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A Single Recurrence Simulates Growth Cycles in Nature

Alexander Harrison

School of Engineering, the University of Newcastle, Callaghan, NSW Australia 2308

Author's Mail Id: alexander.harrison@newcastle.edu.au, Tel.: +61-4-5167-0624

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Abstract—Numerous systems in nature grow or decay over time, with many exhibiting cycloidal or S-shaped trajectories. Competing effects within any predator-prey system may lead to oscillatory and even chaotic trajectories. Natural systems including solar cycles and even ethanol production curves exhibit an S-curve trajectory during the growth cycle. Traditionally, differential equations are required to analyse the dynamics of a large system of distributed particles. An alternative method for describing growth paths in natural systems applies a single modulated discrete-domain recurrence relation to emulate the shape of growth or decay trajectories. The model requires a known initial condition and a growth target value for the particular physical system. Addition of a modulating term within the recursion model enables wave action within growth cycles, without impacting numerical simulation stability. A recurrence relation is developed to simulate the known growth and decay characteristics in real systems such as hurricanes, tornados, ethanol production during fermentation, coronavirus growth, solar cycle count and sunspot dynamics, to mention a few. Simulation by recursion of growth curves for the above-mentioned natural systems each show S-shape appearance, suggesting that systems with limiting growth may also submit to trajectory predictability. Growth curves for any physical systems that evolve from a minimum to a maximum value are amenable to analysis by the recurrence method.

Keywords—Simulation, Dynamics, Recurrence, Nature, Growth, Decay

I. INTRODUCTION

Motion of particles or objects in physical systems including biology, chemistry, physics and earth sciences usually follow predictable and stable dynamic trajectories. A large system of particles or objects could also behave non-linearly, depending the interaction between driving mechanisms and the system's response. Complex and non-linear interactions require more complex analysis to solve growth, decay and cyclical dynamic behaviour.

Growth may be described as an increase in a discrete sample value or count number between successive time intervals in a natural dynamic system. An observation that S-shaped curves describe growth in numerous natural systems is of considerable interest. The evolution of growth from an initial low value to a limiting value indicates that such systems may be governed by a common descriptive procedure. Establishing an underlying mathematical treatment for growth trajectories in different natural systems would be significant.

Modelling the dynamic motion of a system of inter-connected or distributed particles in nature traditionally involves solutions derived from ordinary differential equations (ode). A closed-form solution of an ode's usually means a stable and well-defined trajectory of motion. Non-linear interactions may result in chaotic oscillations and solutions that do not follow smooth trajectories.

In systems where growth is limited, a stable attractor or fixed-point solution can be expected. For stable or limiting growth, an ode may be converted to a difference equation or a recurrence relation. A recurrence relation can be evaluated numerically to produce a set of discrete-time values or counts, suitable for rapid graphing and easy visualization of the system's trajectory. Application of a recurrence relation to numerically solve an ode requires both formulation and use of initial conditions. In addition, recursion permits forward prediction of a system's trajectory in the discrete domain.

Many physical systems exhibit growth and decay curves that are bounded or asymptote to a value in time. Existence of growth curves with a common shape in both microscopic and macroscopic systems would suggest an underlying commonality in nature. Notwithstanding, growth in nature's systems is controlled by complex effects such as temperature variations, chemical reactions, biological competition and solar radiation.

In this paper, Section I contains the introduction, Section II contains related work, Section III outlines the methodology used in the paper to derive recursion equations for growth modelling, Section IV details examples which apply the methodology of recurrence to simulate physical and biological systems in nature, Section V describes results and discussion of the research, and Section VI contains a conclusion with reference to future research directions.

II. RELATED WORK

An objective of the research is the development of a mathematical relationship to simulate trajectories in natural systems that exhibit S-curve growth. Since the 1980s, analysis of growth has been researched from various perspectives. Logistic maps for predator-prey models using recurrence relations have been investigated in the past [1]. Calculus of variations have been used to derive optimal start curves for long elastic belts where an S-shaped velocity curve minimizes dynamic forces and elastic wave amplitudes [2,3].

