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Research Article

Logarithmic Penalty Function Approach for Solving Multi-Objective Geometric Programming Problems

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Abstract— This paper presents a novel approach to solving Multi-Objective Geometric Programming Problems (MOGPP) using a Logarithmic Penalty Function (LPF) method. We introduce a transformation technique that converts the original multi-objective geometric programming problem into an equivalent single-objective problem. The proposed method is shown to satisfy Karush-Kuhn-Tucker (KKT) conditions, ensuring optimality of the solutions. Numerical examples are provided to demonstrate the effectiveness of the approach in comparison to existing methods. The proposed method shows improved convergence and solution quality across a range of test problems.

Keywords— Multi-Objective, Optimization, Geometric Programming, Logarithmic Penalty Method, Pareto Optimality

1. Introduction

Multi-objective optimization is crucial in various real-world applications where decision-making involves conflicting objectives. Geometric programming, with its emphasis on inequalities and power functions, is a powerful mathematical framework for modelling many engineering and economic systems. However, addressing multi-objective geometric programming (MOGP) problem remains challenging due to the complexity of the handling multiple objectives simultaneously.

This paper will examine the following multi-objective geometric programming problem:

Find $x = (x_1, x_2, \dots, x_n)^T$
so as to
min: $f_k(x) = \sum_{t=1}^{T_{k0}} C_{K0t} \prod_{j=1}^{n} x_j^{a_{k0tj}}, k = 1,2,\dots,p$ 1
subject to
$g_i(x) = \sum_{t=1}^{T_i} C_{it} \prod_{j=1}^{n} x_j^{d_{itj}} \le 1, \quad i = 1,2,\dots\dots,m$ 2
$x_j > 0, \qquad j = 1,2,\dots.n$ 3

(P_0)

where C_{K0t} for all k and t are positive real numbers and d_{itj} and a_{k0tj} are real numbers for all i, k, t, j.
T_{k0} = number of terms present in the k^{th} objective function.
T_i = number of terms present in the i^{th} constraint.

The multi-objective geometric program described above includes p minimization objective functions, m inequality constraints, and n strictly positive decision variables.

2. Related Work

Geometric programming problems (GPP) have been a subject of interest and application across various disciplines for over seven decades. The initial efforts to model GPP within the framework of nonlinear optimization can be traced back to the work of [1], who laid the foundational groundwork. Subsequently, [2] extended this work, proposing a specific algorithm aimed at solving GPP.

Since then, GPP has found applications in a wide range of fields, each contributing to its theoretical development and practical implementation. In the realm of communication systems, for instance, [3] explored its implications, while in engineering design, [4] applied GPP principles. Similarly, [5] delved into resource allocation problems, [6] tackled circuit design challenges, [7] addressed project management concerns, and [8] examined inventory management issues, all within the context of GPP.

However, when dealing with multiple objectives within the GPP framework, the problem transitions into the domain of Multi-Objective Geometric Programming Problems (MOGPP). Within the literature, two predominant solving approaches emerge: the fuzzy GPP method and the weighted mean method. [9] contributed to the former.

Despite the breadth of research, tackling optimization problems, particularly constrained ones, remains challenging. To address this, the penalty function method emerged as a pivotal approach. Originating from Zangwill's seminal work in 1967, this method integrates constraints into the objective function by incorporating penalty terms, thereby ensuring adherence to constraints. Subsequent studies by [10] demonstrated the efficacy of this approach, highlighting zero-duality gaps between optimization problems involving invex functions and their Lagrangian dual problems.

Building upon this foundation, researchers further refined the penalty function method. [11] enhanced the Courant-Beltrami penalty function, while [12]) extended its applicability to conic convex programs [13] proposed penalty function approaches for linear bilevel multi-objective problems, expanding the method's utility.

