

Effects of Dissipation on Magneto-Convection Flow over a Shrinking Surface with Chemically Reactive Species under the Influence of Radiation and Internal Heat Generation

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Abstract— The effects of chemical reaction and dissipation on nonlinear MHD flow over a shrinking surface subjected to heat and mass flux under the influence of radiation and internal heat generation have been analyzed. Similarity transformations are utilized to obtain the nonlinear ordinary differential equations from nonlinear partial differential equations. Nachtsheim Swigert shooting iteration scheme together with Runge Kutta fourth order method is applied to the reduced highly nonlinear equations. Effects of pertinent parameters on heat and mass transfer characteristics are thoroughly examined. From the investigation it is noted that the species concentration boundary layer thickness reduces due to increase in Schmidt number and Chemical reaction parameter.

Keywords— Forced convection; thermal radiation; Chemical reaction; Heat and mass flux.

I. INTRODUCTION

The combined effect of heat and mass transfer with chemical reaction has received the attention of researchers in recent years due to its occurrence in various branches of science and technology. The analysis of chemical reaction with heat transfer has enormous engineering applications like tubular reactors, oxidation of solid materials and synthesis of ceramic materials. There are two types of reactions namely, homogeneous reaction and heterogeneous reaction. A reaction that takes place uniformly throughout the given phase is said to be homogeneous, whereas the reaction that occurs in a restricted region or within the boundary of a phase is termed as heterogeneous. The effects of chemical reaction depend on whether the reaction is homogeneous or heterogeneous. In first order chemical reaction, the rate of reaction is directly proportional to concentration itself. In numerous industrial applications such as polymer production, manufacturing of glass ware etc., involves flow and mass transfer over a continuously moving surface. The diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid which can greatly affect the flow thereby affecting the properties and the quality of the final product.

Chambre and Young [1] considered the diffusion of a chemically reactive species in a laminar boundary layer flow which develops over the surface of a body. Later, Anderson et al. [2] investigated the effect of transfer of chemically reactive species in the laminar flow over a stretching sheet. Afify [3] explicated the MHD free convective flow of viscous fluid and mass transfer over a stretching sheet with chemical reaction. Mbeledogu and

Ogulu [4] encountered the effects of chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source. Akyildiz [5] obtained the solution for non-Newtonian fluid flow with chemically reacting species over a stretching sheet embedded in porous medium. Mansour et al. [6] analyzed the effects of chemical reaction and thermal stratification on MHD free convective flow through a porous medium over a vertical stretching surface. Mohamed and Abo-Dahab [7] studied the effect of heat generation, radiation and chemical reaction on heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium. Bhattacharyya and Layek [8] discussed the behaviour of chemically reactive solute distribution in MHD boundary layer flow over a permeable stretching sheet. The effect of chemical reaction on slip MHD liquid flow and heat transfer over non-linear permeable stretching surface was investigated by Yazdi et al. [9].

In recent times, the flow of an incompressible fluid due to a shrinking sheet acquires attention for its abnormal behavior in the flow dynamics. The flow past a shrinking sheet exhibits quite distinct physical phenomena from the forward stretching flow. For this flow configuration, the fluid is stretched towards the slot. It is also revealed that to maintain the flow over the shrinking sheet the mass suction is required. Miklavcic and Wang [10] confirmed the existence and uniqueness of the solution of steady viscous flow over a shrinking sheet. Boundary layer flow over a shrinking sheet with power law velocity was addressed by Fang [11]. Yao and Chen [12] examined analytical solution branch for the Blasius equation with a shrinking sheet.