Van Geert has researched developmental change in children's learning, describing basic principles of growth as a measured gain in value in discrete time [4]. In relation to gradients of growth, basic calculus defines a difference between two closely-separated values on a curve as the curve's gradient, slope or derivative. The 18th century mathematician Euler derived what is known as the "Euler Method" to numerically solve the initial value problem for a first-order ode [5].

Coupling basic concepts of growth with a numerical simulation method is researched in this paper for various systems in nature [6]. The coupling method enables rapid analysis of growth trajectories using graphical visualization of the solution. While recurrence relations are known as a powerful mathematical tool, their application to studying growth dynamics has only been examined by a limited number of researchers.

III. METHODOLOGY AND DERIVATIONS

A proposed methodology for studying growth mechanisms in nature involves the adaptation of the recursion method to simulate changes in system values, such as population or density. A first-order recurrence relation describes growth as a sequence of numbers in discrete time T, i.e., $N_{T+1} = N_T + k$ where N_T is a current value, k is a number, and N_{T+1} is the next value at the next discrete interval. Using an initial value N_0 at T = 0 permits calculation of the next values of the sequence, i.e., $N_1 = N_0 + k$, $N_2 = N_1 + k = N_0 + 2k$, and so on.

The Euler method shows that if $k = \tau dN/dt$, a solution of a first-order ode for any curve N(t) can be recursively iterated using the equation $N_{T+1} - N_T = \tau dN/dt$, where the difference in the time step $((T+1) - T) = \tau$. In general, a discrete sample interval would be unitary, so $\tau = 1$ and k is just the curve's gradient at the point (N(t), t).

In relation to growth, when dN/dt = k, recursion produces arithmetic growth. When $dN/dt = \lambda N$, recursion produces geometric growth with $\lambda > 1$, or geometric decay if $\lambda < 1$.

In nature, most processes multiply faster than linear. During growth, losses may occur, limiting unbounded growth to some value P. A loss factor g = (1 – N/P) is used to modify k, so that for geometric growth $k = \lambda gN$. Accordingly, $k \to 0$ as $N \to P$, i.e., the slope dN/dt approaches zero as N approaches an upper population P. Similarly, when N = 0, k = 0, indicating that the curve's gradient dN/dt has is zero slope. Hence, $dN/dt = \lambda gN$ behaves as an S-curve, also known as a Verhulst curve [6].

Applying the above analysis to the growth problem, a new difference equation is written as:

$$N_{T+1} = N_T + \lambda N_T(1 - N_T/P)\,, \qquad (1)$$

with a form suitable for sequence generation and graphical presentation using recursion. Wave activity in real systems can be accommodated by adding a new modulation term M_T to (1), i.e., $M_T = A\cdot\sin(\omega T)\cdot\exp(-bT) + C$, where A and C are dimensionless values, ω and b have their usual physical meaning. Inserting a modulating term affects the recursion calculation, thereby modifying the sequence and the graphical trajectory of growth. From (1), a single difference equation including modulation is then described by:

$$N_{T+1} = \beta N_T + \lambda g N_T + M_T \qquad (2)$$

or more concisely,

$$N_{T+1} = \gamma N_T - \varepsilon N_T^2 + M_T \qquad (3)$$

where $\gamma = (\beta + \lambda)$ and $\varepsilon = \pm\lambda/P$. Clearly, (3) has a squared term indicating the selection of its coefficients will critically determine stability [7]. In some fields of research such as chaos and non-linear dynamics, (3) is referred to as a logistic map [8].

Numerous situations occur in nature where time delays affect growth. As discussed previously, May researched time delay effects in populations where competition and predator-prey interactions occur between species.

