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Oblique Electromagnetic Ion Cyclotron Instability with A.C. Electric field for Loss-cone Distribution Function

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Abstract – Electromagnetic ion cyclotron (EMIC) wave's play an important role in magnetospheric dynamics and their global distribution has been of great interest. In this paper, the effect of cold plasma injection on oblique propagating Electromagnetic Ion Cyclotron waves in the presence of A.C electric field due to distribution of hydrogen ions in a background plasma having bi-maxwellian and also loss-cone distribution function has been examined in the magnetosphere of Earth. In these cases the effect of cold plasma injection and other parameters have been compared. Applying kinetic approach, expression for dispersion relation and growth rate has been derived. It is found that the A.C frequency has profound effect on growth rate for both bi-maxwellian and loss cone distribution. Growth rate increases with increase in the A.C. frequency. Increase of temperature anisotropy also increases the growth rate, thus it can be concluded that the source of free energy for this instability is not only temperature anisotropy but also A.C. field frequency. There is marginal decrease in growth rate with increase in angle of propagation for bi-maxwellian distribution but for loss cone distribution, growth rate increases significantly. The ratio of cold injected plasma to background plasma (n_c/n_w) reduces the growth rate.

Keywords – Magnetosphere of Earth, Ion-cyclotron waves, Cold Injection, Loss cone distribution Functions.

I. INTRODUCTION

Space surrounding the Earth where the dominant magnetic field is the field of Earth rather than interplanetary space magnetic field is magnetosphere. After investigation of many past centuries, it was found that Earth's magnetic field is quite complex. But still it can be viewed as a dipole with north and South Pole simply acting as a bar magnet. The boundary separating the Earth's magnetic field and solar wind is the magnetopause and it is the constantly moving boundary. The region inside magnetopause is divided into several boundaries. Plasma being the fourth sate of matter plays an important role in the Universe as most of the stars are in the form of plasma. Number of charge carriers present in the magnetosphere makes the plasma electrically conductive. This conductive plasma respond to electromagnetic fields [1].

Cyclotron waves are an important constituent of plasmas in solar corona, solar wind and planetary magnetosphere. As it is well known, the energetic particles (electrons, protons, heavy ions) with anisotropic temperature can excite a wide class of cyclotron wave instabilities. Kinetic theory of electromagnetic cyclotron waves/instabilities in the straight magnetic field plasma is well developed and published by [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. Electromagnetic ion cyclotron (EMIC) waves are one of the cyclotron waves and take place in several space environments with the ion beam interaction. Previous works by [3], [15] indicates that the most favorable region for the generation of EMIC waves is the equatorial region. These waves generally propagate along geomagnetic field lines. The propagation of these waves to high latitudes results in the high inclination of wave normal to the magnetic field and the absorption of wave energy by the interaction of cyclotron resonant with ambient ions [16]. In addition, EMIC waves and ion beam interaction take place in laboratory plasma also. They are generated in equatorial region of magnetosphere of Earth. Electromagnetic ion cyclotron (EMIC) waves are transverse plasma waves and are the important component of magnetospheric plasma. They are generated by ring current protons in the magnetosphere with temperature anisotropy in three bands, less than the gyrofrequency of He^+, H^+, O^+ ions and can be excited by an energy of few KeV by an inverted -V electron beam [17]. The growth of EMIC waves can lead to the isotropization of the proton distribution which are initially unstable and atmospheric loss of protons [3], [18]. These waves can cause rapid pitch angle scattering, radiation belt particles loss [19], [20], and different components of thermal plasma can be heated by these waves [21], [22], [23], [24], [25]. Observations by AMPTE/CCE spacecraft have shown that EMIC waves occur most repeatedly in the outer magnetosphere yonder L=7 [26], [27].

Under geomagnetically disturbed conditions EMIC waves can also be found in inner magnetosphere in a region of L<5 [28], [27]. These waves are assumed to be the cause of ring current loss by wave-particle interaction [28], [29], [30], [31]. Their role in the acceleration and loss of relativistic electrons has also quoted by [32], [33], [31].

II. RELATED WORK

On the basis of the previous literature and work done to understand ion cyclotron waves in Earth's magnetosphere, we made an attempt to study the effect of oblique propagating EMIC waves on the growth rate. The main aim of this study is to investigate the generation of EMIC waves and to see the effect of cold particle injection. The detailed formulation and mathematics used is described in the next section with the dispersion relation incorporated in the study.

III. DISPERSION RELATION

A spatially homogeneous an isotropic, collision less magneto plasma subjected to an external magnetic field $B_0 = B_0\, e_z$ and an electric field $E_0= (E_0 \sin \upsilon\ t\ \hat{e}_x)$ has been considered in order to obtain the dispersion relation. In this case, the Vlasov-Maxwell equations are used. The linear zed equations obtained after neglecting the higher order terms and separating the equilibrium and non- equilibrium parts, following the techniques of [34] and [35] are given as

$$v\frac{\partial f_{s0}}{\partial r}.+\frac{e_s}{m_s}\left[E_o \sin \nu t+(v\times B_o)\right]\left(\frac{\partial f_{so}}{\partial v}\right)=0 \tag{1}$$

$$\frac{\partial f_{s1}}{\partial t}+v.\frac{\partial f_{s1}}{\partial r}+\left(\frac{F}{m_s}\right)\left(\frac{\partial f_{s1}}{\partial v}\right)=S(r,v,t) \tag{2}$$

Where force is defined as $F = m\frac{dv}{dt}$

$$F = e_s\left[E_{0x}\sin\nu t+(v\times B_0)\right] \tag{3}$$

The practical trajectories are obtained by solving the equation of motion defined in equation (3) and S(r,v,t) is defined as.