Fang and Zhang [13] obtained an analytical solution for steady MHD flow over a porous shrinking sheet subjected to applied suction. Noor and Hashim [14] discussed the viscous electrically conducting fluid flow and heat transfer adjacent to a permeable shrinking sheet embedded in a porous medium. MHD flows of a viscous nonlinear shrinking sheet with homotopy analysis have been studied by Nadeem and Hussain [15]. Ali et al. [16] analyzed the MHD viscous flow and heat transfer due to a permeable shrinking sheet with constant heat flux. Fang and Zhang [17] obtained the analytical solution for thermal boundary layers over a shrinking sheet for variable surface temperature and variable surface heat flux. Javed et al. [18] discussed the effect of viscous dissipation on heat transfer over a non-linear shrinking sheet incorporating the effects of applied magnetic field. The shrinking sheet problems are extended by considering the radiation effects. Krishnendu Bhattacharyya and Layek [19] analyzed the effect of suction/blowing and thermal radiation on steady boundary layer stagnation point flow and heat transfer towards a shrinking sheet. The effect of thermal radiation on micropolar fluid flow and heat transfer over a shrinking sheet for the case of constant temperature was studied by Bhattacharyya [20].

The diffusion of species with chemical reaction in the boundary layer flow has huge applications in water and air pollution, fibrous insulation, atmospheric flows and many other chemical engineering problems. Hayat et al. [21] initiated the flow and mass transfer of an upper convected Maxwell fluid past a porous shrinking sheet with chemical reaction species and applied magnetic field. Muhaimin et al. [22] analyzed the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow past a porous shrinking sheet with suction. The exact solutions of chemically reactive solute distribution in MHD boundary layer flow over shrinking surface is considered by Midya [23] without considering mass suction and heat transfer. Numerical solution of MHD flow and heat and mass transfer over a porous shrinking surface was obtained by Khalid et al. [24] considering radiation and viscous dissipation effects. Kamal et al. [25] made an attempt on stability analysis of MHD stagnation-point flow over a permeable stretching/shrinking sheet in a nanofluid with chemical reaction effect. He considered the shrinking surface with constant surface temperature and constant surface concentration.

Based on the above investigated works and its applications, the author has attempted to study the effect of chemical reaction, dissipation, radiation and heat generation on hydromagnetic flow over a shrinking surface prescribed with heat and mass flux, the analysis of which has not been made so far. The aim of the paper is of two fold. (i) Heat transfer analysis is carried out by taking into account of the effects of radiation, heat generation, viscous and Ohmic dissipation and mass transfer analysis is carried out by considering the chemical reaction effect. (ii) The shrinking surface is subjected to heat and mass flux. The numerical

code used in the study is validated by comparing the results with the available results in the absence of certain parameters.

This paper is structured as: Section I contains the introduction and its related work. Section II includes the mathematical formulation of the problem. Section III provides the numerical technique of solving the governing equations. Section IV encloses the graphical representation of the results and its discussion. Section V comprises of concluding remarks with future directions.

II. MATHEMATICAL FORMULATION

Two dimensional, steady MHD boundary layer flow of a viscous, incompressible, electrically conducting and radiating fluid with heat and mass transfer over a shrinking surface with chemical reaction, internal heat generation and dissipation effects is analyzed in this investigation. The x axis is introduced along the shrinking surface with linear velocity u_w and a uniform magnetic field is applied along the y axis.

- The fluid is considered to be a gray, absorbing and emitting radiation but non-scattering medium and Rosseland approximation is used to describe the radiative heat flux in the energy equation.
- The radiation heat flux in the x direction is considered to be negligible in comparison to the heat flux in the y direction.
- A uniform transverse magnetic field B_0 is imposed perpendicular to the sheet. The induced magnetic field due to motion of electrically conducting fluid is assumed to be negligible as the magnetic Reynolds number is considered small.
- It is noted that the electric field due to polarization of charges is assumed to be negligible and as $\text{curl } \mathbf{E} = 0$ and $\text{div } \mathbf{E} = 0$, \mathbf{E} is assumed to be zero.
- The first order chemical reaction takes place in the species concentration equation.
- Soret and Dufour effects are neglected, as C_∞ is infinitesimally small.
- The rate of chemical reaction is constant throughout the fluid.