Adding a time delay ϕ to the loss factor in (1) results in a new loss factor $g = (1 - N_{T-\phi}/P)$. When $\phi = 0$, no delay exists in the population loss factor. For integer $\phi > 0$, the general equation (3) becomes a delay recursion relation:

$$N_{T+1} = \gamma N_T - \varepsilon N_T N_{T-\phi} + M_T \qquad (4).$$

A discrete delay in one sample requires the parameter $\phi = 1$, and in this case (4) becomes a single delay problem in which oscillations and chaotic stability are possible. Selecting a value of $\phi = 2$ in (4), together with particular control parameters, produces a potentially unstable double-delay equation that allows the simulation of complex cyclic systems, including solar cycles, as discussed in following sections. Usually when a double delay is introduced to produce long period oscillatory behaviour similar to solar cycles, the outcome may be stable or chaotic.

IV. APPLICATION AND EXAMPLES

Parameters λ and P shown in (1) are experimentally determined to produce a trajectory which correlates with actual count samples from a naturally occurring system. A selection of examples showing bounded and cyclic growth are now described. Graphs depicting measured count values for tornados, hurricanes, ethanol production and coronavirus growth are simulated by recursion.

For example, variations in the trajectory of coronavirus require the use of a non-zero M_T term in (2). In following sections, sunspot count over one cycle is shown to follow growth and decay according to (1), however employing a double-delay in (4) produces solar cycle oscillations that correlate with measurements over an 11-year interval. Careful selection of appropriate initial conditions is necessary for stable simulation.

A. Hurricanes and Tropical Storms

Hurricane and tropical storm count emanating from the Atlantic Ocean is widely available through organizations including the National Oceanic and Atmospheric Administration (NOAA). In December 2012, Doggett discussed the 2012 hurricane season in perspective, based on Air-Worldwide data. A graph of tropical storm count from 2012 compares with simulation by (1) for experimentally selected parameters β = 1, λ = 0.65, P = 11.3 and N_0 = 0.14. For hurricanes, correlation occurs for β = 1, λ = 0.65, P = 6.2, M_T = 0 and N_0= 0.07. Figure 1 shows simulated tropical storm and hurricane trajectories using (1), overlaid on recorded counts published online by spc.noaa.gov and air-worldwide.com.

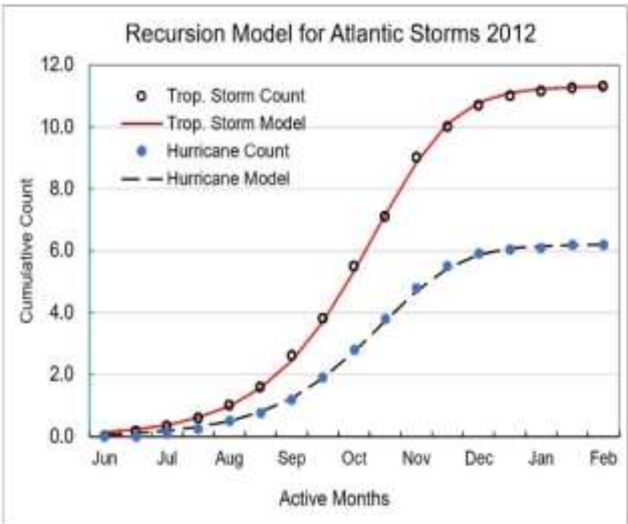

Figure 1. Tropical storm (upper curve) and hurricanes (lower curve) for 2012 North Atlantic Ocean weather, compared to simulation curves using (1).

B. Tornado Count in USA, 2019-2020

Tornado count provided by NOAA for 2019 and 2020 are graphed in Figure 2. A recursive simulation curve generated by (1) shows a reasonable fit to 2019 recorded data for parameters β = 1, λ = 0.85, P = 1402, N_0= 26, whereas for 2020, β = 1, λ = 0.6, M_T = 0 and N_0= 50. Recursion predicts a final count tally of 1020 for 2020, for the given initial values. Recursive modelling shows that actual measured tornado count is followed closely by the predictive curve with the selected parameters. Figure 2 show some variability attributed to reported tornado count. Early in any tornado season, an initial value placed in the simulation permits a prediction of the final P value.

