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Dynamic stability Analysis of a Bisplinghoff, Ashley and Halfman Wing

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Abstract— The present study outlines a procedure to evaluate flutter characteristics of a Bisplinghoff Ashley and Halfman (BAH) wing. The flexibility matrix has been derived using bending, torsion and shear stiffness distribution along the span of the wing. The procedural aspect of deriving the dynamically coupled energy equivalent mass matrix is outlined. The aerodynamic matrix used can be derived using the doublet methods. Nastran package has been used to develop the aerodynamic influence coefficient matrices using the doublet in strips. The final flutter analysis has been computed to verify the procedural aspects of the calculations.

Keywords— Dynamic Stability Analysis, Nastran

I. INTRODUCTION

Dynamic aero-elastic stability problems can be solved in any speed regime by selecting the appropriate aerodynamic theory. The solution involves a series of complex eigen value solutions, the eigen value problem to be solved depends on the way in which the aerodynamic loads are taken into consideration in the equations of motion or whether certain damping terms are included. Theodorsen in 1935 first developed the K-Method of flutter analysis, he introduced aerodynamics into a vibration analysis as complex inertial terms and the flutter analysis became a vibration analysis requiring complex arithmetic [1]. At the same time he introduced an artificial complex structural damping, proportional to the stiffness to sustain the assumed harmonic motion. Frazer in England was attempting to solve the flutter problem using aerodynamic stability derivatives [2]. This approach introduced the aerodynamic loads into equations of motion as frequency dependent stiffness and damping terms. In this representation it should be noted that the aerodynamic terms are slowly varying functions of the reduced frequency in contrast to the K method. Lawrence and Jackson gave a comparison of the British Method and the American Method [3]. Hassig introduced an alternate variation of the British method in which the aerodynamic loads are considered as complex springs; Hassig called this method the P-K method [4].

II. INFLUENCE COEFFICIENT MATRIX

The wing structure can be considered as made up of number of strips. The BAH wing is made into five strips. each strip will have a tapering section along the span and in the chord wise direction only inter torque box is considered to be effective in carrying the load. The chord section properties are expressed along the elastic point of the mid-section of the strip. It is necessary to know the location of the elastic point. The elastic axis is the locus of all the elastic points along the span of the wing. The elastic axis used for the computing bending and torsional stiffness is 35% of the chord.

A. Matrix – Bending Influence Coefficients

The bending influence coefficients are obtained numerically by evaluating following equations:

$$C_{ZZ}(y,\eta) = \int_0^y \frac{(\eta-\lambda)(y-\lambda)}{EI} d\lambda + \int_0^y \frac{d\lambda}{GK} \quad (\eta \geq y) \qquad (1)$$

$$C_{ZZ}(y,\eta) = \int_0^y \frac{(\eta-\lambda)(y-\lambda)}{EI} d\lambda + \int_0^y \frac{d\lambda}{GK} \quad (\eta \leq y) \qquad (2)$$

Expanding equation 1 and 2 and inserting $y = y_i$ and $\eta = y_j$ we obtain the expression for the bending influence coefficient C_{ij}. The bending stiffness matrix evaluated using the bending stiffness EI at the chosen stations which lie at the midpoint of the strips. Putting y_j =90 we can compute the first column of the bending influence coefficients where y_i is 90, 186, 268, 368 and 458 [5].

B. Matrix - Torsional Influence Coefficient

The torsional influence coefficients are obtained from the equation

$$C_{\theta\theta}(y,\eta) = \int_0^y \frac{d\lambda}{GK} \quad (\eta \geq y) \tag{3}$$

$$C_{\theta\theta}(y,\eta) = \int_0^y \frac{d\lambda}{GK} \quad (\eta \geq y) \tag{4}$$

From equation 4 and 3 the torsional influence coefficient is calculated taking values of GJ from [5]. The required $C_{\theta z}$ matrix can be obtained by utilizing the following equation (5).

$$C_{\theta z}(y,\eta) = \int_0^{y/cos\wedge} \frac{\frac{\eta}{cos\wedge} - \lambda}{EI}\, d\lambda \tag{5}$$

The C_{zz} and $C_{\theta\theta}$ matrix refers to the mid points which lie on the intersection of the strip and elastic axis. These are the span wise stations at which we introduce two control points. Then the required C_{hh} matrix will be of the order of 10 × 10 for 5 stations, we can get the C_{hh} matrix by working with C_{zz} $C_{\theta\theta}$ and $C_{\theta z}$.