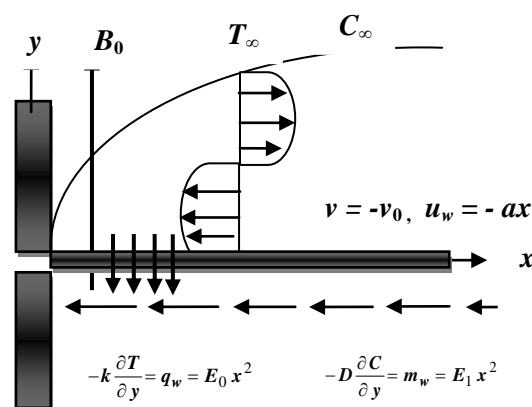


Figure 1. Schematic diagram of the problem

Under the usual boundary layer approximation and the above mentioned assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration in the presence of radiation, magnetic field, viscous and Joule's dissipation effects, heat generation and chemical reaction are specified by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q(T - T_\infty) + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \sigma B_0^2 u^2 \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \tag{4}$$

Boundary conditions pertaining to the above equations are

$$\left. \begin{aligned} \text{At } y = 0 : \quad & u = u_w = -ax, \quad v = -v_0, \\ & -k \frac{\partial T}{\partial y} = q_w = E_0 x^2, \quad -D \frac{\partial C}{\partial y} = m_w = E_1 x^2 \\ \text{As } y \rightarrow \infty : \quad & u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \end{aligned} \right\} \tag{5}$$

where u, v are the x and y components of velocity, T and C are the fluid temperature and concentration respectively, ν is the kinematic viscosity, σ is electrical conductivity, B_0 is the strength of the magnetic field, ρ is the density of the fluid, C_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, T_∞ is temperature of the fluid far away from the sheet, μ is the coefficient of viscosity, q_r is the radiative heat flux, Q is the volumetric rate of heat generation, D is the molecular diffusivity, k_1 is the rate of chemical reaction, $a > 0$ is a dimensional constant called shrinking rate, v_0 is the constant suction velocity, E_0 and E_1 are the constants, q_w and m_w denote heat and mass flux at the wall and C_∞ is the concentration far from the plate.

Similarity transformations

In order to solve the continuity equation (1) and the momentum equation (2) with boundary conditions in (5), the following similarity transformations are introduced.

The stream function ψ is defined by $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$

$$\left. \begin{aligned} \text{where } \eta = y \left(\frac{a}{\nu} \right)^{\frac{1}{2}}, \quad \psi(x, y) = \sqrt{a\nu} x F(\eta) \\ u = ax F'(\eta), \quad v = -\sqrt{a\nu} F(\eta) \end{aligned} \right\} \tag{6}$$

From the choice of u and v it is noted that the continuity equation (1) is identically satisfied. Employing the equation (6), the partial differential equation (2) with boundary condition in (5) is reduced to the following ordinary differential equations

$$F''' + F F'' - (F')^2 - M^2 F' = 0 \tag{7}$$

with boundary conditions

$$F(0) = S, \quad F'(0) = -1 \quad \text{and} \quad F'(\infty) = 0 \tag{8}$$

where the derivative symbol “'” represents $\frac{d}{d\eta}$.

$M^2 = \frac{\sigma B_0^2}{\rho a}$ is the Magnetic parameter,

$S = \frac{v_0}{\sqrt{a\nu}}$ is the Suction parameter ($v_0 > 0$).

Heat and Mass Transfer Analysis

The radiative heat flux term in equation (3) is simplified by using the Rosseland approximation for radiation for an optically thick layer, and one can write (Brewster [26]),

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{9}$$

where σ^* denote the Stefan Boltzman constant and k^* represent the mean absorption coefficient. It is assumed that the temperature difference within the flow is such that T^4 can be expressed as a linear function of temperature (Raptis et al. [27]). This is accomplished by expanding T^4 in Taylor series about T_∞ and neglecting higher order terms, T^4 can be expressed as $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$ and hence

$$q_r = -\frac{16\sigma^*}{3k^*} T_\infty^3 \frac{\partial T}{\partial y} \tag{10}$$

The energy equation (3) can be written in the following form using the radiative heat flux

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \sigma B_0^2 u^2 \quad (11)$$

and the concentration equation is

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \quad (12)$$

The boundary conditions are

$$\left. \begin{aligned} \text{At } y = 0: & \quad -k \frac{\partial T}{\partial y} = q_w = E_0 x^2, \quad -D \frac{\partial C}{\partial y} = m_w = E_1 x^2 \\ \text{As } y \rightarrow \infty: & \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \end{aligned} \right\} \quad (13)$$