Unlike regression curve fitting, simulation forward-predicts a trajectory using only a few initial data points. The general shape of the curve is defined by an initial condition and its growth parameter, thus setting the curve shape.

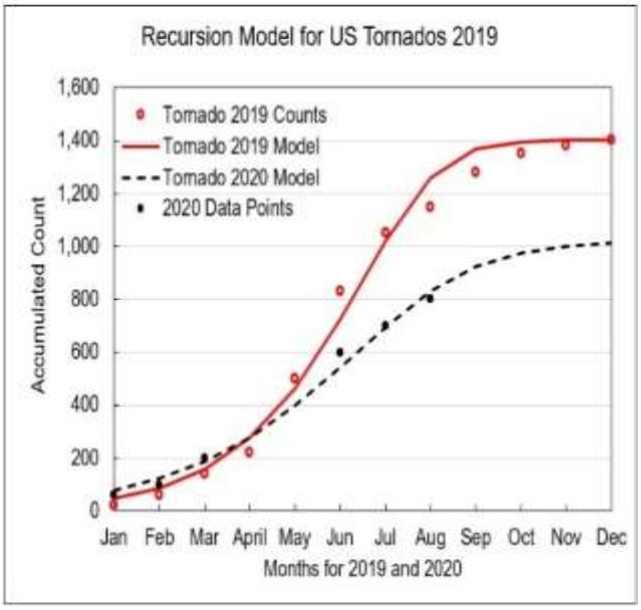

Figure 2. Tornado count from NOAA records compared to model trajectory from (1).

C. Ethanol Growth during Fermentation

Curves for ethanol generation in a sugar-yeast mix are widely available in the literature. Guadalupe et. al. published measurements related to ethanol generation during beer production and summarized the general curves expected for ethanol yield [9].

An almost infinite set of ethanol production curves exist due to the various brewing techniques. Alcohol production curves depend on pressure, temperature, sugar type, water and material purity, and yeast type. In general, yeast numbers multiply as sugars are converted to alcohol, in accordance with general published curves. The cited reference indicates that all measurements of alcohol production, when normalized against each other, generally follow similar trajectories to one another.

Figure 3 illustrates a simulation curve for beer ethanol production, based on (1). Alcohol percentage (%w/w) plotted against discrete hourly intervals correlate with the simulation model for parameters experimentally set to λ = 0.53, β = 1, P = 3.3, M_T = 0 and N_0 = 0.047.

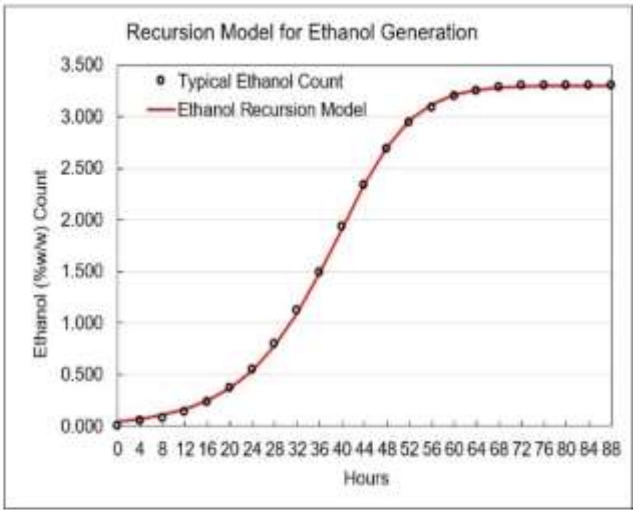

Figure 3. Curve for ethanol production in beer.

D. Coronavirus Count for USA 2020

Coronavirus Covid-19 growth curves during 2020 are examined for the USA due to its large number of cases. Figure 4 shows cumulative coronavirus cases compared to simulation curves using (2). Harrison indicates that infection levels of 10%, 14% and 18% in the population were common in past pandemics [10].