$$x_0 = x+\left(\frac{v_y}{\omega_{cs}}\right)+\left(\frac{1}{\omega_{cs}}\right)\left[v_x\sin\omega_{cs}t' - v_y\cos\omega_{cs}t'\right]+\left(\frac{\Gamma_x}{\omega_{cs}}\right)\left[\frac{\omega_{cs}\sin\nu t' - \nu\sin\omega_{cs}t'}{\omega_{cs}^2-\nu^2}\right]$$

$$y_0 = y+\left(\frac{v_x}{\omega_{cs}}\right)-\left(\frac{1}{\omega_{cs}}\right)\left[v_x\cos\omega_{cs}t' - v_y\sin\omega_{cs}t'\right]-\left(\frac{\Gamma_x}{\nu\omega_{cs}}\right)\left[1+\frac{\nu^2\cos\omega_{cs}t' - \omega_{cs}^2\cos\nu t'}{\omega_{cs}^2-\nu^2}\right]$$

$$z_0 = z - v_z t' \tag{4}$$

and the velocities are

$$v_{x0} = v_x\cos\omega_{cs}t' - v_y\sin\omega_{cs}t' + \left\{\frac{\nu\Gamma_x(\cos\nu c' - \cos\omega_{cs}t')}{\omega_{cs}^2-\nu^2}\right\}$$

$$v_{y0} = v_x\sin\omega_{cs}t' + v_y\cos\omega_{cs}t' - \left\{\frac{\Gamma_x(\omega_{cs}\sin\nu t' - \nu\sin\omega_{cs}t')}{\omega_{cs}^2-\nu^2}\right\}$$

$$v_{z0} = v_z \tag{5}$$

Where $\omega_{cs} = \frac{e_s B_0}{m_s}$ is the cyclotron frequency of species s and $\Gamma_x = \frac{e_s E_0}{m_s}$ and a.c. electric field is varying as

$E = E_{0x} \sin \nu t$, ν being the angular a-c frequency.

$$S(r,v,t) = \left(-\frac{e_s}{m_s}\right)[E_1 + v \times B_1]\left(\frac{\partial f_{so}}{\partial v}\right) \tag{6}$$

Where s denotes species and E_1, $B_1$and f_{s1} are perturbed and are assumed to have harmonic dependence in f_{s1}, B_1 and E_1 =~exp I(k, r-ωt).The method of characteristic solution is used to determine the perturbed distribution function.f_{s1}, which is obtained from equ. (2) by

$$f_{s1}(r,v,t) = \int_{0^\circ}^{\infty} S\{r_0(r,v,t^`), v_0(r,v,t^`), t - t^` dt^`. \tag{7}$$

The phase space coordinate system has been transformed from (r,v,t) to $(r_0,v_0,t\text{-}t^`)$.The particle trajectories which have been obtained by solving eq.(3) for the given external field configuration and wave propagation $k = [k_\perp e_{x,} 0, k_\parallel e_z]$.After doing some lengthy algebraic simplification and carrying out the integration, the perturbed distribution function f_1 is written as [4].

$$f_{s1}(r,v,t) = -\frac{e_s}{m_s \omega} \sum_{m,n,p,q}^{\infty} \frac{J_p(\lambda_2) J_m(\lambda_1) J_q(\lambda_3) e^{i(k.r - \omega t)}}{\{\omega - k_\parallel v_\parallel - (n+q)\omega_{cs} + p\nu\}} \left[E_{1x} J_n J_p \left\{ \left(\frac{n}{\lambda_1}\right) U^* + D_1\left(\frac{p}{\lambda_2}\right) \right\} - iE_{1y} \left\{ J_n^{'} J_p C_1 + J_n J_p^{'} D_2 \right\} + E_{1z} J_n J_p W^* \right]$$

Where the Bessel identity

$$e^{\lambda \sin\theta} = \sum_{k=-\infty}^{\infty} J_k(\lambda)\, e^{ik\theta}$$

has been used, the arguments of the Bessel functions are

$$\lambda_1 = \frac{k_\perp v_\perp}{\omega_{cs}}, \lambda_2 = \frac{k_\perp \Gamma_x \nu}{\omega_{cs}^2 - \nu^2}, \lambda_3 = \frac{k_\perp \Gamma_x \omega_{cs}}{\omega_{cs}^2 - \nu^2}$$

Where

$$C_1 = \frac{1}{v_\perp}\left(\frac{\partial f_0}{\partial v_\perp}\right)(\omega - k_\parallel . v_\parallel) + \left(\frac{\partial f_0}{\partial v_\parallel}\right) k_\parallel$$

$$U^* = C_1\left[v_\perp - \left\{ \frac{\nu \Gamma_x}{\omega_{cs}^2 - \nu^2} \right\} \right]$$

$$W^* = \left[\left(\frac{n\omega_{cs} v_\parallel}{v_\perp} \right) \left(\frac{\partial f_0}{\partial v_\perp} \right) - n\omega_{cs} \left(\frac{\partial f_0}{\partial v_\parallel} \right) \right] + \left[1 + \left\{ \frac{k_\perp \nu \Gamma_x}{\omega_{cs}^2 - \nu^2} \right\} \{(p/\lambda_2) - (n/\lambda_1)\} \right]$$

$$D_1 = C_1\left\{ \frac{\nu \Gamma_x}{\omega_{cs}^2 - \nu^2} \right\}, D_2 = C_2\left\{ \frac{\omega_{cs} \Gamma_x}{\omega_{cs}^2 - \nu^2} \right\}$$

$$J_n^{'} = \frac{dJ_n(\lambda_1)}{d\lambda_1}, J_p^{'} = \frac{dJ_p(\lambda_2)}{d\lambda_2} \tag{8}$$