The nondimensional temperature and concentration are defined as follows

$$\left. \begin{aligned} T - T_\infty &= \frac{E_0 x^2}{k} \left(\frac{\nu}{a} \right)^{1/2} \theta(\eta) \quad \text{and} \\ C - C_\infty &= \frac{E_1 x^2}{D} \left(\frac{\nu}{a} \right)^{1/2} \phi(\eta) \end{aligned} \right\} \quad (14)$$

On using (6) and (14) the nondimensional energy and concentration equations are given as follows

$$\left(\frac{3Rd+4}{3RdPr} \right) \theta'' + F \theta' - (2F' - Hs)\theta = -Ec \left[(F'')^2 + M^2 (F')^2 \right] \quad (15)$$

$$\phi'' + Sc F \phi' - 2Sc F' \phi - Sc \gamma \phi = 0 \quad (16)$$

with its corresponding boundary conditions as

$$\theta'(0) = -1, \quad \phi'(0) = -1 \quad \text{and} \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0 \quad (17)$$

The parameters involved in the temperature and concentration equations are $Pr = \frac{\mu C_p}{k}$ is the Prandtl

number, $Rd = \frac{k k^*}{4\sigma^* T_\infty^3}$ is the Radiation parameter,

$Hs = \frac{Q}{a \rho C_p}$ is the Heat generation parameter,

$Ec = \frac{a^2}{C_p \left(\frac{E_0}{k} \sqrt{\frac{\nu}{a}} \right)}$ is the Eckert number, $Sc = \frac{\nu}{D}$ is the

Schmidt number, $\gamma = \frac{k_1}{a}$ is the Chemical reaction parameter.

The main physical quantities of interest are the values of skin friction coefficient, nondimensional wall temperature and nondimensional wall concentration which are defined as

$C_f = \frac{\tau_w}{\rho u_w^2 / 2}$ where the wall shear stress τ_w is given as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

Hence, the skin friction coefficient is obtained using (6) as

$$\sqrt{Re_x} \frac{C_f}{2} = F''(0) \quad (18)$$

where $Re_x = \frac{a x^2}{\nu}$ is the local Reynolds number.

The dimensionless wall temperature and wall concentration can be determined from

$$T_w = T_\infty + \frac{E_0 x^2}{k} \left(\frac{\nu}{a} \right)^{1/2} \theta(0) \quad (19)$$

$$C = C_\infty + \frac{E_1 x^2}{D} \left(\frac{\nu}{a} \right)^{1/2} \phi(0) \quad (20)$$

III. NUMERICAL SOLUTION

Equations (7), (15) and (16) represent a highly nonlinear boundary value problem of third and second order respectively. The most efficient shooting technique along with Fourth Order Runge Kutta integration algorithm is employed to solve the above nondimensional equations. These equations of third order in F and second order in θ and ϕ have been transformed to a system of seven simultaneous equations of first order. In order to solve the system, seven initial conditions are needed whilst only four initial conditions $F(0)$, $F'(0)$ on F and two initial conditions $\theta'(0)$ and $\phi'(0)$ on θ and ϕ are available. The remaining three initial conditions $F''(0)$, $\theta(0)$ and $\phi(0)$ which are not prescribed are found satisfying the asymptotic conditions using Nachtsheim Swigert shooting iteration technique. The success of the method depends completely on the initial guess made to initiate the shooting process. Once these values are found, the system is solved using Fourth Order Runge Kutta method and the numerical values of dimensionless velocity, temperature and concentration are

obtained. The favorable comparisons with the limiting case of the problem lend confidence in the numerical method employed and the numerical solutions are obtained for various values of Physical parameters. Numerical values for Skin friction coefficient, wall temperature and wall concentration are also obtained.

IV. RESULTS AND DISCUSSION

A hydromagnetic boundary layer flow problem with heat and mass transfer over a shrinking surface taking into account of radiation, internal heat generation, dissipation and chemical reaction effects is examined in the presence of a transverse magnetic field when the surface is subjected to prescribed heat and mass flux. The boundary layer equations of momentum, energy and concentration are solved numerically using Nachtsheim Swigert shooting iteration technique along with Fourth Order Runge Kutta method so as to satisfy the asymptotic boundary conditions. Numerical solutions have been carried out for various values of Physical parameters to have the physical insight of the problem.