Virus propagation trajectories were plotted by setting equation (2) parameters to correlate with initially reported growth counts from Worldometers.com. A simulation of the cumulative value of counts over time allows a forward projection by the model, relative to recorded counts. Parameters for the 14% infection curve are experimentally determined for initial recorded case numbers, using $\beta = 1$, $\lambda = 0.09$, P= 46 (in millions), $N_0 = 0.01$ and a sine modulation of $M_T = 0.2 \cdot \sin(2\pi T/15) \cdot e\text{^}(-T/100) + 0.0125$. Given a US population of 330 million people, the target populations for 10%, 14% and 18% infection levels for the entire population correspond to P = 33, 45 and 60 million, respectively. Covid-19 data for China modelled with logistic maps is of interest and applicable to the study [11].

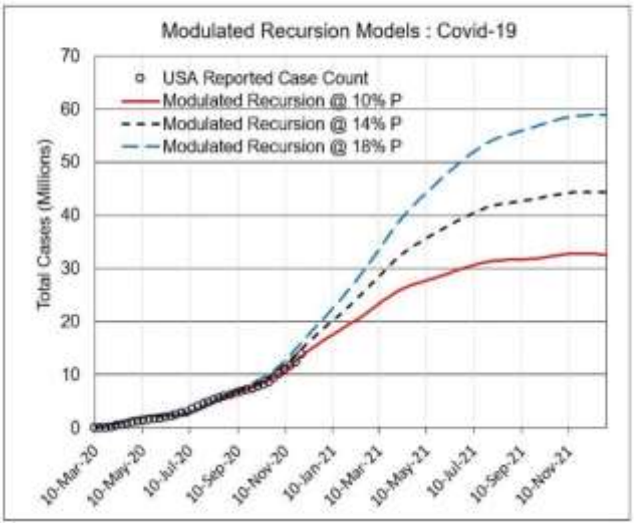

Figure 4. Coronavirus count for USA, compared to the model using equation (2), for three possible infection rates.

Modulation provides the mechanism for adding wave activity to simulate growth oscillations, while P shifts the infectable target population in (2). From November 2020, the trajectory of cases surged closer to the 18% simulated infection curve after society restrictions were eased, resulting in M_T values $\lambda = 0.125$, c = 0.13 and $N_0 = 0.05$.

E. Sunspot Count during Solar Cycles

Previously, four examples of the application of (1) and (2) have been discussed, where naturally occurring processes exhibit a saturated or limiting growth count. For the four previous examples of growth in nature, recursive simulations are shown to match experimentally measured values when initial conditions are set for each type of natural system. However oscillatory growth and decay cycles in nature are treated by introducing a time delay to (3), forming (4) as a solution for recursive simulatiom.

Solar cycles or sunspot count over an 11-year period are now examined using a selected solar cycle 21 (1976-1987). Sunspot counts for this period are published at justinweather.com. A plot of sunspot count for each year in solar cycle 21 shows that the single equation (1) is piece-wise valid for a growth (+λ) and decay (-λ) using the equation-pair:

$$Nr_{T+1} = Nr_T + \lambda Nr_T \,(1 - \varepsilon Nr_T) \qquad 5(a)$$

$$Nf_{T+1} = Nf_T - \lambda Nf_T \,(1 - \varepsilon Nf_T). \qquad 5(b)$$

Equation 5(a) describes the growth in sunspot count Nr for cycle 21, with the initial condition $\beta = 1$, $\lambda = 1.3$, $\varepsilon = 1/P = 1/250$ and $Nr_{T=0} = 23$, rising towards 250 over the discrete range (years) $0 < T < 12$.