Additionally, the logarithmic penalty function method emerged as a robust tool for addressing nonlinear programming problems. Another study introduced a logarithmic penalty function approach tailored to handle irregular features in optimization problems, albeit limited to equality constraints. In the context of multi-objective geometric programming, the logarithmic penalty function method has garnered attention due to its versatility and applicability across various domains. However, challenges such as parameter selection and sensitivity to objective function scaling persist [14].

To address these challenges, researchers have proposed innovative methodologies. For instance, a generalized logarithmic penalty function method for smooth nonlinear programming problems involving invex functions, showcasing its effectiveness compared to alternative approaches was initiated by [15].

Despite these advancements, further research is warranted to explore the full potential of penalty function methods in addressing multi-objective geometric programming problems. Drawing inspiration from prior works, our approach combines multi-objective functions with the logarithmic penalty function method, emphasizing the need for tailored parameterization to the original geometric programming problem.
In this paper, we propose a logarithmic penalty function method to tackle MOGP problem efficiently.

3. Theory/Calculation

3.1 Preliminary definitions.

Definition 1: $x^* \in X$ is a pair to optimal solution of MOGPP (1) if there does not exist another feasible solution $\bar{x} \in X$ such that.
$f_k(\bar{x}) \leq f_k(x^*),\ k = 1,2,\dots,p \ and\ f_j(\bar{x}) < f_j(x^*)$ at least one j.

Definition 2: $x^* \in X$ is a weakly pare to optimal solution of MOGPP (1.1) if there does not exist another feasible solution $\bar{x} \in X$ such that $f_k(\bar{x}) < f_k(x^*), k = 1,2,\dots,p$

Definition 3: A continuous function p : R^n →R satisfying the following conditions:
(a) p(x) = 0 if x is feasible (in other word, if $g_j(x) \leq 0$)
(b) p(x) > 0 otherwise (in other word, if $g_j(x) > 0$)

Is said to be a penalty function for constrained optimization problem.

Conventionally, a penalty function approach introduced by Zang will work for both equality and inequality constraints was popularly known as absolute value penalty function, it is of the following form:

$$p(x) = \sum_{j=1}^{m}\left[g_j^+(x)\right] + \sum_{k=1}^{l}|h_k(x)| \quad 4$$

Note that: $g_j^+(x) = \max\{0, h_j(x)\}$ and $h_k(x) = 0$ (equality constraints), $\forall\, k \in K = \{1,2,\dots,l\}$

3.2 The logarithmic penalty method

A constrained optimization problem can be converted into a single unconstrained problem in single-objective programming or into a sequence of unconstrained problems for multi-objective optimization using a penalty function. By adopting the logarithmic penalty function proposed by Hassan and Baharum, we modified the Courant–Beltrami penalty function for equality constraints into the following form

$$p(x) = \sum_{j=1}^{m}[g_j^+(x)]^2 \quad 5$$

for inequality constraints, the modified Courant– Beltrami penalty should be constructed as follows:

$$p(x) = \sum_{j=1}^{m}\ln[\left(g_j^+(x)\right)^2 + 1^j] \quad 6$$

This leads to the following logarithmic penalized optimization problem for multi-objective fractional programming(P);

$$\text{Minimize } P_c(x) = f_k(x) + c\sum_{j=1}^{m}\ln[\left(h_j^+(x)\right)^2 + 1^j]. \quad 7$$

The solutions to the minimization problem (P_0) can be fully described in terms of the minimizers of the logarithmic penalty function when the penalty parameter exceeds a certain appropriate threshold. For a sufficiently large value of c, and under appropriate assumptions on the functions in (P_0), a KKT point minimizes the auxiliary function $P_c(x)$ if and only if it also minimizes the optimization problem (P_0).