III. THE MATRIX EQUATION

The deflection integral equation in terms of surface pressure for a cantilevered beam can be written as h=[a]f, where the force matrix must include both the integral and aerodynamic forces. A suitable division of the surface plan form must be made that is amenable to analysis of both mass and aerodynamic distribution. Such a division is a series of chord wise strip parallel to the free stream direction. The mass distribution of each strip may be replaced by forward and aft concentrated masses and the aerodynamic lift and pitching moment may be replaced by forward and aft concentrated forces at the same point. If we define a complex matrix of oscillatory aerodynamic influence coefficient such as

$$F_a = \omega^2(M + \rho b_r^2 s[C_h])h \tag{6}$$

If we substitute h=[a]f into equation 6 and divide the flexibility coefficient by (1+ig) to account for the structural damping necessary to sustain the assumed harmonic motion, we obtain the matrix equation for flutter.

$$h = \omega^2 / 1 + ig([a]([M] + \rho b_r^2 s[C_h]h) \tag{7}$$

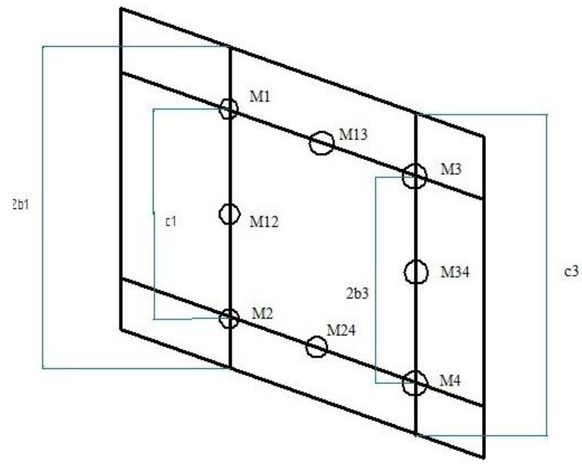

Figure 1: Surface Geometry and Mass Distribution

Equation 7 can be solved for the complex mode shape (h) and complex eigen value (1 + ig)/ω^2 by complex matrix iteration from the eigen value.

$$\lambda = \lambda_r + i\lambda_I = (1 + ig) / \omega^2 \tag{8}$$

and we obtain the flutter frequency as

$$\omega = i / \sqrt{\lambda_r} \tag{9}$$

and structural damping is given as,

$$g = \lambda_I / \lambda_r \tag{10}$$

from a series of solutions for different reduced frequencies a damping velocity curve can be constructed for a specific altitude.

A. Mass Matrix

Here the number of degree of freedom remains the same as the number if control points. Figure 1 shows the configuration with degrees of freedom (an illustration). We now derive the inertia force at first control points from Lagrange's equation as an example to derive the mass matrix. To simplify kinetic energy expressions we place the intermediate masses at the mid-point.

$$2T = M_1h_1^2 + \ldots + M_4h_4^2 + M_{12}\left[\frac{(h1+h2)}{2}\right]^2 + M_{13}\left[\left(\frac{h1+h3}{2}\right)\right]^2 + M_{24}\left[\left(\frac{h2+h4}{2}\right)\right]^2 + M_{34}\left[\left(\frac{h3+h4}{2}\right)\right]^2 \tag{11}$$

From the kinetic energy equation given in [11] we find equation [12]

$$F_1 = -\frac{d}{dt}\left(\frac{dt}{dh1}\right) = -(M_1 + 0.25M_{12} + 0.25M_{13})h_1 - 0.25M_{12}h_2 - 0.25M_{13}h_3 \tag{12}$$

In the same manner the resultant inertia force can be derived at each control point yielding the dynamically coupled mass matrix for two strip surface. In a similar manner the mass matrix for the entire wing can be derived.