The simulation curve for 5(a) is shown on Figure (5) by the left-side dashed curve. Equation 5(b) describes the fall or decay in sunspot count Nf for the trailing part of solar cycle 21, shown by the long-dashed line on the right side of the graph. An initial condition for 5(b) is $Nf_{T=0} = 250$, falling to zero over the same discrete range $0 < T < 12$.

Separate curves produced by 5(a) and 5(b) are shown individually in Figure 5, along with measured sunspot counts during cycle 21. A single pulse-like curve for solar cycle 21 is indicated from recorded measurements of sunspot count. A pulse is modelled for one solar cycle by applying a mathematical product of two independent simulations defined by 5(a) and 5(b):

$$S_{T+1} = (Nf_{T+1} \,.\, N_{T+1})/250 \qquad (6).$$

Comparing sunspot counts for cycle 21 on the graph of Figure 5 with the piece-wise product method allows a tractable simulation of solar cycle 21.

Simulation of multiple solar cycles is more challenging. Equation (4) is capable of producing oscillatory behaviour, including, long-cycle chaotic oscillations for delay $\phi > 1$.

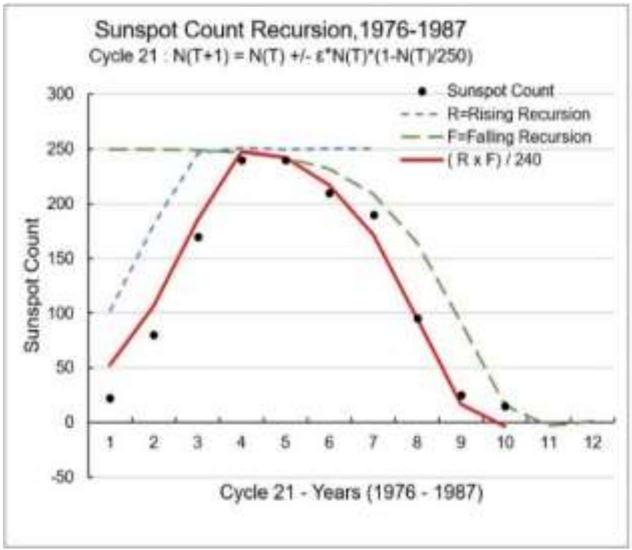

Figure 5. Sunspot count compared to the piece-wise model results using 5(a) and 5(b), showing a graph of their simulation product of the general model based on (1) and (6).

When describing delay effects for $\phi = 2$, the second last iteration of the recurrence calculation affects the current value, termed a double delay, meaning the underlying ode is of second order. A delay product $(N_T \, . \, N_{T-2})$ in the logistic map (4) produces long periodic oscillations.

Simulation of multiple solar cycles requires (4) with $\phi = 2$, $N_0 = 15.9$, $\lambda = 1.5$ and $\beta = 0.27$. The combined variables are $\gamma = 1.77$, $\rho = 1.5/P$ and $P = 210$, the peak sunspot count in cycle 22. A 23-year peak variance is simulated using the modulation function $M_T = -0.1 * N_T \, (1 - \cos(\pi (T-2)/46)$.

Figure 6 shows the graph of actual measured and simulated sunspot count during solar cycles 22 to 24. Close inspection the model graph shows non-repeating counts typical of second order effects and slightly chaotic instabilities.

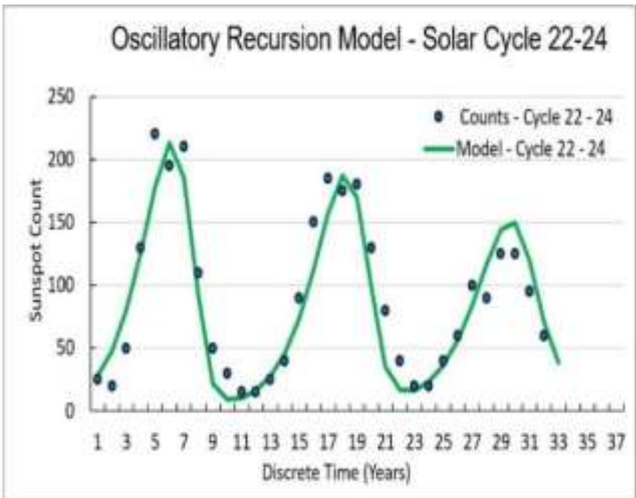

Figure 6. Reported sunspot count for three solar cycles peaking in 1992, 2003 and 2014, including a graph of delay by (4) that simulates oscillation in sunspot count for cycles 22 to 24.

