1712	-11.685601
9.5	42.551	-7.1918475	37.5974	-12.0782	45.774999	-7.434628	34.92768	-11.895845	39.17136	-12.185601	39.1712	-12.185601
10	41.8499	-7.6429123	37.5974	-12.5782	45.774999	-7.934628	34.8	-12.392921	39.17136	-12.685601	39.1712	-12.685601

From tables 1.1 to 1.4 autocorrelation affected the optimum values of mean and expected profit. In several values of autocorrelation profit is found negative which indicated loss. Sensitive analysis of parameters there was no change found in optimum value of mean in case parameter b and i but change was found their expected profits. Rest of the parameters a, r, c, s affected the optimum values of mean and expected profit.

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