4. Experimental Method/Procedure/Design

4.1 Kuhn–Tucker multiplier for logarithmic penalty function

In any nonlinear optimization problem, the Karush–Kuhn–Tucker (KKT) conditions represent the first-order necessary criteria for optimality, provided that certain constraint qualifications are met. However, Courant–Beltrami penalty function may not be differentiable at a point $h_j(x) = 0$ for some $i \in I$. But for the constrained optimization problem both objective function and constraints may be partially differentiable on R^n while at the same time the penalized problem is not, being differentiability is not among the properties of $\max\{0, g_j(x)\}$. Therefore, some additional hypothesis may be imposed on the constraint function $h_j(x)$, i.e. if the constraint $g_j(x)$ has continuous first-order partial derivatives on R^n, for this reason $[g_j^+(x)]^2$ admit the same. Therefore,

$$\frac{\partial}{\partial x_r}[g_j^+(x)]^2 = 2[g_j^+(x)]\frac{\partial}{\partial x_r}g_j^+(x) \quad 8$$

Where r is the multi-variable indexes.
Considering Equation (1.8), if p(x) : $R^n \rightarrow R$ is a logarithmic penalty functions and the constraints $h_j(x)$ has continuous first-order partial derivative on R^n,then

$$\nabla p(x) = \sum_{i=1}^{m} \nabla[\ln([g_j^+(x)]^2 + 1^i)] = \sum_{i=1}^{m} \frac{2g_j^+(x)\nabla h_j(x)}{([g_j^+(x)]^2+1^i)} \quad 9$$

From (9), we can define Kuhn–Tucker multiplier as follows:

$$\mu_j = \frac{2g_j^+(x)}{([g_j^+(x)]^2+1^i)} \quad 10$$

Theorem 1: Let $\bar{x}$ be the optimal solution in the problem(P_o) and assume that any suitable constraint qualification in (2.0) be satisfied at $\bar{x}$. Then there exists a Lagrange multiplier $\bar{\mu} \in R^m$ such that;

$$\sum_{i=1}^{p} w_i \nabla f_0(\bar{x}) + \sum_{j=1}^{m} \bar{\mu}_j \nabla g_j(\bar{x}) = 0, \quad 11$$

(i) $g_j(\bar{x}) \leq 0$,
(ii) $\bar{\mu}_j g_j(\bar{x})=0, j \in J$,
(iii) $\bar{\mu} \geq 0$
(iv) $\sum_{i=1}^{p} w_i = 1$

The conditions (11) are necessary for a local minimum of problem. The conditions are called Kuhn-Tucker conditions. The Kuhn tucker conditions satisfy the necessary and sufficient conditions for local optimum point to be a global optimum point.

4.2 The logarithmic penalty method for a geometric optimization problem

The general formation of a multi-objective geometric programming problem MOGPP can be expressed as;

$$\text{Minimize } f_i(x) = f_1(x), f_2(x), \ldots, f_m(x) \qquad (P_1)$$
$$\text{Subject\_to; } g_i(x) \leq 0$$
$$x_i > 0$$

Where $f: R^n \rightarrow R$, and $f_i(x)$ are the objective functions, $g_i(x)$ are the constraint functions, and x_i are the non-zero vector of decision variables. i.e $x \in R^n$, $i \in I = \{1,2,\ldots,m\}$, To transform the constraints multi-objective problems into an unconstrained form, we consider the generalize logarithmic penalty function for equality and inequality constraints introduce by Hassan and Baharum, (2019a). The generalize logarithmic penalty function defined as:

$$p(x) = \sum_{i=1}^{m} \ln\left((g_i^+(x))^2 + 1^i\right) \quad i \in I = \{1,2,\ldots m\}$$

where $g_i^+(x) = \max\{0, g_i(x)\}$

We construct a scalar objective function by combining the multiple objectives: Transforming a constrained optimization to a single unconstrained problem by using weighted sum method:

$$F(x) = \sum_{i=1}^{p} w_i f_i(x) \quad 12$$

Subject to; $g_i(x) \leq 1, \quad i = 1,2,\ldots p$
$\sum_{i=1}^{p} w_i = 1, \quad w_i > 0, \quad i = 1,2,\ldots p$
$x \in R^n$

This leads to the following logarithmic penalized optimization problem for multi-objective geometric programming (P_1);

$$P_{c_k}(x) = F(x) + c_k \sum_{i=1}^{m} \ln((g_i^+(x))^2 + 1^i) \quad 13$$

By updating the parameter c_k, the sequence solution will approach the Pareto optimal set of the original multi-objective geometric optimization problem.

