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Predicting US Covid-19 Infection Rate using Time-delay Recurrence Simulation

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Abstract—A recursion model is developed to describe the growth of Covid-19 cases in the USA. An essential requirement of any model is confidence in its predictive capability. Published growth rates for Covid-19 in the USA are shown to correlate well with recursive time-delay simulations. Data for six months after March 2020 is compared to predictions from known logistic equations and modified time-delay relations. Simple logistic equations do not show a correlated trajectory with case numbers, whereas a time-delay recurrence equation can be calibrated to follow actual data. Modelling predicts over 10 million infections by January 2021. Growth curves projected to mid-2022 are examined and discussed. Infection totals in the USA is predicted to approach 35 million cases by the end of 2021 without human control.

Keywords—US Covid-19, Simulations, Prediction, Growth Rates, Recurrence Model

I. INTRODUCTION

Population modelling attempts to predict the outcome of competition between species. A study of natural laws of growth and decay applies to numerous scientific disciplines including ecology, environment, physics, engineering, chaos and biology. A virus in the community is considered a predator and so lends itself to predictive growth analysis.

Many researchers have postulated mathematical descriptions of the growth and decay of organisms. Ordinary differential equations (ODE) describe the way a virus population might change over time. The first known use of an ODE equation to describe population growth was published in 1845 by Verhulst [1]. Verhulst's equation $dN/dt = \beta.N(t)\ [\ (P - N(t))/P]$ describes a logistic equation where N(t) is the population at time t, P is the maximum population, and β is a rate constant.

In 1926, Volterra further developed growth equations with two variables to describe prey-predator in an ecosystem [2]. Volterra's model assumes a maximum possible population P that can be infected by a predator such as a virus. A more general growth model was proposed by Richards in 1959, allowing population growth to be broken into three phases, i.e., the initial, middle and final stages. Richard's model resulted in solutions similar to the logistic model [3].

Substantial literature exists on Covid-19 virus pandemic. A recent paper by Wu et.al. describes the use of the logistic equation to model growth rates in China and other parts of the world [4]. More recently a paper by Mathews describes the rate of transmission of the virus by proximity to infected persons [5]. Use of the logistic model alone does not explain the trajectory of virus growth in the USA. Fundamentals of Covid-19 growth are examined in this paper using recurrence relations. Calibration of the model uses published virus growth figures to predict trajectories of growth into the future.

Section I has introduced he problem of finding a mathematical method that most accurately describes Covid-19 growth paths. Section II contain related work, Section III contains the methodology adopted for this research, including development of recursion models for geometric and arithmetic growth with modulated recursion, Section IV describes results and discussion and Section V concludes research work with future directions.

II. RELATED WORK

Known research related to the propagation of Covid-19 has, in general, relied on traditional methods to investigate the problem. Methods used to simulate Covid-19 growth have manly examined various logistic-type equations to predict growth. As with all predictive methods, the aim in this case is to determine the duration and the final levels of infection in the community. Data available for USA Covid-19 case numbers have shown poor correlation with simple logistic or statistical analyses.

Research was required to develop a method of analysis that provides a closer correlation between virus transmission trajectory and actual data. To my knowledge, no one has investigated the use of time-delay modulated recursion methods to describe the trajectory of coronavirus spread in the United States.

III. METHODOLOGY

Logistic and time-delay difference equations applied to predator-prey and population growth is used in many fields of modelling [6] [7]. In this paper, notation (N(T) and N(T+1) define the number of Covid-19 cases at discrete time interval T and the next interval (T+1), respectively. The time step interval is T=1 week, and N(T) is the summed 7-day average in the discrete interval T. Numerical analysis by recursion is considered applicable to the analysis of Covid-19 infection rates in the USA.

Curves and recorded data for US virus growth over time can be obtained from the internet site Worldometer. Countries including USA, Canada, Germany, UK, Russia, Australia and South Africa all show a "general" growth trajectory of the type presented in Figure 1.