Following [2],[36] the conductivity tensor $\|\sigma\|$ is written as

$$\|\sigma\| = -\sum \frac{e_s^2}{m_s \omega} \sum_{m,n,p,q}^{\infty} \int \frac{J_q(\lambda_3) S_{ij} d^3 v}{\omega - kv - (n+q)\omega_{cs} + p\nu}$$

where

$$S_{ij} = \begin{Vmatrix} v_\perp \frac{n}{\lambda_1}(J_n)^2 J_p A & iv_\perp J_n B & v_\perp W^* \frac{n}{\lambda_1} J_n^2 J_p \\ v_\perp J_p A J_n J'_n & v_\perp J'_n B & iv_\perp W^* J_p J_n J'_n \\ v_\parallel J_n^2 J_p A & -iv_\parallel J_n B & v_\parallel W^* J_n^2 J_p \end{Vmatrix}$$

$$A = \left(\frac{n}{\lambda_1}\right) U^* + \left(\frac{p}{\lambda_2}\right) D_1 , \qquad B = J'_n J_p C_1 + J'_n J_n D_2$$

From $J = \|\sigma\|.E_1$ and two Maxwell's curl equations for the perturbed quantities, we have

$$\left[k^2 - k.k - \frac{\omega^2}{c^2}\varepsilon(k,\omega)\right].E_1 = 0$$

*** An anisotropic collision less plasma in presence of electric field $E_{ox} = E_o \sin(\upsilon t)\hat{e}_x$ and an external magnetic field $B_o = B_o \hat{e}_z$ is assumed. The Vlasov-Maxwell equations are linearized for spatially homogeneous plasma. Conductivity tensor is written as

$$\| \sigma \| = -\sum \left(\frac{e_s}{m_s \omega}\right) \sum_{m,n,p,q=-\infty}^{\infty} \int d^3 v \left[\frac{J_q(\lambda_3) \| S_{ij} \|}{\omega - k_\parallel v_\parallel - (n+q)\omega_{cs} + p\upsilon}\right] \tag{9}$$

And dielectric tensor as

$$\varepsilon_{ij}(k,\omega) = 1 + \sum_s \frac{4 e_s^2 \pi}{m_s \omega^2} \sum_n \sum_p J_p(\lambda_2) J_q(\lambda_3) \int \frac{d^3 v \| S_{ij} \|}{\omega - k_\parallel v_\parallel - (n+q)\omega_{cs} + p\upsilon} \tag{10}$$

Where

$$\| S_{ij} \| = \begin{vmatrix} N^2 \cos^2\theta_1 + \varepsilon_{11} & \varepsilon_{12} & N^2 \cos\theta_1 \sin\theta_1 + \varepsilon_{13} \\ \varepsilon_{21} & N^2 + \varepsilon_{22} & \varepsilon_{23} \\ N^2 \cos\theta_1 \sin\theta_1 + \varepsilon_{31} & \varepsilon_{32} & N^2 \sin^2\theta_1 + \varepsilon_{33} \end{vmatrix}$$

After using the limits in above tensor $k_\perp = k \sin\theta_1 \to 0$ and $k_\parallel = k\cos\theta_1$, the generalized dielectric tensor becomes simplified tensor:

$$\begin{vmatrix} -N^2 + \varepsilon_{11} & \varepsilon_{12} & 0 \\ -\varepsilon_{21} & -N^2 + \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{vmatrix} \tag{11}$$

$$-N^4-2\varepsilon_{11}N^2+\varepsilon_{11}^2+\varepsilon_{12}^2=0 \tag{12}$$

Neglecting the higher order terms of N, the relation becomes:

$$\varepsilon_{11}\pm\varepsilon_{12}=N^2\cos^2\theta \tag{13}$$

The unperturbed generalized distribution function is written as:

$$f_o(v)=\frac{n_0 v_\perp^{2j}}{\pi^{3/2}\alpha_{\perp s}^{2(j+1)}\alpha_{\parallel s}\, j!}\exp\left[-\left(\frac{v_\perp}{\alpha_{\perp s}}\right)^2-\left(\frac{v_{\parallel s}}{\alpha_{\parallel s}}\right)^2\right] \tag{14}$$

where s for species, j= loss-cone index; $\alpha_{\perp s}$ and $\alpha_{\parallel s}$ are thermal velocities; for j =0 and it reduces to an-isotropic bi-Maxwellian and further for $\alpha_{\perp s}=\alpha_{\parallel s}$ it becomes a isotropic Maxwellian. After some algebraic manipulations and velocity integrations the resulting dispersion relation is written as parallel propagation as [34].

$$D(k,\omega)=1-\frac{k^2c^2}{\omega^2}+\sum\frac{8\pi e_s^2 n_0}{m_s\omega^2}\sum\frac{J_p(\lambda_2)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)}j!}\left[X_1\frac{\omega}{k_\parallel\ \alpha_{\parallel s}}Z(\xi_s)+X_2\left(1+\xi_s Z(\xi_s)\right)\right] \tag{15}$$