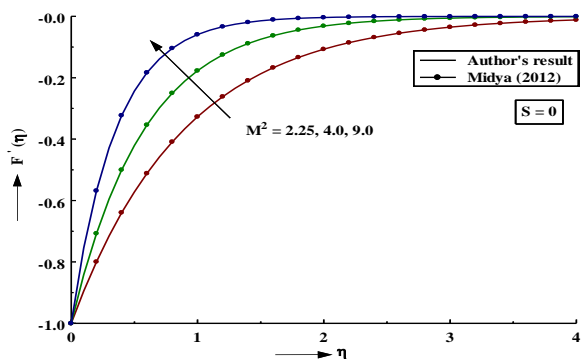


Figure 2. Comparison graph showing the effect of M^2 over the dimensionless velocity

Figure 3.

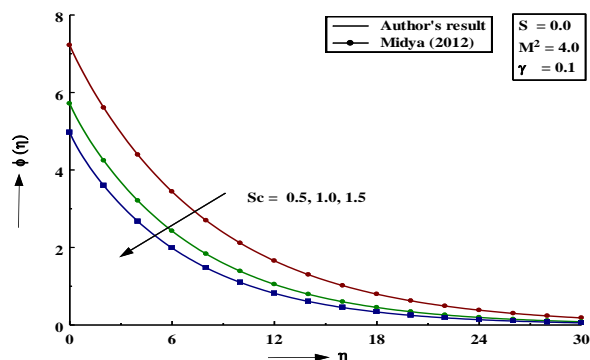


Figure 4. Comparison graph showing the effect of Sc over the dimensionless concentration

