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Laplace Transformation of Integral Equations of convolution type

Dinesh Verma[1*], Rohit Gupta[2]

[1, 2]Department of Applied Sciences, Yogananda College of Engineering and Technology, Jammu (J&K)

**Corresponding Author: drdinesh.maths@gmail.com, Tel.: +8082002045*

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***Abstract*:** This paper presents the application of Laplace transformation method for solving integral equations of convolution type which arise in most of the scientific and engineering problems like diffraction problems, scattering in quantum mechanics, conformal mapping etc. Generally, Laplace transformation has been applied to obtain the solutions of ordinary and partial differential equations. By solving integral equations of convolution type via means of Laplace transformation method, it becomes easy to solve most of the scientific and engineering problems.

Keywords- Laplace Transformation, Integral equations.

I. INTRODUCTION

The Laplace transformation has been applied in science and engineering applications. It is a powerful mathematical tool in solving ordinary and partial differential equations. It has been applied to Ordinary linear differential equations with constant coefficients as well as with variable coefficients without finding their general solutions and the arbitrary constants. It has some applications in nearly all engineering disciplines, like application in electric circuit theory, application in power system load frequency control, analysis of electronic circuits, Nuclear Physics, diffraction problems, scattering in quantum mechanics etc. In this paper, Laplace transformation method is applied solving integral equations of convolution type which arise in most of the engineering problems.

II. BASIC DEFINITION

The Laplace transformation of a function F (t), t ≥ 0 is defined as $L\{F(t)\} = f(p) = \int_0^\infty e^{-pt} F(t)dt$, provided that the integral exists. Here p is a parameter which may be real or complex and L is the Laplace transformation operator.

III. INTEGRAL EQUATIONS

The integral Equation is of the form:

$$y(t) = f(t) + \int_c^d K(t,u)y(u)\,du$$

In this integral equation $K(t,u)$ is called the kernal. If c & d are constants, the equation is called a Fredholm integral equation. If c is a constant and d = t, then the equation is called a Volterra integral equation.

If the kernal $K(t,u)$ is of the form $K(t-u)$, then the integral $\int_0^t K(t-u)f(u)\,du$ represents convolution. Thus, we have

$$y(t) = f(t) + \int_c^d K(t,u)y(u)\,du = f(t) + (K * y)(t)$$

This equation is called convolution type integral equations.

Taking Laplace Transformation on both sides of the equation, we have

$$L\{y(t)\} = L\{f(t)\} + L\{(K*y)(t)\}$$

$$L\{y(t)\} = L\{f(t)\} + L\{K(t)\}L\{y(t)\}$$

$$L\{y(t)\} = \frac{L\{f(t)\}}{[1 - L\{K(t)\}]}.$$

$$y(t) = L^{-1}\left[\frac{\bar{f}(p)}{[1 - \bar{K}(p)]}\right]$$

A. To solve the integral equation

$$223t + 331t^2 = \int_0^t \frac{f(u)}{(t-u)^{1/4}}du$$

We can write this equation

$223t + 331t^2 = \int_0^t f(u)(t-u)^{-1/4}\,du \ldots\ldots..(1)$

Comparing this with

$$y(t) = f(t) + \int_c^d g(t,u)y(u)\,du = f(t) + (g*y)(t)$$

Here, $g(t-u) = (t-u)^{-1/4}$

$$g(t) = (t)^{-1/4}$$

$$L\{g(t)\} = l\{(t)^{-1/4}\}$$

$\frac{\Gamma 3/4}{p^{3/4}} = \bar{g}(p)$ ………… (2)