Where

$$X_1=\frac{\alpha_{\perp s}^{2(j+1)}j!}{2}-\frac{\nu\Gamma_{xs}}{\omega_c^2-\nu^2}\frac{\alpha_{\perp s}^{2(j+1)}}{4}\left(j-\frac{1}{2}\right)!$$

$$X_2=\frac{\alpha_{\perp s}^{2(j+1)}j!}{2}\left((j+1)\frac{\alpha_{\perp s}^2}{\alpha_{\parallel s}^2}-1\right)-\frac{\nu\Gamma_{xs}}{\omega_c^2-\nu^2}\frac{\alpha_{\perp s}^{2(j+1)}}{4}\left((2j+1)\frac{\alpha_{\perp s}^2}{\alpha_{\parallel s}^2}-1\right)\left(j-\frac{1}{2}\right)!$$

$$\xi_s=\frac{\omega-n\omega_c-q\omega_c+p\nu}{k_\parallel\ \alpha_\parallel} \tag{16}$$

The above dispersion relation is now approximated in Ion-cyclotron range of frequencies. In this case the electron temperatures are assumed as $T_{e\perp}=T_{\parallel e}=T_e$ and assumed to be magnetized with $|\omega_r+i\gamma|<<\omega_{ce}$ while ions are assumed to have

$T_{\perp i}>T_{\parallel i}$ and $|k_\parallel\ \alpha_{\parallel i}|<<|\omega_r\pm\omega_{ci}+i\gamma|$. The approximated dispersion relation is

$$D(k,\omega_r+i\gamma)=1-\frac{k^2c^2}{(\omega_r+i\gamma)^2}+\sum\frac{J_p(\lambda_2)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)}j!}\left[\frac{2\omega_{pe}^2}{\omega_{ce}^2}-\frac{2\omega_{pe}^2}{(\omega_r+i\gamma)(\pm\omega_{ce})}\right]X_{1e}+$$
$$\sum\frac{J_p(\lambda_2)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)}j!}\frac{2\omega_{pi}^2}{(\omega_r+i\gamma)^2}\left[X_{1i}\frac{\omega_r+i\gamma}{k_\parallel\ \alpha_{\parallel i}}Z(\xi_i)+X_{2i}\left(1+\xi_i Z(\xi_i)\right)\right] \tag{17}$$

Where

$$X_{1i} = \frac{\alpha_{\perp i}^{2(j+1)} j!}{2} - \frac{\nu\Gamma_{xi}}{\omega_c^2 - \nu^2} \frac{\alpha_{\perp i}^{2(j+1)}}{4}\left(j - \frac{1}{2}\right)!$$

$$X_{2i} = \frac{\alpha_{\perp i}^{2(j+1)} j!}{2}\left((j+1)\frac{\alpha_{\perp i}^2}{\alpha_{\parallel i}^2} - 1\right) - \frac{\nu\Gamma_{xi}}{\omega_c^2 - \nu^2} \frac{\alpha_{\perp i}^{2(j+1)}}{4}\left((2j+1)\frac{\alpha_{\perp i}^2}{\alpha_{\parallel i}^2} - 1\right)\left(j - \frac{1}{2}\right)! \tag{18}$$

$$X_{1e} = \frac{\alpha_{\perp e}^{2(j+1)} j!}{2} - \frac{\nu\Gamma_{xe}}{\omega_c^2 - \nu^2} \frac{\alpha_{\perp e}^{2(j+1)}}{4}\left(j - \frac{1}{2}\right)!$$

$$\omega_{ps}^2 = \frac{4\pi e_s^2 n_0}{m_s}$$

Applying the condition $\frac{k^2c^2}{\omega^2} >> 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}$ and the charge neutrality condition $\frac{\omega_{pe}^2}{\pm\omega_{ce}} = -\frac{\omega_{pi}^2}{\pm\omega_{ci}}$ the dispersion relation reduces to

$$D(k, \omega_r + i\gamma) = -k_\parallel^2 c^2 + \sum \frac{2J_p(\lambda_2)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)} j!}\left(\frac{\omega_{pe}^2}{\omega_{ce}^2} X_{1e}\right) +$$

$$\sum \frac{2J_p(\lambda_2)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)} j!}\left[\omega_{pi}^2 X_{1i}\left\{\frac{\omega}{k_\parallel \alpha_{\parallel i}} Z(\xi_i) + \frac{X_{2i}}{X_{1i}}\left(1 + \xi_i Z(\xi_i)\right)\right\}\right]$$

$$= -\frac{k_\parallel^2 c^2}{\omega_{pi}^2} + \sum \frac{2J_p(\lambda_2)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)} j!}\left(\frac{\omega}{\pm\omega_{ci}} X_{1e}\right) + \sum \frac{2J_p(\lambda_2)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)} j!} X_{1i}\left[\left\{\frac{\omega}{k_\parallel \alpha_{\parallel i}} Z(\xi_i) + \frac{X_{2i}}{X_{1i}}\left(1 + \xi_i Z(\xi_i)\right)\right\}\right] \tag{14}$$

Now using the asymptotic expansion of $Z(\xi_i)$ from [37] and for n=1 the real and imaginary parts of dispersion relations are:

$$\mathrm{Im}D(k,\omega) = -\sum \frac{2J_p(\lambda_1)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)} j!} X_{1i} \frac{1 - X_3 + X_4}{k}\left[\frac{X_{2i}}{X_{1i}} - \frac{X_3}{1 - X_3 + X_4}\right]\sqrt{\pi}\exp\left[-\left(\frac{1 - X_3 + X_4}{k}\right)^2\right]$$ where

$$X_3 = \frac{\omega_r}{\pm\omega_{ci}} \qquad X_4 = \frac{q\omega_{ci} - p\nu}{\pm\omega_{ci}} \qquad k' = \frac{k_\parallel \alpha_{\parallel i}}{\pm\omega_{ci}} \tag{19}$$

$$\mathrm{Re}D(k,\omega) = -\sum \frac{2J_p(\lambda_1)J_q(\lambda_3)}{\alpha_{\perp s}^{2(j+1)} j!} \frac{X_{1i} k''^2}{2(1 - X_3 + X_4)^2}\left[\frac{X_{2i}}{X_{1i}} - \frac{X_3}{1 - X_3 + X_4}\right] - \frac{2X_3(1 - X_3 + X_4)}{k''^2}$$
$$+ \frac{2X_3(1 - X_3 + X_4)^2 K_1 X_{1e}}{k''^2 X_{1i}}$$