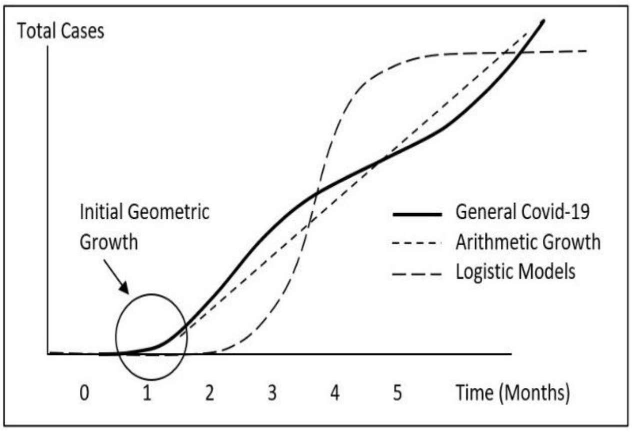

Figure 1: General curve of Covid-19 growth trajectories, compared to logistic models

A typical curve of Verhulst's logistic model in Figure 1 shows delayed initial growth, followed by a trajectory to an asymptotic value, the maximum infectable population.

Pandemic growth such as the 1918 flu (a H1N1 swine flu virus) resulted in a 33% infection rate, while the same virus in 2009 infected between 11-21% of the population. Previously, the Hong Kong flu (H3N2) of 1968 infected >14% of the population while the Asian flu (H2N2) of 1957 infected 14% of the population. Based on historical data, SARS-CoV-2 (Covid-19) may infect up to 12% of the US population (β = 1.12), without a greater level of research to control virus spread.

Figure 2 shows a graph of total reported US Covid-19 cases to mid-August 2020. Virus growth clearly does not conform to that expected for the logistic equation [4]. The initial 4 weeks of virus spread shows an exponential increase in cases (geometric growth), followed by a linear rise for 6 weeks (arithmetic growth). After 10 weeks of growth, a slowdown followed by a rise in cases occur, a variance that requires further analysis. Note that by late August 2020, countries such as India, Mexico and Brazil are still in a general geometric growth phase, showing classic exponential trajectories.

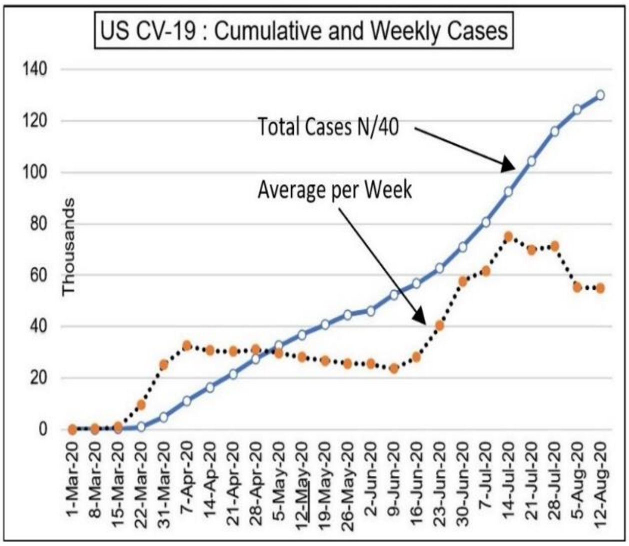

Figure 2: Graph of US cumulative cases and discrete weekly numbers

A general ODE that describes the rate of change of a population N(t) at time t as being proportional to N(t) is dN(t)/dt = βN(t), where "β" is a constant or a growth rate coefficient. In discrete time T, the growth can be represented by a difference equation N(T+1) = βN(T), which exhibits geometric or exponential growth. An initial condition at T = 0 would be N = N(0).

Discrete time equations lend themselves to iterative or numerical simulation by recursion, because the next value N(T+1) at time step (T+1) is related to the previous value N(T) at time T by the difference equation. Logistic model growth (Verhulst) is written as by the recursion equation:

$$N(T+1) = \beta N(T)[1 - N(T)/P] \quad (1)$$

where P = 330 million and β =1.12. In the simulations that follow, T = 0 begins on 29-February-2020. Application of the logistic equation to US data predicts that N(T+1) exceeds 30 million all-time cases, occurring at T = 90 which equating to the end of October 2021. The curve of equation (1) is discussed further in relation to Figure 5.

Numerous difference equations exist to describe physical growth or decay [6]. For analysis of US Covid-19 infection growth rate, a general time-delay recurrence relation is formulated with the form:

$$N(T+1) = \beta[N(T) - m(T)] - \alpha N(T-1) \quad (2)$$

where m(T) is a new modulating function introduced to accommodate case number control, such as lockdown and un-lockdown. In the case where both m(T) = 0 and α = 0, equation (1) reverts to N(T+1) = β (N(T), i.e. geometric growth. In the recursive process, N(1) = β N(0), N(2) = β N(1) or N(2) = β^2N(0), and so on. A solution to the recursive analysis of a geometric series is N(n) = β^n N(0).

