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Review Paper

Fractal Geometry and its Application in Geographic Information Science

Thomas U. Omali[1]

[1]Bio-resources Development Centre (BIODEC) Isanlu [National Biotechnology Development Agency (NABDA)], Nigeria

Author's Mail Id: t.omali@yahoo.com

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Abstract— The revolutionary article about the length of the Britain's coastline was published over a half-century ago by the late mathematician Benoît Mandelbrot. The article directly culminated in the birth of fractal geometry, which has been significantly applied in many disciplines including Geographic Information Science (GIScience). The plethora of recent research demonstrate how potent fractal geometry is in GIScience. Thus, this study is a review of fractal geometry and its application in GIScience. The relevant academic literature were identified through a harmonised search in the electronic records. Furthermore, the search result was analysed using various keywords. The result presents the selected principles upon which fractal geometry is based, including scale measure, self-similarity, self-affinity, and fractal dimension. Other results of the review present a method of generating fractals, and the fractal-based analysis in GIScience.

Keywords— Complex, dimension, scale, self-affinity, self-similarity, shape

1. Introduction

The development and study of geometry have been over two thousand years since Euclid, a Greek mathematician first developed Euclidean geometry. Of course, geometry has been an important tool for understanding, describing, and interrelating with space [1]. The framework of Euclidean Geometry is customarily used for mapping geographic features [2,3] using the geometric features [4,5]. Regrettably, it cannot define the irregular outlines of the geographic world precisely, and it generates a series of problems regarding the measuring scale. As a result, fractal geometry emerged to solve scale problems and capture the fractal characteristics of the geographic structures.

With fractal geometry, it is possible to study the characteristics of objects that have irregular [6] or fragmentation in spatial patterns [7,8], broken lines, fuzzy boundaries and fractional dimensions [9]. Fractal geometry is essentially applied in defining the geometry of nature within several natural entities and phenomena including chains of mountains and rivers [10], etc. Since its emergence in the mid-1970s, its utilization has been pervasive in several disciplines. This include landscape mapping [see 11-16], river networks assessment [see 17,18], sedimentology [see 19-22], and rock assessment [see 23], computer graphics [see 24-26], characterization of urban growth [see 27-31], and others. Thus, this study aimed at reviewing fractal geometry with a specific interest in its principles, methods, and application in GIScience.

This paper is written in seven sections: the introduction is in section 1, the literature review is in Section 2, the study method is explained in Section 3, some principles of fractal geometry are described in Section 4, the methods of generating fractals are given in Section 5, GIScience application of fractal geometry is presented in Section 6, and Section 7 concludes the study.

2. Literature Review

Several studies on fractal geometry relative to various interests are in the literature. In many cases, the fractal dimension is computed by different methods. For instance, the box-counting technique can be used (see [32,33]). Box-counting involves the use and counting of grids that cover the whole image or map. Also, the area-perimeter fractal dimension has been utilised as a landscape metric for characterizing urban forms [34], capturing the structure of urban growth [35], and enhancing accuracy of LULC classification using aerial photographs.

Fractal geometry can show urban intricacy [36]. Initially, urban fractal studies concentrated on the impervious land [37], roads [38,30], traffic movement [39]. Fractal can be mono-fractal at different positions, and the growth is in an anisotropic way [40]. Similarly, there are random fractals with bi-fractals, indicating different multi-scale processes at diverse growth directions [41]. Nie et al. [42] studied the complexity and the modifications of the impervious area using the fractal principle. The study also covers urban assessment in line with Purevtseren, Tsegmid, Indra, and

Sugar [43]. The fractal dimensions of the LULC spatial form were estimated using the cell-counting in combination with the least squares regression technique.

Furthermore, Mo, Liu, and Lv [44] conducted a GIS-based fractal assessment of Shenzhen's township roads. The Hausdorff technique was used to compute the road index and to analysed the computed results. Similarly, Karpinski et al. [45] proposed the geospatial application for assessing road network based on the fractals theory. The box-counting and GIS analysis were proposed for calculating the fractal dimension.

In another study, Shahsavari, Afza, and Hekmatnejad [46] identified and then determined the geochemical discrepancies for assessment of Pb, Zn, and Cu mineralization in UDMA, Central Iran using Fractal and LOLIMOT Neuro-Fuzzy modelling. It was shown that the LOLIMOT algorithm signify anomalies in the areas with no litho-geochemical samples. Also, Chunzhong et al. [47] used the fractal dimensions to describe the spatial extents of lineaments in the Northeastern Yunnan Province, China.