Now, Taking Laplace Transformation of (1)

$$L\{223t + 331t^2\} = L\{\int_0^t f(u)(t-u)^{-1/4}\,du\}$$

$$L\{223t\} + L\{331t^2\} = L\left\{\int_0^t f(u)(t-u)^{-1/4}\,du\right\}$$

$$223L\{t\} + 331L\{t^2\} = L\{f * g\}(t)$$

$$223\frac{1}{p^2} + 331\frac{2}{p^3} = \bar{f}(p)\bar{g}(p)$$

$$\frac{223}{p^2} + \frac{662}{p^3} = \bar{f}(p)\frac{\Gamma 3/4}{p^{3/4}}\ , \qquad from(2)$$

$$\bar{f}(p) = \frac{p^{3/4}}{\Gamma 3/4}\left[\frac{223}{p^2} + \frac{662}{p^3}\right]$$

$$\bar{f}(p) = \frac{1}{\Gamma 3/4}\left[\frac{223\,p^{3/4}}{p^2} + \frac{662\,p^{3/4}}{p^3}\right]$$

$$=L\{f(t)\} = \frac{1}{\Gamma 3/4}\left[223\,p^{-5/4} + 662\,p^{-9/4}\right]$$

$$f(t) = \frac{1}{\Gamma 3/4}\left[223L^{-1}\{p^{-5/4}\} + 662\,L^{-1}\{p^{-9/4}\}\right]$$

$$f(t) = \frac{1}{\Gamma 3/4}\left[223L^{-1}\{\frac{1}{\{p^{5/4}}\} + 662\,L^{-1}\{\frac{1}{\{p^{9/4}}\}\right]$$

$$f(t) = \frac{1}{\Gamma 3/4}\left[223\frac{t^{1/4}}{\Gamma 5/4} + 662\ \frac{t^{5/4}}{\Gamma 9/4}\right]$$

$$f(t) = \frac{1}{\Gamma 3/4}\left[892\frac{t^{1/4}}{\Gamma 1/4} + \frac{10592}{5}\frac{t^{5/4}}{\Gamma 1/4}\right]$$

$$\frac{t^{1/4}}{\Gamma 1/4\ \Gamma 3/4}\left[892 + \frac{10592}{5}t\right]$$

$$\frac{1}{\pi\sqrt{2}}t^{1/4}\left[892 + \frac{10592}{5}t\right]$$

B. To solve the integral equation

$2654\text{Sin } 22\text{t} = \int_0^t f(u)f(t-u)\,du$

Taking Laplace Transformation, on both sides

$\text{L}\{2654\text{Sin } 22\text{t}\} = L\left[\int_0^t f(u)f(t-u)\,du\right]$

$2654\ \text{L}\{\text{Sin } 22\text{t}\} = L\left[\int_0^t f(u)f(t-u)\,du\right]$

$2654\ \frac{22}{\text{p}^2+484} = \bar{f}(p)\bar{f}(p)$

$$\frac{58388}{\text{p}^2 + 484} = [\bar{f}(p)]^2$$

$$\bar{f}(p) = \text{L}\{\text{f(t)}\} = \frac{\sqrt{58388}}{\sqrt{\text{p}^2 + 484}}$$

$$\text{f(t)} = \text{L}^{-1}\frac{\sqrt{58388}}{\sqrt{\text{p}^2 + 484}}$$

$$\text{f(t)} = \sqrt{58388}\ \text{J}_0(22\text{t})$$

C. To solve the integral equation

$y(t) = 1 + t^2 + 21t^3 + 71t^4 + \int_0^t y(u)Sin(t-u)\,du$

Taking Laplace Transformation on both sides,

$L\{y(t)\} = L\{1\} + L\{t^2\} + L\{21t^3\} + L\{71t^4\} +$
$L\left[\int_0^t y(u)Sin(t-u)\,du\right]$

$$\bar{y}(p) = \frac{1}{p} + \frac{2}{p^2} + 21\frac{6}{p^3} + 71\frac{24}{p^4} + L\{y * g\}(t)$$

$$\bar{y}(p) = \frac{1}{p} + \frac{2}{p^2} + \frac{126}{p^3} + \frac{1704}{p^4} + \bar{y}(p)\bar{g}(p) \ldots\ldots\ldots (1)$$