The penalty function algorithm involves solving a sequence of unconstrained sub problems to converge to the constrained optimum. The algorithm iteratively minimizes the penalized objective function while adjusting the penalty parameter.

Algorithms

Step 1: Initialization
• Choose an initial guess $x_0 \in R^n$
• Set an initial value for the penalty parameter,$c_0 > 0$.
Step 2: Iterative Optimization Process
1. Define the Penalized Objective Function:
• For each iteration k, define the penalized objective function: $P_{c_k}(x)$
2. Solve the Unconstrained Problem:
• Minimize the penalized objective function $P_{c_k}(x)$ using fmincon in Matlab.
3. Check Constraints:
• Evaluate the constraints at the current solution: $g_i(x_k)$
4. Update Penalty Parameter:
• If any constraints are violated (i.e., if any $g_i(x_k) > \varepsilon$):
* Increase the penalty parameter by $\beta > 0$, set $c_{k+1} = \beta * c_k$
• If all constraints are satisfied:
* Optionally decrease the penalty parameter to allow exploration of more feasible regions.
5. Convergence Check:
• Determine if the algorithm has converged by checking if changes in the solution or objective values are below a predefined threshold. If convergence is achieved, stop the process.
Step 3: Output Results
• Once convergence is reached, return the optimal solution $\bar{x}$

and its corresponding objective values.

5. Results and Discussion

5.1 Numerical examples.

Example 1: To illustrate the proposed model, we consider the following problem which is also used by Ojha and Biswal (2010).

Min: $G_1(x)=4x_1 + 10x_2 + 4x_3 + 2x_4$
Max: $G_2(x)=x_1x_2x_3$
subject to

$$\frac{x_1^2}{x_4^2} + \frac{x_2^2}{x_4^2} \le 1$$
$$\frac{100}{x_1 x_2 x_3} \le 1$$
$$x_1, x_2, x_3, x_4 > 0$$

Now the problem can be written as:
Min: $f_1(x) = 4x_1 + 10x_2 + 4x_3 + 2x_4$
Min: $f_2(x) = x_1^{-1}x_2^{-1}x_3^{-1}$
subject to
$g_1(x) = x_1^2 x_4^{-2} + x_1^2 x_4^{-2} \le 1$
$g_2(x) = 100x_1^{-1}x_2^{-1}x_3^{-1} \le 1$
$x_1, x_2, x_3, x_4 > 0$

Now, we construct the unconstrained multi-objective geometric programming based on logarithmic penalized optimization problem as in Equation (2.4)

$$\text{Minimize } P_{c_k}(x) = \left(F(x) + c_k \sum_{j=1}^{m} \ln[\left(g_j^+(x)\right)^2 + 1^j]\right)$$

Where
$f_1(x) = 4x_1 + 10x_2 + 4x_3 + 2x_4$, $f_2(x) = x_1^{-1}x_2^{-1}x_3^{-1}$.
Therefore, we are to find a pareto optimal for the following unconstrained objective functions:

$$\begin{aligned} \text{Min } P_c(x) = {} & w_1(4x_1 + 10x_2 + 4x_3 + 2x_4) + w_2(x_1^{-1}x_2^{-1}x_3^{-1}) + \\ & c\ln[(max\{0, x_1^2 x_4^{-2} + x_1^2 x_4^{-2} - 1\})^2 + 1] + \\ & c\ln[(max\{0, 100x_1^{-1}x_2^{-1}x_3^{-1} - 1\})^2 + 1], \end{aligned} \quad (4.1)$$

Where $w_1 + w_2 = 1$
Now (4.1) will be solved using LPF approach.