(20)

By using the standard definition of growth rate for p=1 and q=0, $\sum J_p(\lambda_1)J_q(\lambda_3)=1$

$$\gamma/\omega_{ci} = \frac{-\mathrm{Im}D(k,\omega)}{\omega_{ci}\dfrac{\partial \mathrm{Re}D(k,\omega)}{\partial\omega}}$$

$$\frac{\gamma}{\omega_{ci}} = \frac{\dfrac{\sqrt{\pi}}{k''}\left(\dfrac{X_{2i}}{X_{1i}}-K_4\right)K_3^3\exp\left(-\left(\dfrac{K_3}{k''}\right)^2\right)}{1+X_4+\dfrac{(1+X_4)k''^2}{2K_3^2}-\dfrac{k''^2}{K_3}\left(\dfrac{X_{2i}}{X_{1i}}-K_4\right)-\dfrac{X_{1e}}{X_{1i}}K_1K_3^2} \tag{21}$$

$$\text{where}\quad K_3 = 1-X_3+X_4\ ,\ K_3 = \frac{X_3}{1-X_3+X_4}$$

From real part of the dispersion relation (20) set to zero, one gets for p=1, n=1 and q=0, $\sum J_p(\lambda_1)J_q(\lambda_3)=1$

$$X_3 = \frac{\omega_r}{\omega_c} = \frac{k''^2}{\beta}\left[\frac{K_2X_{1i}(1+X_4)}{X_{1i}-X_{1e}K_1(1+X_4)}+\frac{X_{2i}}{X_{1i}}\frac{\beta\left(X_{1i}(1+X_4)\right)}{2(1+X_4)^2\left(X_{1i}-X_{1e}K_1(1+X_4)\right)}\right]$$

$$A_T = \frac{\alpha_\perp^2}{\alpha_\parallel^2}-1\quad,\quad \beta=\frac{K_BT_\parallel\mu_0n_0}{B_0^2}\quad,K_2=\frac{\alpha_\perp^{2(j+1)}j!}{4X_{1i}}\qquad,K_1=\frac{\alpha_{\perp i}^{2(j+1)}}{\alpha_{\perp e}^{2(j+1)}},k''=\tilde{k}\cos\theta$$

IV. PLASMA PARAMETERS

To analyse the effect of cold injections in magnetosphere of Earth on growth rate of EMIC waves, following set of parameters are considered with electric field $E_o = 4\times10^{-3}$ V/m. EMIC waves are primarily excited by the anisotropic protons within the energy range of 10 to 50 KeV [3, 16].

Background bi-Maxwellian plasma with j=0 is assumed to have temperature anisotropy with $T_{\perp i}/T_{\|i}$ = 2, number density n_o= 5×10^6 m^{-3}, AC frequency ν = 8 Hz, energy density of ions K_BT_{li}= 5KeV and energy density of electrons $K_BT_{\|e}$= 0.3eV.

Injected cold plasma with loss-cone index j=1 is assumed to have temperature anisotropy with $T_{\perp i}/T_{\|i}$ = 2, number density n_o= 5×10^7 m^{-3}, AC frequency ν = 8 Hz, energy density of ions K_BT_{li}= 5KeV and energy density of electrons $K_BT_{\|e}$= 0.3KeV with n_c/n_w=10. Also magnetic field is considered to be $B_o = 2\times10^{-7}$ nT.

V. DISCUSSION WITHOUT BEAM AND WITH BEAM

Numerical solution of the dispersion relation for EMIC left hand polarized wave propagating along the magnetic field line have been made in the equatorial region of magnetospheric plasma at L=7.Various parameters used are as per [38]. The calculations are performed for generalized distribution function reducible to bi-maxwellian for j=0 and loss cone for j=1.

The results without beam are found and are shown below.

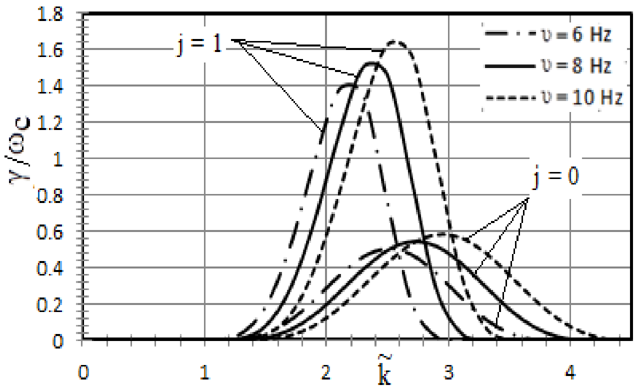

Figure 1. Variation of growth rate γ/ω_c versus $\tilde{k}$ for different values of A.C frequency and other parameters are $B_o = 2\times10^{-7}$ T, $n_o = 5\times10^6$ m^{-3}, $T_{\perp i}/T_{\parallel i}$ = 2, $K_BT_{\parallel i}$= 5 KeV, $K_BT_{\parallel e}$= 0.3eV, $E_o = 4\times10^{-3}$ V/m.