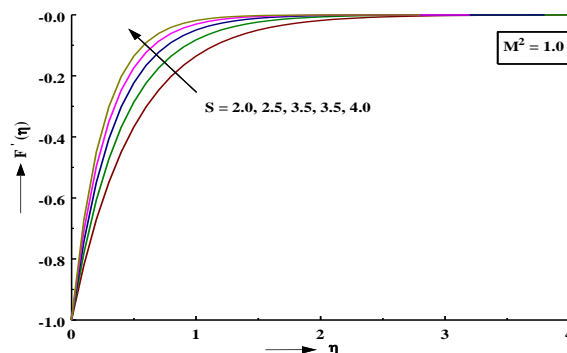


Figure 5. Effect of Suction parameter over the dimensionless velocity

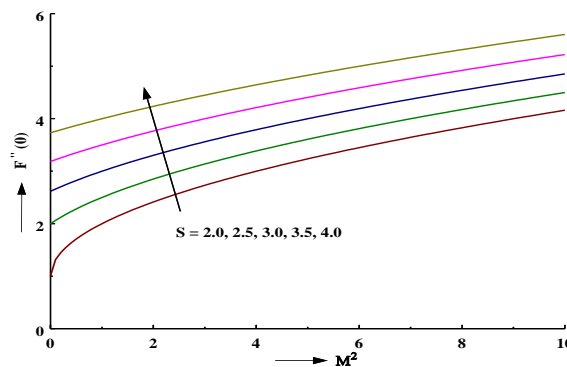


Figure 6. Skin friction coefficient for different values of S

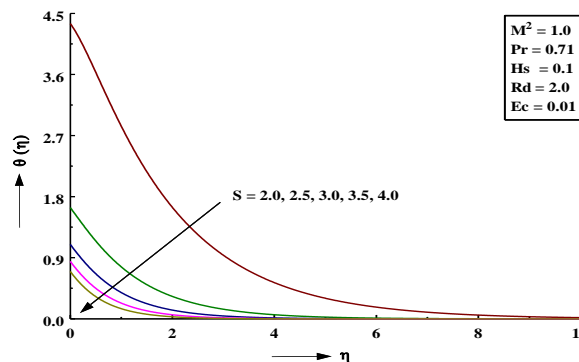


Figure 7. Dimensionless temperature profiles for different values of S

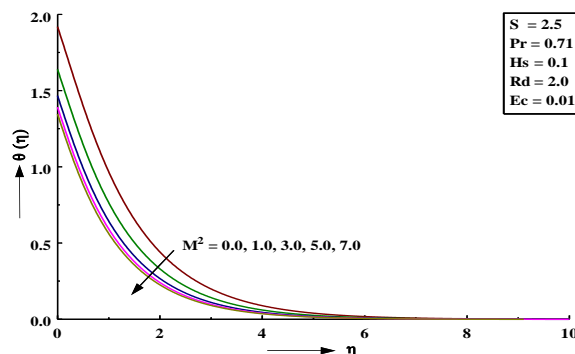


Figure 8. Dimensionless temperature profiles for different values of M^2

Figures 2 and 3 elucidate the comparison of the results with that of Midya [23]. It is seen from these graphs that author's results of dimensionless velocity for various values of the Magnetic parameter, dimensionless concentration for various values of the Schmidt number are identical to that of Midya [23] in the case of impermeable surface.

Figure 4 discloses the dimensionless velocity for different values of the Suction parameter. It is noted that the effect of Suction parameter is to accelerate the dimensionless velocity due to which the hydromagnetic boundary layer thickness reduces. Variation in skin friction coefficient for different values of Suction parameter is elucidated in Figure 5. It is viewed from the figure that the value of skin friction coefficient is more for higher values of Suction parameter.

Dimensionless temperature distribution for different values of Suction parameter is illustrated in Figure 6. An increase in Suction parameter (which means the fluid is brought closer to the wall) has the tendency to reduce the temperature and hence the thermal boundary layer thickness becomes smaller. Figure 7 implies that the temperature is suppressed due to increase in value of Magnetic parameter. It is further noted from the graph that the thickness of thermal boundary layer becomes thinner by the increase in the strength of magnetic field.