For the weights w1 = w2 =0.5

$$\begin{aligned} \text{Min } P_c(x) = {} & 0.5(4x_1 + 10x_2 + 4x_3 + 2x_4) + 0.5(x_1^{-1}x_2^{-1}x_3^{-1}) \\ & + c\ln[(max\{0, x_1^2 x_4^{-2} + x_1^2 x_4^{-2} - 1\})^2 + 1] + c\ln[(max\{0, 100x_1^{-1}x_2^{-1}x_3^{-1} - 1\})^2 + 1] \end{aligned}$$

By applying algorithm with a termination scalar $\varepsilon = 10^{-5}$.
- a starting point x_0 =(1,1,1,1).
- a penalty parameter c_0=1
- a scalar β =10. In each iteration we calculate the minimum (4.2) depending on the current c value. Every optimal solution is a starting point in the next iteration. The loop will stop when violation is smaller than ϵ.

All calculations are done in *MATLAB* using *fmincon* minimization function. It can be seen in the Table1 and Figure 1 as the parameter increases the solution converge at c = (10^6), where the constraints satisfied and algorithm terminate.
Finally At iteration 6 the solution converges and satisfy (2.1) KKT condition at $\bar{x}$ =(5.0800, 2.6841 , 7.3339, 5.7455) with

lagranges multiplier μ=[1.4271, 7.2199] which is the pareto optimal solutions points of the given MOGPP for the weight w_1 = 0.5 and w_2 = 0.5 and optimal values are f_1=87.9873,

f_2=0.0100.

Table 1: Solution for w_1 = 0.5 and w_2 = 0.5, using the logarithmic penalty function approach.

Iterations	c	x_1	x_2	x_3	x_4	f_1	f_2
0	1	1.1836	0.5048	1.2794	0.2265	15.3525	1.3083
1	10	4.1888	2.1650	5.8585	4.2315	70.3018	0.0188
2	100	4.8974	2.5774	7.0359	5.4615	84.4307	0.0113
3	1000	5.0611	2.6697	7.2962	5.7139	87.5537	0.0101
4	10000	5.0817	2.6812	7.3287	5.7448	87.9432	0.0100
5	100000	5.0839	2.6824	7.3319	5.7481	87.9833	0.0100
6	1000000	5.0800	2.6841	7.3339	5.7455	87.9873	0.0100