In Figure 1 dimensionless growth rate of EMIC waves has been plotted with respect to wave number for different values of AC frequency (υ). Fixed plasma parameters are described in the captions of the graph. As the graph shows AC frequency affects the growth rate significantly. The maxima shift to higher values of $\tilde{k}$ as frequency changes from 6 Hz to 10 Hz. This figure shows that increase in growth rate of EMIC waves with increasing frequency. As AC frequency increases from 6 Hz to 10 Hz, the growth rate increases from 0.505033 to 0.582652 with slight shift in peak value from $\tilde{k}$ =2.5 to 2.9 for bi-maxwellian distribution and from 1.378895 to 1.628726 with shift in peak value from $\tilde{k}$ = 2.1 to 2.5. Similar results have been shown by [39].

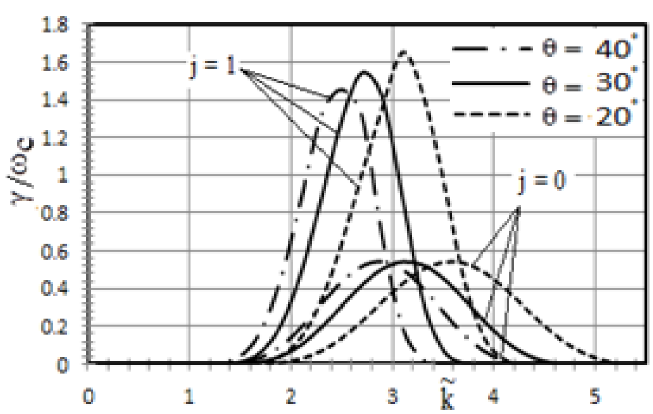

Figure 2. Variation of growth rate γ/ω_c versus $\tilde{k}$ for different values angle of propagation and other parameters are B_o = 2×10^{-7} T, $n_o = 5\times10^6$ m^{-3}, $T_{\perp i}/T_{\parallel i}$ = 2, $K_BT_{\parallel i}$= 5 KeV, $K_BT_{\parallel e}$= 0.3eV, ν = 8 Hz, $E_o = 4\times10^{-3}$ V/m.

Figure 2 show the variation in growth rate of EMIC waves with respect to wave number for different values of propagation angle (θ). The study considers propagation of these waves at angle of 20^o , 30^o and 40^o with respect to ambient magnetic field of Earth. This figure shows the change growth rate of EMIC waves in cold bi-Maxwellian background plasma as angle of propagation changes. The maximum growth rate decreases from 0.56389 to 0.534476 as the angle of propagation changes from 20^o to 40^o with k shifting from 2.9 to 3.3 for bi-maxwellian distribution and it increases from 1.649696 to 1.454213 as angle of propagation changes from 20^o to 40^o with k shifting from 2.5 to 3.1.

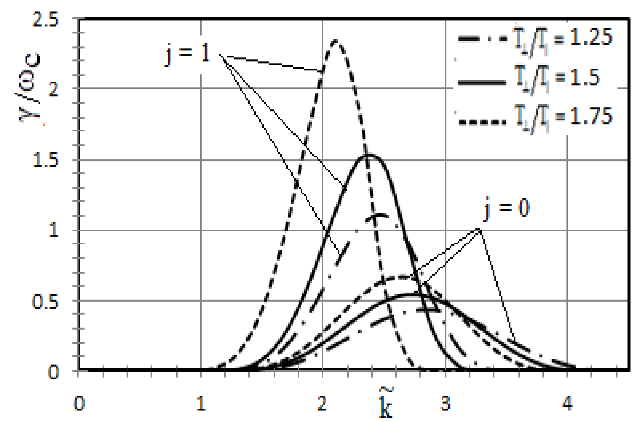

Figure 3. Variation of growth rate γ/ω_c versus $\tilde{k}$ for different values of $T_{\perp i}/T_{\parallel i}$ and other parameters are B_o=2×10^{-7} T, n_o= 5×10^6 m^{-3}, $K_BT_{\parallel i}$= 5 KeV, $K_BT_{\parallel e}$= 0.3 eV, ν = 8 Hz, E_o = 4×10^{-3} V/m.

Temperature anisotropy ($A_T = T_{\perp i}/T_{\parallel i}$ -1) is one of the prime factors governing the dynamics of magnetospheric plasma. Figure 3 shows the variation in growth rate of EMIC waves with respect to wave number for different values of temperature anisotropy in magnetosphere of earth. For electron cyclotron waves the value of temperature anisotropy is assumed to be ~1 and for ion cyclotron waves, value of $T_{\perp i}/T_{\parallel i}$ is taken ~2 [36]. Therefore, in this paper growth rate of EMIC waves is calculated for $T_{\perp i}/T_{\parallel i}$ = 1.25, 1.5 and 1.75. **Figure 3** is plotted for growth rate of waves in background plasma using bi-Maxwellian and loss cone distribution function. For $T_{\perp i}/T_{\parallel i}$ = 1.25, peak value γ/ω_c =0.434248 at $\tilde{k}$ = 2.9, for $T_{\perp i}/T_{\parallel i}$ = 1.5, peak value γ/ω_c =0.544041 at $\tilde{k}$ = 2.7 and $T_{\perp i}/T_{\parallel i}$ = 1.75, peak value γ/ω_c =0.665216 at $\tilde{k}$ = 2.7. For j=1, $T_{\perp i}/T_{\parallel i}$ =

1.25, peak value γ/ω_c =1.106488 at $\tilde{k}$ = 2.5, for $T_{\perp i}/T_{\parallel i}$ = 1.5, peak value γ/ω_c =1.504767 at $\tilde{k}$ = 2.3 and $T_{\perp i}/T_{\parallel i}$ = 1.75, peak value γ/ω_c =2.340911 at $\tilde{k}$ = 2.1. Thus, the significant shift in lower wave number side and increase in magnitude of growth rate with increasing value of $T_{\perp i}/T_{\parallel i}$, is very evident from the graph. The results shown above are in accordance with [40].