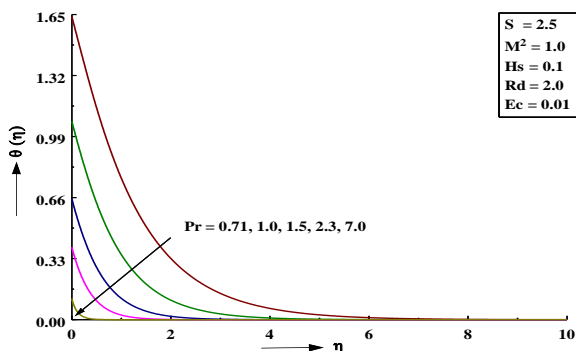


Figure 9. Dimensionless temperature profiles for different values of Pr

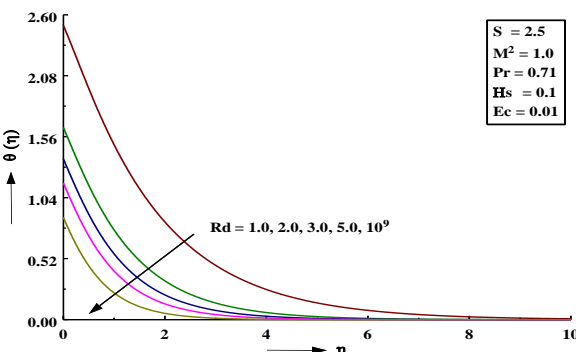


Figure 10. Dimensionless temperature profiles for different values of Rd

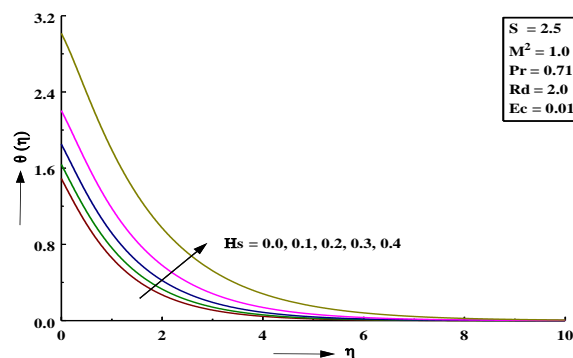


Figure 11. Dimensionless temperature profiles for different values of Hs

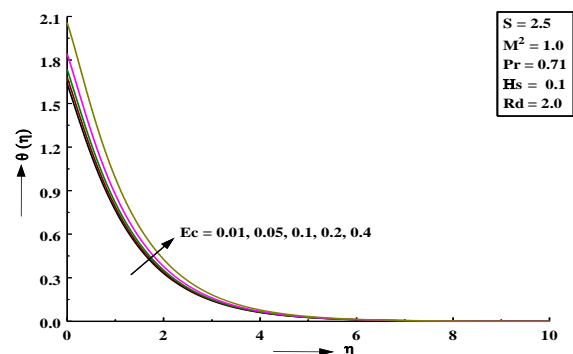


Figure 12. Dimensionless temperature profiles for different values of Ec

The effect of Prandtl number is to reduce the thermal boundary layer thickness and the dimensionless temperature is displayed through Figure 8. Figure 9 elucidates that the radiation parameter has significant effect so as to decrease the temperature for its increasing values with higher temperature at the wall.

Figure 10 demonstrates that heat energy generated in thermal boundary layer causes the temperature to increase with an increase in magnitude of Heat generation parameter. The temperature of the fluid is found to increase apparently due to significant increase in Eckert number and it increases the thermal boundary layer thickness is presented in Figure 11.

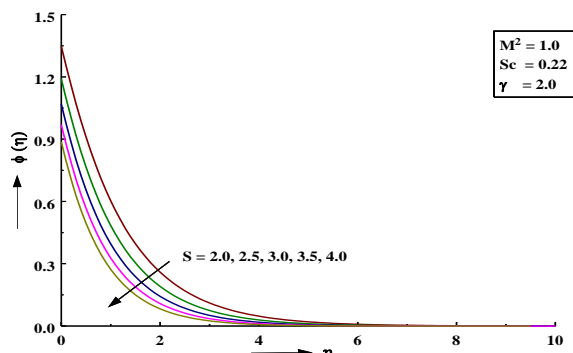


Figure 13. Dimensionless concentration profiles for various values of S

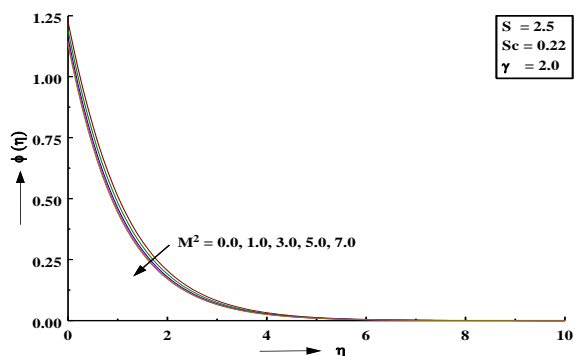


Figure 14. Dimensionless concentration profiles for various values of M^2

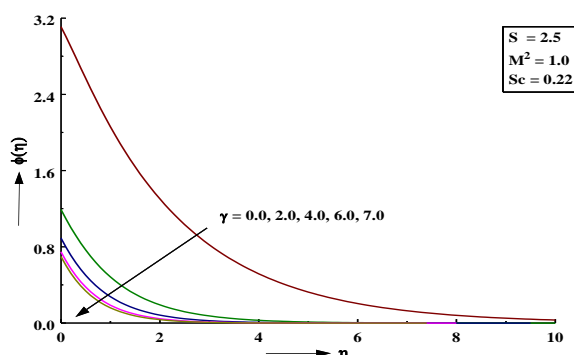


Figure 15. Dimensionless concentration profiles for various values of γ

The effect of Suction parameter over dimensionless concentration is disclosed in Figure 12. It is noted that the rise in Suction parameter leads to the fall in the dimensionless concentration of the fluid. The dimensionless concentration decreases with an accompanying decrease in solute boundary layer thickness by increasing the values of Magnetic parameter is illustrated in Figure 13. The dimensionless concentration for various values of Chemical reaction parameter is represented in Figure 14. It is observed that the effect of increasing Chemical reaction parameter is to lower the dimensionless concentration of the fluid and hence there is a reduction in solute boundary layer thickness.