M_{11}	$0.25M_{12}$	$0.25M_{13}$	0
$0.25M_{12}$	M_{22}	0	$0.25M_{24}$
$0.25M_{13}$	0	M_{33}	$0.25M_{34}$
0	$0.25M_{24}$	$0.25M_{34}$	M_{44}

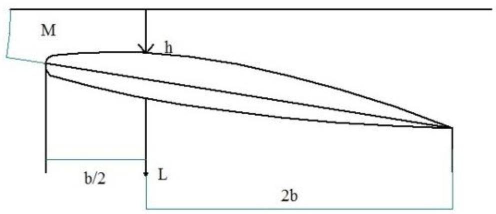

Figure 2: The lift and moment at quarter chord point

B. Aerodynamic Matrix

The aerodynamic matrix is a set of influence co-efficients by which the aerodynamic force at any control points as defined in the equations. We begin with a consideration of strip theory because of its traditional use in flutter analysis. This covers high aspect ratio low mach and low aspect ratio high mach surfaces. The two dimensional oscillatory aerodynamic co-efficient are generally tabulated as lift and moment co-efficient referred to the airfoil quarter chord. Since the problem at hand is to replace this co-efficient by force co-efficient at the section of forward and aft control points which lie on the surface of the quarter chord line. Hence it becomes convenient if the forward control point lies on the surface quarter chord point line. The positions of the aft control point can be arbitrary of the rear spar. Figure 2 shows the lift and moment at quarter chord the oscillatory lift on the inboard strip of the shown figure is given by equation [13]

$$L = \pi\cos\Lambda\rho\omega^2 b_1^2 \Delta y_1(L_h(h/b_1) + L_\alpha\alpha) \quad (13)$$

$$L = F_1 + F_2 \quad (14)$$

And the moment

$$M = \pi\cos\Lambda\rho\omega^2 b_1^4 \Delta y_1(M_h(h/b_1) + M_\alpha\alpha) \quad (15)$$

We know that $\alpha = (h_2 - h_1)/c_1$ hence

$$F_1 = \pi\cos\Lambda\rho\omega^2 b_1^2\Delta y_1[[(L_h/b_1) - (L_\alpha/b_1) - (b_1/c_1)((Mr/b_1) - (M_\alpha/c_1))]h_1 + ((L_\alpha/c_1) - (b_1/c_1^2)M_\alpha)h_2] \quad (16)$$

$$F_2 = \pi\cos\Lambda\rho\omega^2 b_1^4(\Delta y_1/c_1)[((M_h/b_1) - (M_\alpha/c_1))h_1 + (M_\alpha/c_1h_2)] \quad (17)$$

The above equations 16 and 17 can also be written in matrix form. In the above equations the subscript on the oscillatory coefficients indicates the dependent on the inboard strip reduced frequency. By applying the same developed equations to the outboard forces which can be combined with the inboard forces into the partitioned matrix equation. The matrix aerodynamic influence coefficients at control points for the BAH wing for k=0.8 can be computed for 10 control points, which will be a 10×10.

IV. RESULTS

Solving the developed matrices gives us results for structural damping and velocity for 2 mode shapes and we can compare this with the structural damping and velocity from the assume mode method described by Robert et al[5].