IV. RESULTS AND DISCUSSION

Figure 1 describes general types of mathematical growth model curves along with a typical Covid-19 growth curve. Application of equation (2) in recursive calculations show that trajectories of actual case numbers for US Covid-19 correlate under the following constraints:

Geometric growth

$$N(T+1) = 2.3\,N(T) \quad \text{(3-a)}$$

Conditions: $0 < T < 5$, $m(T) = 0$, $\alpha = 0$
(T = 0 at 1-March-2020)

Arithmetic growth

$$N(T+1) = 2\,N(T) + D\delta \quad \text{(3-b)}$$

Conditions: $4 < T < 14$, $\beta = 2$, $D = 30{,}000$/day, $\delta = 7$

Modulated growth

$$N(T+1) = 2[N(T) - m(T)] - N(T-1) \quad \text{(3-c)}$$

Equations (3-a) and (3-b) are first-order linear difference equations, whereas equation (3-c) is a second-order time-delay difference equation. Equation (3-b) has a basis in the Taylor series expansion of $N(T+\delta) = N(T) + (dN/dt).\delta$ where δ is a discrete interval. Equation (3-c) equates to the homogeneous part of an ODE $y'' - r\,y' + y = f(t)$, where $r = 2(1+k\ \lambda/P)$, k is typically a modulation multiplier with values near +/- 1, D is the daily case number averaged over 1 week, and $\lambda \approx D/4$.

Modulating functions m(T) provide a method of controlling variations to the model at any time step. Variations include effects of lockdown on growth, transmission issues in crowds, general opening-up activity and vaccine application [5].

Graphing the trajectories of Covid-19 cases uses calculated recursive values from the above equations. For example, equation (3-b) contains the term D = dN/dt which is calculable from Worldometer data. If the daily infection value of Covid-19 in the US on day-1 was 27,000/day, a daily increase by 1,000 for 7 days yields D = 30,000 cases/day on average for that week.

Initial Virus Growth Trajectories

Equations (3-a) and (3-b) are now applied to US Covid-19 data analysis. Figure 3 shows US Covid-19 growth curves for reported total cases, compared to simulated values using the geometric recursion of equation (3-a). The simulation closely follows actual growth up to T = 8 (March-24-2020). Beyond this time, progression predicted by equation (3-a) begins to rise faster than reported figures. Linear growth after T = 8 suggests the arithmetic model of equation (3-b) would be more applicable at larger case numbers. Figure 4 shows that Covid-19 growth is relatively linear between March-24 to May-19-2020.

Using the gradient D = dN/dt =30,000/day defined above, case numbers at week T = 8 sets the initial conditions of the model for equation (3-b), i.e., the value at 31-March-2020 is N(4) = 194,114 cases and the next value N(5) = 444,000 cases. Predicted trajectory of growth by Equation (3-b) is in good agreement with the recorded data in the range 4 < T <14.

Change in the trajectory of US Covid-19 total cases from geometric to arithmetic growth after T = 8 is generally mirrored in data from other countries. A more complex model is therefore required to simulate observed growth, rather than considering zonal growth patterns suggested by Richards [3].

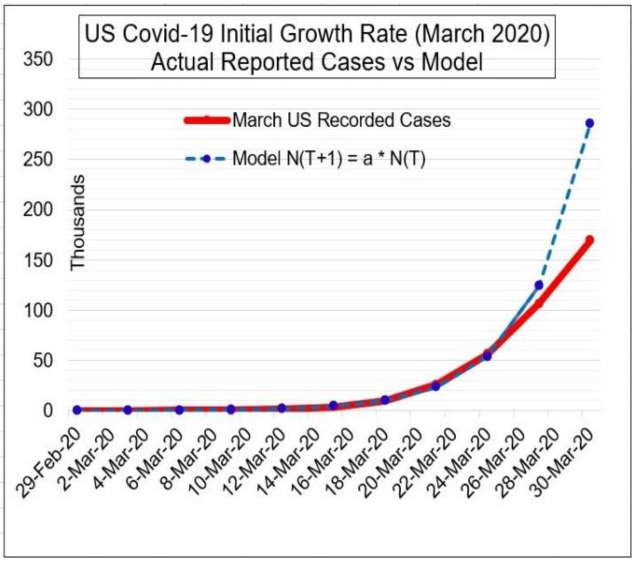

Figure 3: Initial total case curves of US Covid-19 According to equation (3-a)