Table 1. Wall temperature $\theta(0)$ for different Pr with $M^2=5.0, S=3.0, Rd=10^9, Ec=Hs=0.0$

Pr	$\theta(0)$	
	Present result	Ali et al. (2010)
0.7	0.504404	0.5044
10	0.033669	0.0337
15	0.022376	0.0224
20	0.016755	0.0168
30	0.011151	0.0112

Table 1 shows the comparison of the results for wall temperature for various values of Prandtl number. The table values show that the present results are identical to the results of Ali et al. (2010) in the case of two dimensional flows when the surface is prescribed with constant heat flux

in the absence of radiation, dissipation effects and internal heat generation.

Table 2. Wall temperature $\theta(0)$ for different S, M^2, Pr, Hs, Rd and Ec when $Sc=0.22$ and $\gamma=2.0$

S	M^2	Pr	Hs	Rd	Ec	$\theta(0)$
2.0	1.0	0.71	0.1	2.0	0.01	4.35124
2.5						1.64018
3.0						1.09785
3.5						0.84784
4.0						0.69904
2.5	0.0	0.71	0.1	2.0	0.01	1.91997
	1.0					1.64016
	3.0					1.46751
	5.0					1.39209
	7.0					1.34676
2.5	2.0	0.71	0.1	2.0	0.01	1.64016
		1.00				1.07332
		1.50				0.65479
		2.30				0.39314
		7.00				0.11418
2.5	2.0	0.71	0.0	2.0	0.01	1.49251
			0.1			1.64016
			0.2			1.85318
			0.3			2.20573
			0.4			3.01627
2.5	2.0	0.71	0.1	1.0	0.01	2.51111
				2.0		1.64016
				3.0		1.37270
				5.0		1.16706
				10^9		0.87335
2.5	2.0	0.71	0.1	2.0	0.01	1.64016
					0.20	1.68310
					0.40	1.73676
					0.60	1.84407
					0.80	2.05870

Table 3. Wall Concentration for various values of S, M^2, Sc, γ when $S=2.0, M^2=1.0, Sc=0.22$ and $\gamma=2.0$

S	M^2	Sc	γ	$\phi(0)$
2.0	1.0	0.22	2.0	1.34896
2.5				1.19176
3.0				1.06976
3.5				0.97090
4.0				0.88853
2.5	0.0	0.22	2.0	1.22690
	1.0			1.19176
	3.0			1.16077
	5.0			1.14392
	7.0			1.13253
2.5	1.0	0.22	2.0	1.19176
		0.50		0.66362
		0.62		0.56078
		0.78		0.46505
		1.30		0.29840
2.5	1.0	0.22	0.0	3.11121
			2.0	1.19176
			4.0	0.89304
			6.0	0.74842
			7.0	0.69890

Tables 2 and 3 display the numerical values of wall temperature and wall concentration by varying the pertinent parameters involved in the study. It is revealed that the

wall temperature reduces due to increasing values of Suction parameter, Magnetic parameter, Prandtl number, Radiation parameter whereas it gets enhanced for increasing Heat generation parameter and Eckert number.

The numerical values in Table 3 infer that the parameters namely Suction parameter, Magnetic parameter, Schmidt number and Chemical reaction parameter have the same effect on wall concentration so as to reduce it.

V. CONCLUSION AND FUTURE SCOPE

The effects of first order chemical reaction, internal heat generation and radiation on nonlinear hydromagnetic boundary layer flow over a shrinking surface subjected to prescribed heat and mass flux with viscous and Joule's dissipation effects are analyzed in this investigation. A parametric study on dimensionless velocity, temperature, concentration, skin friction coefficient, wall temperature and wall concentration are carried out.

The important findings of the present investigation are:

- The obtained numerical results show good agreement with the results available in the literature under some limiting cases.
- The effect of Eckert number (due to viscous and Joule's dissipation) alters the rate of heat transfer significantly.
- The mass transfer is strongly influenced by Schmidt number (decrease of molecular diffusivity) and Chemical reaction parameter.

The future scope of the current analysis is that the fluid flow described by law of conservation of mass, momentum and energy which are governed by nonlinear partial differential equations can be modified to incorporate the effects that are neglected in this study.

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