Kabo-Baha et al. [48] evaluated the importance of fractal terrain structures on the general Erosion Potential Mapping (EPM) in the Loess Plateau of the Northern Shaanxi, China. Also, Mahmoodi, Nourbakhsh and Kusky [49] investigated the deformation pattern of land surfaces by applying ASTER GDEM through a combined geo-information and fractal approach. They extracted the earth's surface fractal dimension using the covering divider method. This was eventually used to estimate the surface roughness of the earth's topography through GIS.

3. Method and Tools

This study started with a synchronized search for literature on fractals and GIScience in the prominent electronic databases. The search retrieval was analysed with diverse keywords, which led to the best enquiry that fit the GIScience papers on fractals, that is, journal articles with "fractal*" and either "GIScience"*" without any time limits (i.e., (KEY (fractal* AND GIScience*))). After extensive experimentation, this query was identified as the most effective to gather the relevant articles concerning the aim of this study.

4. Basic Principles of Fractal Geometry

Fractal images are common in mathematics [50]. The word fractal was derived from the Latin word fructus which is the past participle of the word *frangere*. *frangere* refers to: break, crumble or fragment in free translation. *Fractus* is the root of the English word *fractured*.

According to Jiang and Brandt [51], fractals are classic representation of multifarious systems. Fractal geometry is present in several natural objects and phenomena like the mountain chains and rivers [52]. Of course, geographic phenomena have various properties relating to fractals. The goal of this section is to review some important features of fractal geometry.

4.1 Scale Measure

It is essential to define the scale to which certain shapes are defined at the beginning of each landscape assessment. Increasing the map scale will result in a larger number of smaller shapes being displayed on the map. Such shapes do not depend on each other, and they have a non-hierarchical scale [53]. The scale is one of the most significant aspects of spatial analysis. It affects the geometric extent of a geographic object, and also has a direct influence on the size of the symbols on the map.

In fractal geometry, scale is mainly defined in such a way that a series of scales are connected to one another in a scaling hierarchy. For instance, a coastline is a set of recursively defined curves that forms the scaling hierarchy of far more small curves than large ones. Therefore, the definition of "fractal" is explicitly based on the view of far more small things than large ones [54,55].

4.2 Self-similarity

The fundamental notion of fractal geometry is self-similarity. This implies the properties of an object resemble itself irrespective of which portion is observed, and irrespective of the number of times it is enlarged (see Fig. 1). A fractal is made up of an endless number of parts that resembles the entire object. Thus, fractal structures are said to be self–similar when portion of the object resembles the whole object [56, 57].

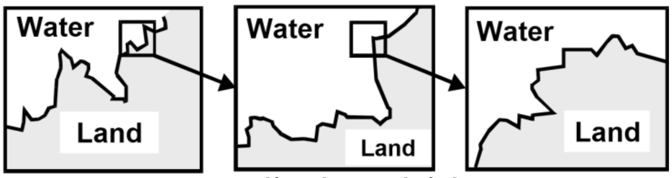

Figure.1. Self-similar Fractal of Objects.

The self-similar fractals are isotropic meaning that they have the identical characteristics and their parameters' values do not depend on the orientation of x and y axes [58]. They resist affine transformations, meaning that the fractal shape is constant regardless of how the cutout fractal shape is displayed extends/diminishes, rotates or shifts.

4.3 Self-affinity

Self-affinity in the 2-d space is mathematically defined based on the relationship between points F and F´, where F(x, y) is statistically similar to point F´(rx, r^{Ha}y). In this case, r is an affine transformation and Ha is the Hausdorff measure [43]. Unlike the self-similar fractal, the self-affined fractals are not isotropic.

4.4 Fractal Dimension

The dimension of an object is conventionally measured using the topological dimension. For a typical Euclidian geometry, four forms of this topological dimension exists including 0-dimension, 1-dimension, 2-dimension, and 3-dimension (see figure 2).