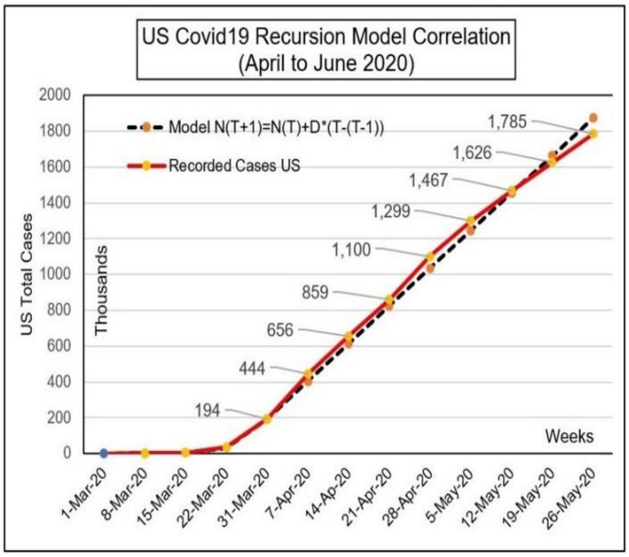

Figure 4: US Covid-19 growth curves from March to May 2020 as defined by equation (3-b)

Time-delay Recurrence Model

Recorded actual case numbers between May-31-2020 and August-30-2020 show a modulation or variation in the trajectory of total cases, as seen in Figure 5. Applying the time-delay recurrence equation (3-c) to simulate Covid-19 growth in the US produces a graph that more closely follows actual case numbers.

No Modulation Delay Model

A limiting growth function m(T) is defined and applied to equation (3-c) as follows:

$$m(T) = k\lambda[1 - N(T)/P] \quad (4)$$

where λ = 7200, k = 1 and P =330 million, the population that could be 100% infected. The value of D used for the start of simulation is 28,800 cases/day.

A graph for T < 32 is shown by the "dashed line" in Figure 5. During the early phase of the pandemic, N(T) is small and m(T) is very small by comparison to P. For N(T) large, m(T) = 0, and the population growth plateaus.

Figure 5 shows the graphs of actual US case data plotted from 5 February 2020 to mid-October 2020. A simulation curve for the delay model (no modulation) uses a unity λ value is used in equation (4). Included is an expanded view of the modulated curve, discussed in the next section.

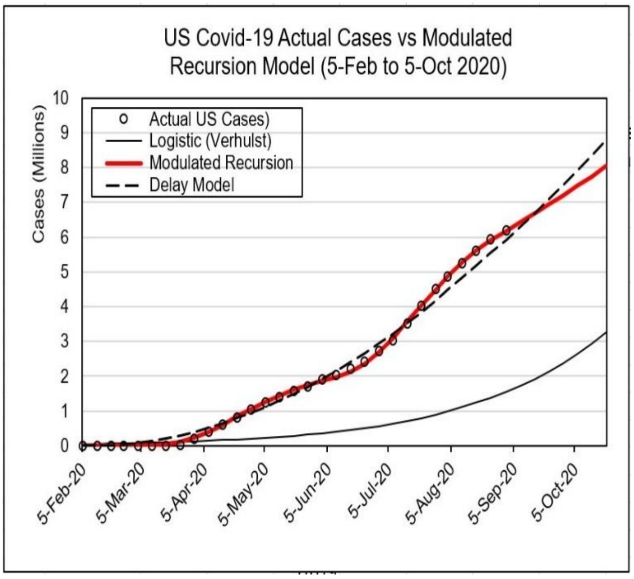

Figure 5: Recursive model projections for US Covid-19 to October 2020 per equation (3-c)

Examination of the simulation graph in Figure 5 shows a gradual rise to about 6.5 million cases by September-5-2020, which compares reasonably to the actual case numbers shown by "circles" in the figure. However, a Verhulst logistic graph shows the population slowly rising to 1.5 million cases by the beginning of September 2020, well below the expected 6.5 million cases.

Model Graph - Modulated Recursion

Equation (4) applies a λ value to simulate known growth behaviour in equation (3-c), from which future trajectories may be speculated in relation to social conditions. Table 1 lists λ values for equation (4) that produce a correlation between case numbers and the modulated recursion model solution, shown in expanded form in Figure 5..