After studying the effect of EMIC waves on growth rate without injecting any particle, a further study on the growth rate with the cold particle injection has been done and is shown.

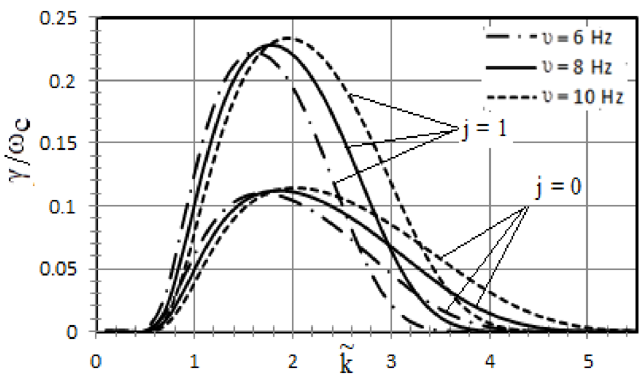

Figure 4. Variation of growth rate γ/ω_c versus $\tilde{k}$ for different values of A.C frequency and other parameters are $B_o = 2\times10^{-7}$ T, $n_o = 5\times10^6$ m^{-3}, $T_{\perp i}/T_{\parallel i}$ = 2, $K_BT_{\parallel i}$= 5 KeV, $K_BT_{\parallel e}$= 0.3eV, $E_o = 4\times10^{-3}$ V/m, n_c/n_w = 10.

In Figure 4 dimensionless growth rate of EMIC waves has been plotted with respect to wave number for different values of AC frequency (υ). Fixed plasma parameters are described in the captions of the graph. **Figure 4** displays the significant decrease in growth rate of EMIC waves after the cold injection has taken place. The graph shows the behavior of EMIC waves interacting with cold injected particles. The maxima for 6 Hz occurs at $\tilde{k}$ =1.7 with γ/ω_c as 0.110062, for 8Hz at $\tilde{k}$ =1.9 with γ/ω_c as 0.11222 and for 10Hz at $\tilde{k}$ =2.1 with γ/ω_c as 0.114143 for j=0 and for j=1, the maxima for 6 Hz occurs at $\tilde{k}$ =1.7 with γ/ω_c as 0.220737, for 8Hz at $\tilde{k}$ =1.7 with γ/ω_c as 0.227016 and for 10Hz at $\tilde{k}$ =1.9 with γ/ω_c as 0.233235 with noticeable shift in spectra as $\tilde{k}$ shifts. It is inferred that injection affects the lower wave number range of spectrum.

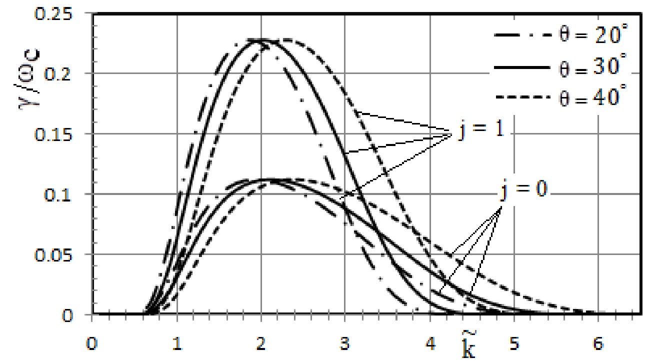

Figure 5. Variation of growth rate γ/ω_c versus $\tilde{k}$ for different values angle of propagation and other parameters are B_o = 2×10^{-7} T, $n_o = 5\times10^6$ m^{-3}, $T_{\perp i}/T_{\parallel i}$ = 2, $K_BT_{\parallel i}$= 5 KeV, $K_BT_{\parallel e}$= 0.3eV, ν = 8 Hz, $E_o = 4\times10^{-3}$ V/m, n_c/n_w = 10.

Figure 5 show the variation in growth rate of EMIC waves with respect to wave number for different values of propagation angle (θ). The study considers propagation of these waves at angle of 20^o , 30^o and 40^o with respect to ambient magnetic field of Earth. **Figure 5** is the study of growth rate of EMIC waves considering cold injection in magnetospheric plasma of Earth. The graph thus plotted, shows maximum peak value as γ/ω_c =0.112236 for θ =20^o , γ/ω_c =0.112083 for θ =30^o and γ/ω_c =0.112031 for θ =40^o at $\tilde{k}$ =1.9 to 2.3 for bi-maxwellian distribution and for loss cone distribution, the growth rate shows maximum peak value as γ/ω_c =0.227976 for θ =20^o , γ/ω_c =0.227331 for θ =30^o and γ/ω_c =0.226101 for θ =40^o for the same shift in k as in bi-maxwellian distribution. This result is in agreement with the result shown by [41], they concluded that there will be decrease in the growth rate as the wave becomes more oblique either due to landau damping or cyclotron resonant frequency and the broadness of frequency range increases.