V. DISCUSSION OF RESULTS

Recursion relations apply to both bounded and cyclic growth in natural systems. A single recurrence equation generates a sequence of numbers that simulate measured counts or growth values, demonstrated using examples from nature. Calculated values may be readily plotted to show a simulated growth trajectory, suggesting the method has useful application to forward prediction.

More generally, the shape of recursion curves is parametrically adjustable, enabling experimental matching to measured values of a system. With only a few initial values, a recursion can generate a family of curves that predict trajectories for slightly different initial conditions. In most growth systems, only the variables λ and P are required to define a curve trajectory, since β is usually unity, except in (2).

In situations where the final target population or count P is not known, λ values known from similar systems together with some initially measured counts on the trajectory, are used to generate a curve that is forward projected to limit P. The procedure was used in Figure 2 where six values published for 2020 tornado count requires a $\lambda = 0.6$, resulting in a forward prediction to a final tornado count of 1020 known for the year 2020.

The paper has demonstrated that natural processes such as growth or sample count of organisms, cyclones, storms, solar flares and sunspot activity can be simulated and easily graphed, allowing inspection of trajectory evolution. A single equation describing growth of an entity with an upper limit P can be applied to numerous natural systems as illustrated in Section 2. Equation (2) requires an initial value and two trajectory-forming variables λ and β, to define a curve that simulates measured values.

Coronavirus case numbers reported for the USA total 14 million on 1-December-2020, with a daily count of 143,188. Reported data suggests that if the infection rate were only 10%, a count of 116,300 cases/day would be indicated., whereas a 14% infection curve in Figure 4 more realistically follows the case data for the original strain to a target of 45 million cases. Wave action observed in virus case numbers is accommodated by a sinusoidal modulation function added to (2) and (3). For coronavirus curves, an amplitude of 0.2 applies to early 2020 data. From December 2020 the amplitude is 0.3.

Sunspot activity and its modelling has been examined. Sunspot count is generally predicable over an 11-year period. Analysis shows that a general expression can be developed to describe the sunspot count numbers. In solar cycle 21 (1976-1987) the peak count recorded in 1981 by various institutions was between 245 and 250. During cycles 22, 23 and 24, sunspot peaks reached 210, 180 and 130 counts (in 2014), respectively.

An 11-year peak sunspot count shows variation, simulated using the modulation method in (4). Use of a modulating function M_T within the recursion suggests a slightly unstable or chaotic process affecting solar cycle dynamics. Table 1 provides the experimentally determined set of parameters that enable simulation of selected systems that exhibit S-shaped growth trajectories and cyclic behaviour. Included is the piece-wise method used to describe sunspot growth simulation, based on 5(a) and 5(b), where two initial conditions are required to predict both the growth and decay curves in (6). Parameters for sunspot cycles 22 and 24 were experimentally set to ensure long-period stable oscillations.

Table 1. List summarizing the range of parameters in (1) to (6) for the various examples in nature.

System	λ	β	N_0	**P**
T. Storms	0.65	1	0.14	11.3
Hurricanes	0.65	1	0.07	6.2
Tornados	0.6	1	50	1020
Ethanol	0.53	1	.047	3.3
Covid 14%P	0.09	1	0.02	33x10^6
Covid 18%P	0.09	1	0.05	60x10^6
Sunspot 5(a)	1.3	1	23	250
Sunspot 5(b)	1.3	1	250	250
Cycle 22-24	1.5	0.27	15.93	210

A single recurrence equation can define a natural process and its dynamics, once the initial conditions are established. Applying parameters with typical values near those in Table 1 allows a quick and approximate selection of a starting point for simulation of similar systems in nature with limited growth. Further research will uncover numerous other diverse systems with growth trajectories that can be defined by the single equation.