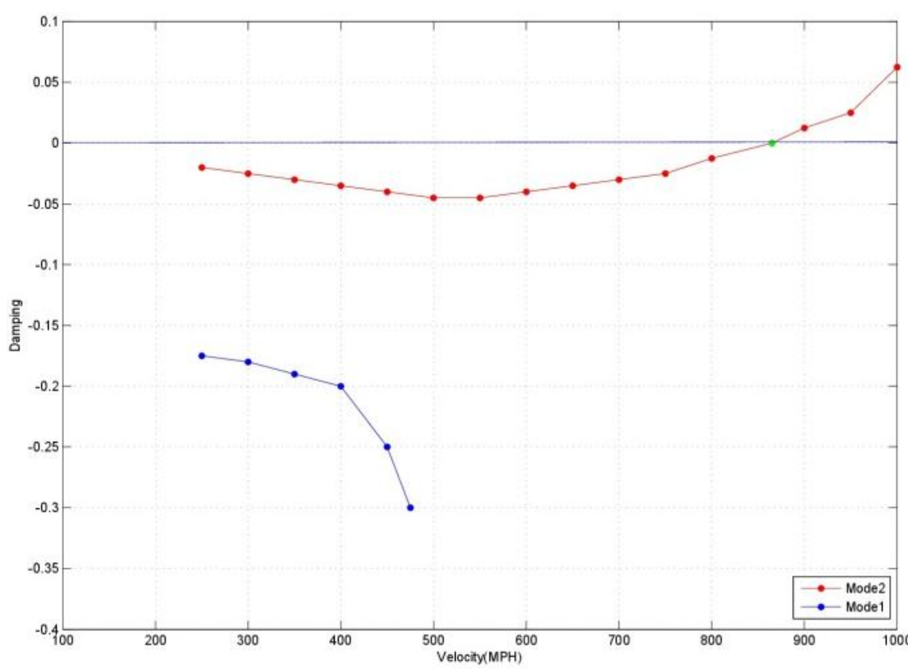

Figure 3: Graph of Damping vs Velocity using the developed matrix methodology

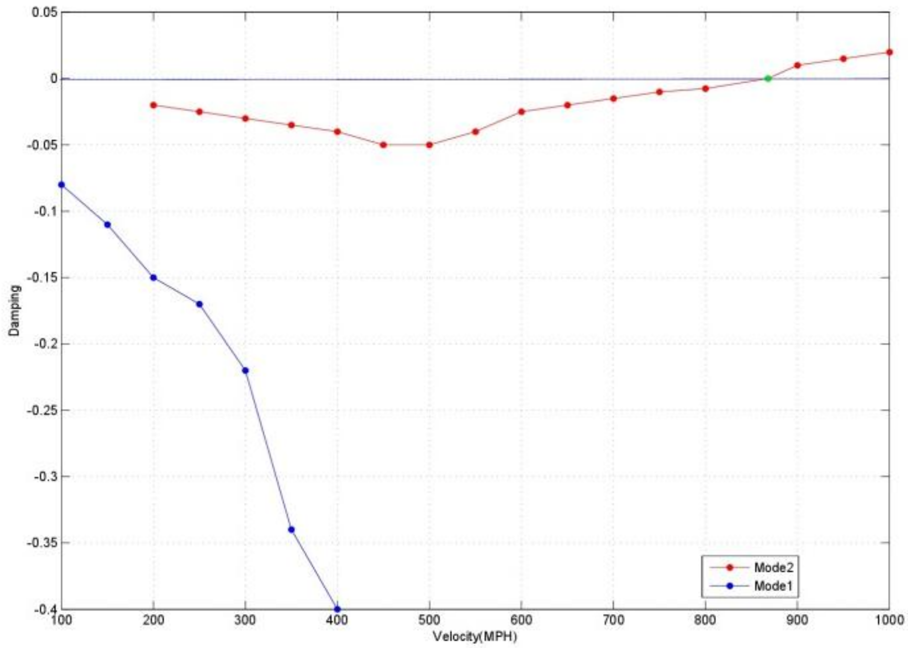

Figure 3: Graph of Damping vs Velocity from the bending torsion analysis from the assumed mode method as directed in [5]

Figure 3 shows the graph of Damping vs Velocity developed from the present derived methodology and figure 4 shows the graph of Damping vs Velocity from the bending torsion analysis from the methodology given in [5], where the green dot indicates the change of sign of damping indicating the critical flutter speed. The mode shapes of the standard BAH wing is as shown in Ref [5]. The critical speed of which is 865MPH obtained by applying the assumed mode method of flutter analysis. Thus by comparing the results of flutter analysis we can say that the results to calculate the critical speed are very close. The result by the matrix method is 868MPH and the result from the method described in [5] is 865MPH.

V. CONCLUSION

This method offers advantages over the other used methods. This type of formulation gives a three dimensional solution applicable to low and high aspect ratio surfaces. Other advantage is that large number of degrees of freedom can be handled using high speed computing, and also that the flexibility, mass and aerodynamic data are kept separate, we can then change the one by keeping the other two the same. The present analysis can be easily extended to box type of structure. The accuracy between experimental and theoretical flexibility matrix would enable the present work to be extended to dynamic response and stability analysis.

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