Table 1: Variables for the general model in equation (4)

T	< 6	< 12	< 18	< 24	< 32	< 40	> 40
λ	0	2.8	-1.4	5.6	-1.4	0	1

Projection to 2022

Growth of Covid-19 in the US in the future can be broadly inferred from known data, and from asymptotic limits of the above logistic models. By the end of August 2020 over 6 million infections have been recorded (in 7 months). Infection rate averages 250,000 cases per week or 30,000 cases per day. At present an average value of 29,815/cases day has not changed significantly since early March 2020.

Figure 6 shows a graph of the time-delay recurrence equation (3-c) projected to May 2022. Both modulation and no modulation curves are based on amplitude and frequency set according to Table 1. Beyond 30 million infections after August 2021 (T > 84), time-delay solutions begin to "flatten" due to action of the terms in equation (4).

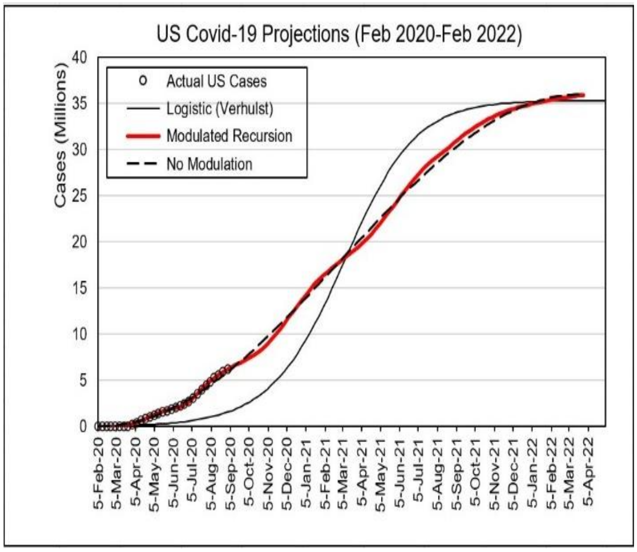

Figure 6: Recursion model solutions projected until natural saturation in March 2022

Discussion

A hypothesis of the research was that a predictive method could be developed to simulate Covid-19 growth curves. To develop the hypothesis, research on recurrence relations appear applicable due to their known ability to model predator-prey relationships in large population studies. Nevertheless, equations did not exist to describe or predict trajectories of US Covid-19.

Given that the US case numbers continue to grow, any research on projection of case numbers is worthwhile. At present, the US is about one-quarter the way through the pandemic trajectory. A vaccine or major social isolation could logically reduce prediction of final case numbers.

Based on the current research, further investigations are required to understand the dynamics of long-term transmission, for T > 100 weeks. Simulation using the derived time-delay equation (3-c) suggests that larger modulation amplitudes are possible for T > 40 weeks in Table 1. Any increase in λ values will cause larger swings and surges in infection rates.

The US has experienced widespread virus growth without real control. From simulations, an average 14-week cycle in peaks and troughs occur, indicating a surge, lull and surge cycle that may be linked to how the virus transmits in the community. Clearly, the entire US count case is investigated, which does not take into account localised variations in high density regions. One surmises that infected case numbers for the entire 50 US states is the simple summation of each state's infected cases.

Four main points are noteworthy from the research, a) infections should average 30,000/day until an external influence forces its reduction, b) lockdown activity typically reduces growth by 15,000/day, c), economic opening raises cases 40,000/day, d) peak-to-peak wave cycles of 28 weeks exist in the data.

V. CONCLUSION AND FUTURE SCOPE

The paper provides a new understanding of the dynamics of Covid-19 growth rates in the USA. A mathematical description of the growth in total cases examines geometric, arithmetic and time-delay recursion methods. Models are calibrated to published case numbers and used to forward-predict trajectories of the virus spread in the community until mid-2022.

Solutions presented in graphical form compare well with reported case numbers to date. US data to mid-August 2020 shows the growth trajectory contains a modulation, simulated by including loss or gain in the modulated recursion model. A modulated recursion method results in a good correlation with actual case numbers up to late August 2020. Correlation enables a projection towards 2022 with some confidence.

Recursion models have not included external human influence on virus transmission. A time-delay recursion model predicts total infection levels exceeding 30 million cases by early 2022. Projections for Covid-19 case number in the US is necessary to allow proper planning and allocation of economic resources. Development of a modulated recursion model that inherently generates natural cyclical functions may provide a new insight into the way pandemics and epidemics propagate in the US population.

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