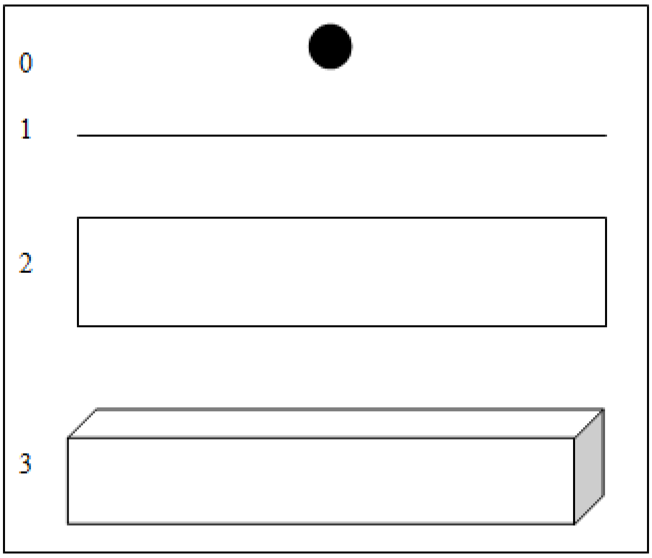

Figure.2. The dimensions shapes of the Euclidian Geometry

The 0-dimensional space represents a point, 1-dimensional space represents a line, 2-dimensional space represents a plane, 3-dimensional space represents a cube. The topological dimension can barely capture fractals and geographic phenomena because fractals are too irregular, both locally and globally. This gave to the idea of fractal dimension.

Many definitions of fractal dimension exist for the quantitative description of fractal geometric measurement. The ‘box-counting dimension’ is commonly used for estimating fractals [59]. It computes the number of grid cells needed to cover an object completely. The box-counting technique is employed for data—subset X of the map in r scale. Here, the box-counting dimension is defined by counting the number of unit boxes which intersect X:

> for any r > 0, let N(r) signify the least number of n-dimensional cells of linear scale r (side length) needed to occupy X [60].

Now, X has dimension D if N(r) with constant c satisfies the power law Formula (see equation 1).

$$N(r) \approx c\,(1/r)^{D} \qquad (1)$$

Thus, X with dimension D is in the following Formula.

$$D = \lim_{r \to 0}[-\log m / \log r] \qquad (2)$$

Based on this principle, Purevtseren, Indra, and Otgongerel [61] computed land-use fractal dimension by employing the most general Formula (see equation 3):

$$D = \frac{2 \log P/4}{\log A} \qquad (3)$$

D is a fractal dimension
P is the perimeter of the land use
A is the area of land use.

5. Generating Fractals

There are many techniques for generating fractals. Reynoso [62] strongly believes that results obtained by different methods frequently differ. This may be attributed to the method employed and the software, which calculates the fractal dimension. By and large, the three major methods for generating fractals comprise escape-time fractals, iterated function systems, and random fractals.

6. GIScience Application of Fractal Geometry

GIScience is concerned with studying spatial data and advanced methods for their production, computational analysis, visualization, and theorization. It cuts across several application domains in both human and physical environments and also connects various fields that study geographic information. Specifically, it comprises how geographic information denotes real world occurrences. GIScience is connected with geoinformatics, information science, data science [63], quantitative geography, and geoscience. The main focus of GIScience encompasses spatial assessment, visualization, and the depiction of uncertainty. This is achievable through the application of GIS technology, which offers tools for visualizing, integrating, and. analysing spatial data, and a distinctive ability to combine information from various sources.

Spatial analysis of geographic phenomenon using fractal algorithms has become popular. It is applicable in classification and assessment of the variations in Earth’s topography [64]. Of course, Earth’s topography entails its partial self-similarity repeating fractal structure of the landscapes at various dimensions. Fractals are also used to assess non-linear variability in geophysics, for universal graphical simulations or terrain generation and modelling. It is similarly used for geomorphological mapping, texture analysis and classification, and cartographic plotting. Additionally, the fractal/multi-fractal simulations are potent tools for geochemical exploration (see [65,66]).

The earliest quantification of landscape pattern utilized fractal analysis. Of course, the fractal dimension is used for studying ecological processes such as the spatial patterns of disturbance [67]. Also, the theoretical basis of fractal geometry has contributed to the widespread use of fractals for urban studies (see [68-71]).

7. Conclusion

This paper presents the fundamental of fractal geometry and its potential usage in GIScience. Fractals have more and more applications in GIScience as they can define the real world better than traditional methods. The complexity or irregularity of several objects can be measured using fractal analyses. Of course, the fractal geometry have attracted researchers mainly due to its capacity to define the irregular or fragmented shape of natural features and other multifaceted objects. The traditional Euclidean geometry could not analyse these objects adequately. Hence, the fractal-based analyses it is important for various application in GIScience.

Data Availability
Note applicable.

Conflict of Interest
The author declare no conflict of interest.

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Authors' Contributions
T.U.O. researched literature, conceived the study, and performed data analysis, wrote, reviewed, and accepted the final version of the manuscript.

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AUTHORS' PROFILE

Thomas U. Omali is presently a staff of Bio-resources Development Centre (BIODEC) Isanlu; National Biotechnology Development Agency (NABDA), Nigeria. He has published about 25 papers and book chapters in reputable journals and books respectively. His latest research focus is on ecology, global change, sustainable development, crustal deformation and earthquake, applied remote sensing, GIS/LIS, spatial analysis, and GPS application.