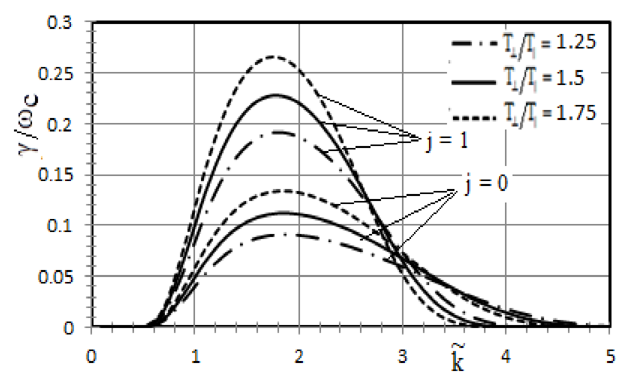

Figure 6: Variation of growth rate γ/ω_c versus $\tilde{k}$ for different values of $T_{\perp i}/T_{\parallel i}$ and other parameters are B_o=2×10^{-7} T,

$n_o = 5\times10^6$ m^{-3}, $K_BT_{\parallel i} = 5$ KeV, $K_BT_{\parallel e} = 0.3$ eV, $\nu = 8$ Hz, $E_o = 4\times10^{-3}$ V/m, $n_c/n_w = 10$.

Also, we analyze effect of temperature anisotropy after the interaction of EMIC waves with cold injected particles in magnetosphere of Earth in **figure 6**. It shows that maxima for j=0 for $T_{\perp i}/T_{\parallel i}$ =1.25 lies at $\tilde{k}$ =1.9 with peak value γ/ω_c as 0.091283, for $T_{\perp i}/T_{\parallel i}$ = 1.5, peak occurs at $\tilde{k}$ =1.9 with γ/ω_c =0.112224 and for $T_{\perp i}/T_{\parallel i}$ = 1.75, maxima lies at $\tilde{k}$ =1.9 with peak value of γ/ω_c = 0.13389 and for loss cone distribution, growth rate shows maxima for $T_{\perp i}/T_{\parallel i}$ =1.25 at $\tilde{k}$ =1.9 with peak value γ/ω_c as 0.190599, for $T_{\perp i}/T_{\parallel i}$ = 1.5, peak occurs at $\tilde{k}$ =1.7 with γ/ω_c =0.227016 and for $T_{\perp i}/T_{\parallel i}$ = 1.75, maxima lies at $\tilde{k}$ =1.7 with peak value of γ/ω_c = 0.265168. It is clearly seen that, bandwidth decreases and growth rate increases with increasing value of $T_{\perp i}/T_{\parallel i}$. Therefore, it can be said that injection events assuming the Loss-cone distribution of particles, affect the lower wave numbers of spectra. Also [41] indicated that the growth rate increases for the EMIC waves with increase in temperature anisotropy.

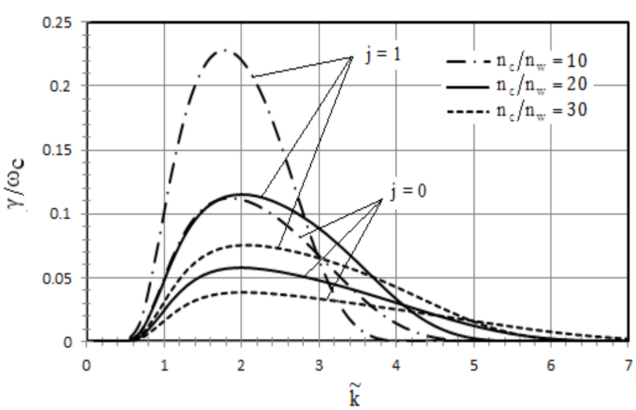

Figure 7. Variation of growth rate γ/ω_c versus $\tilde{k}$ for different values of n_c/n_w and other parameters are $B_o=2\times10^{-7}$ T, $n_o= 5\times10^6$ m^{-3}, $T_{\perp i}/T_{\parallel i}$ = 2, $K_BT_{\parallel i}$= 5 KeV, $K_BT_{\parallel e}$= 0.3 eV, ν = 8 Hz, $E_o = 4\times10^{-3}$ V/m.

Figure 7 show the variation in growth rate of EMIC waves with respect to wave number for different values of n_c/n_w. The study considers behavior of these waves at n_c/n_w of 10, 20 and 30 with respect to ambient magnetic field of Earth. **Figure 7** shows the change growth rate of EMIC waves when the magnetosphere of earth is injected by cold particles. The maximum growth rate increases from0.057708 to 0.112224 as the ratio changes from 30 to 10 with k shifting from 2.1 to 1.9 for bi-maxwellian distribution and it increases from 0.075553 to 0.227016 as the ratio changes from 30 to 10 with k shifting from 2.1 to 1.7 for loss cone distribution.

VI. CONCLUSION

In present paper electromagnetic ion-cyclotron waves, propagating obliquely with respect to magnetic field of Earth are investigated in presence of cold injections. Using liner dispersion relation, expression for growth rate has been derived for waves undergoing wave-particle interaction. Mathematical model using ring distribution function provided the dispersion relation, thus leading to expressions of growth rate and real frequency. It can also be used to study various types of instabilities in planetary magnetospheres. Calculations show that growth rate increases with increasing value of temperature anisotropy and AC frequency. Also, the growth rate increases as the angle of propagation with respect to Bo increases. After the interaction of EMIC waves with cold injected particles in magnetosphere of Earth, It is clearly seen that, bandwidth and growth rate both decreased with respect to the growth rate in case of no injection. But the growth rate increases as A.C. frequency, temperature anisotropy and angle of propagation increases. Also, the growth rate decreases with increases in the ratio of cold electrons to warm electrons with a shift to higher wave number side. Therefore, it can be said that injection events decrease the growth rate of particles and effects the lower wave numbers of spectra.

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