VI. CONCLUSIONS AND FUTURE SCOPE

Specific examples of bounded and cyclic growth cycles in nature have been described by a single recursion equation. A modified Euler Method using initial conditions and parameter values like those in Table 1 generate a sequence of numbers that, when graphed, match growth cycles from six example systems in nature. Selection of parameters for recursive simulation has been discussed. Equations (2) and (4) provide a unique descriptor of a growth trajectory in a natural system. Modulation of the recurrence relation allows wave action to perturb the model, enhancing correlation with measured growth trajectories.

An example of a complex pulse-like trajectory in nature has been modelled by combining a pair of single growth and decay equations. Whenever oscillatory behaviour is measured in nature, a time-delay term in (4) generates stable oscillatory solutions, such is the case with sunspot count over three solar cycles. Undoubtedly, numerous other natural growth systems are amenable to similar recursion analysis to visualize and predict trajectories.
Simulation of solar cycle 22 – 24 presents stable solutions within a very narrow band of parameters. Equation (4) contains a time delay which can cause inherent instability and chaotic behaviour, susceptible to explosive and chaotic oscillations when only a small variation in parameter values occur. An understanding of stability manifolds in non-linear mechanics assists the researcher in finding the stable points for recursion simulations. More research is required to find alternative methods for selecting parameters relevant to other natural systems.

Unlike regression fitting methods, a single recursion equation enables both a curve fit and a forward prediction capability for discrete data points. Once an initial condition and a few count numbers are known, projections can be developed leading to a final or target count.

Methods that produce a forward prediction of trajectory, based on initial conditions for a natural system's state, are invaluable when analyzing trends and limits of or decay. Predicting a final outcome of growth from a potentially dangerous event such as coronavirus spread and hurricane prediction is essential when attempting to prepare for an emergency in society.

Limitations exist in the method that can be improved. For example, a simulated curve may or may not exactly follow actual data along an S-curve path towards an upper or limiting count. At present, parameters in Table 1 represent a good guide for simulating similar natural regimes. In the case of coronavirus growth, each country has its own dynamics and initial conditions. Model parameters for other countries need to be developed to permit simulation agreement and forward prediction of growth curves.

Future research should address the above-stated limitations, such as the development of an algorithm that perturbs the simulation to calculate an outcome above and below the initial experimental point or count value. A convergence to the actual measured value would result, with the output representing the shape parameters for the system with particular initial conditions.

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AUTHORS PROFILE

Alexander Harrison, BA(Hons) Maths and Physics, BEng (Electrical), MSc (Physics), PhD (MechEng), Fellow of The Royal Society NSW, FIEAust, APEC Engineer. Currently Adj. Professor of Engineering, The University of Newcastle Australia. Formerly a PRS (Principal Research Scientist), CSIRO Division of Applied Physis, Sydney Australia, Professor of Mechanical Engineering (Newcastle, Australia) and Adj. Prof. Colorado School of Mines, USA. President of Scientific Solutions Inc. USA and Manager of Conveyor Technologies Ltd. LLC, Denver, Colorado USA, between 1993 and 2015. Manager of Conveyorscience.com, developed code and software solutions including data logging and signal processing, developed commercial use of digital jitter algorithms for noise removal. Author of 160 papers on mathematical analysis of dynamic systems, including long conveyor systems, development of analytical methods in mechanical engineering, applied chaos theory and non-linear dynamics. Non-destructive Evaluation (NDE) of mechanical and physical systems has been fully commercialized using non-contact sensors for NDT monitoring, including NDT sensor theory and drive control with application to the mining, mechanical and electrical industries.