Here, $g(t-u) = Sin\ (t-u)$

$$g(t) = Sin\ (t)$$

$$L\{g(t)\} = L\{Sin\ (t)\}$$

$$L\{g(t)\} = L\{Sin\ (t)\}$$

$$\bar{g}(p) = \frac{1}{p^2 + 1}$$

From (1) ,

$$\bar{y}(p) = \frac{1}{p} + \frac{2}{p^2} + \frac{126}{p^3} + \frac{1704}{p^4} + \bar{y}(p)\frac{1}{p^2 + 1}$$

$$\bar{y}(p)\left[1 - \frac{1}{p^2 + 1}\right] = \frac{1}{p} + \frac{2}{p^2} + \frac{126}{p^3} + \frac{1704}{p^4}$$

$$\bar{y}(p)\left[\frac{p^2}{p^2 + 1}\right] = \frac{1}{p} + \frac{2}{p^2} + \frac{126}{p^3} + \frac{1704}{p^4}$$

$$\bar{y}(p) = \frac{p^2 + 1}{p^2}\left[\frac{1}{p} + \frac{2}{p^2} + \frac{126}{p^3} + \frac{1704}{p^4}\right]$$

$$\bar{y}(p) = \left[\frac{p^2 + 1}{p^3} + \frac{2(p^2 + 1)}{p^4} + \frac{126(p^2 + 1)}{p^5} + \frac{1704(p^2 + 1)}{p^6}\right]$$

$$\bar{y}(p) = \frac{1}{p} + \frac{1}{p^3} + \frac{2}{p^2} + \frac{2}{p^4} + \frac{126}{p^3} + \frac{126}{p^5} + \frac{1704}{p^4} + \frac{1704}{p^6}$$

$$\bar{y}(p) = L\{y(t)\} = \frac{1}{p} + \frac{2}{p^2} + \frac{127}{p^3} + \frac{1706}{p^4} + \frac{126}{p^5} + \frac{1704}{p^6}$$

$$y(t) = L^{-1}\frac{1}{p} + L^{-1}\frac{2}{p^2} + L^{-1}\frac{127}{p^3} + L^{-1}\frac{1706}{p^4} + L^{-1}\frac{126}{p^5} + L^{-1}\frac{1704}{p^6}$$

$$y(t) = 1 + 2t + \frac{127}{2}t^2 + \frac{853}{3}t^3 + \frac{21}{4}t^4 + L^{-1}\frac{853}{60}t^5$$

IV. CONCLUSION

In this paper, we have presented the Laplace Transformation method to obtain the solutions of integral equations of convolution type. Laplace transformation method is found to be very useful mathematical tool to make complex problems like integral Equations of convolution type which arise in most of the scientific and engineering problems, simpler.

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Author's Profile

Dr. Dinesh Verma is currently Associate Professor in Mathematics, Department of Applied Sciences, Yogananda College of Engineering and Technology, Gurha Brahmana (Patoli, Akhnoor Road, Jammu (J&K, India). He has done Ph.D. at M.J.P. Rohilkhand University, Bareilly (U.P.) in 2004. He has been teaching UG Classes for well over two decades. He has to his credit four books for Engineering and Graduation level which are used by students of various universities. He has to his credit twenty four Research Papers. He has attended fourteen Workshop/Conferences/FDP during his Twenty years' experience of teaching. Also, he has a membership with ISTE (Indian Society for Technical Education) and ISCA (Indian Science Congress Association).

Mr. Rohit Gupta is currently Lecturer in physics, Department of Applied Sciences, Yogananda College of Engineering and Technology, Gurha Brahmana (Patoli), Akhnoor Road, Jammu (J&K, India). He has done M.Sc. Physics from University of Jammu (J& K) in the year 2012. He has been teaching UG Classes for well over six and a half years. He has to his credit eighteen Research Papers. He has attended five Workshop/Conferences/FDP during his six and half years' experience of teaching.