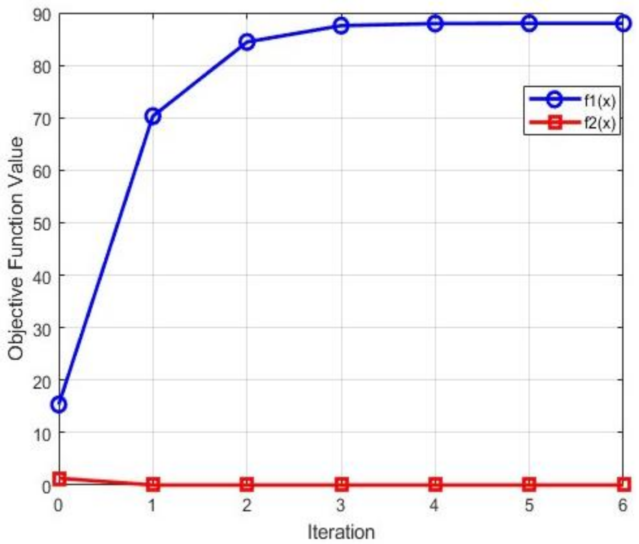

Figure 1: Convergence Plot

Table 2: The solution from the numerical approach

w_1	w_2	x_1	x_2	x_3	x_4	f_1	f_2
0.1	0.9	5.0834	2.6822	7.3335	5.7477	87.9873	0.0100
0.2	0.8	5.0831	2.6827	7.3323	5.7477	87.9867	0.0100
0.3	0.7	5.0837	2.6827	7.3322	5.7481	87.9861	0.0100
0.4	0.6	5.0839	2.6826	7.3321	5.7482	87.9862	0.0100
0.5	0.5	5.0800	2.6841	7.3339	5.7455	87.9873	0.0100
0.6	0.4	5.0838	2.6824	7.3318	5.7480	87.9824	0.0100
0.7	0.3	5.0832	2.6825	7.3322	5.7475	87.9815	0.0100
0.8	0.2	5.0835	2.6825	7.3316	5.7477	87.9806	0.0100
0.9	0.1	5.0836	2.6823	7.3316	5.7477	87.9797	0.0100

5.2 Comparison with Weight Mean Method

As shown in Tables 3, The comparison highlight that all methods provided valid solutions, the logarithmic penalty function's ability to yield a slightly better result indicate its potential for further exploration in similar optimization problems.

Table 3: Comparison of Solutions with weighted mean method

		Variables								Combine Objective Values	
Weights		Weight Mean Method				Logarithmic Penalty Function method				Weighted Mean	LPF Method
w_1	w_2	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4	Z	P
0.1	0.9	5.08405	2.68255	7.33231	5.74836	5.0834	2.6822	7.3339	5.7477	8.80776	8.8072
0.2	0.8	5.08405	2.68255	7.33231	5.74836	5.0831	2.6830	7.3323	5.7477	17.60555	17.6053
0.3	0.7	5.08405	2.68255	7.33231	5.74836	5.0837	2.6827	7.3322	5.7481	26.40333	26.4031
0.4	0.6	5.08405	2.68255	7.33231	5.74836	5.0839	2.6826	7.3321	5.7482	35.20111	35.2007
0.5	0.5	5.08405	2.68255	7.33231	5.74836	5.0839	2.6825	7.3321	5.7482	43.99888	43.9983

The comparison highlight that all methods provided valid solutions, the logarithmic penalty function's ability to yield a slightly better result indicate its potential for further exploration in similar optimization problems. The logarithmic penalty function approach provided solutions that were equivalent to those obtained using the weighted mean method, demonstrating high accuracy. The approach showed faster convergence. making it more efficient for large-scale problems. The penalty function approach effectively handled constraint violations, ensuring feasible solutions throughout the optimization process.

6. Conclusion and Future Scope

In this paper, we presented a novel logarithmic penalty function approach for solving multi-objective geometric programming problems. The proposed method transforms the original MOGP problem into an equivalent single-objective problem and applies a logarithmic penalty function to handle constraints effectively.

We proved that the proposed approach satisfies the KKT conditions, ensuring the optimality of the solutions. Numerical examples demonstrated the effectiveness of our method across a range of problem complexities. Comparisons with existing methods showed that our approach consistently achieves optimal solutions while maintaining competitive computational efficiency.

Future work could focus on extending this approach to handle more complex constraints and exploring its applicability to real-world engineering and economic problems.

Data Availability
The research is based on LPF and numerical simulation has provided all the data needed for clarity.

Study Limitations: None

Conflict of Interest
All Authors declare that they do not have any conflict of interest.

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Authors' Contributions
All authors reviewed and edited the manuscript and approved the final version of the manuscript.

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References

[1] T. Antczak, "The l1 penalty function method for non-convex differentiable optimization problems with inequality constraints", Asia-Pacific Journal of Operational Research, Vol.**27**, No.**5**, pp.**559–576, 2010.**

[2] R., Azari, S. Garshasbi, "Multi-objective optimization of building envelope design for life cycle environmental performance", Energy and Buildings, Vol.**126**, No.**2**, pp.**524–534, 2016.**

[3] M. Chiang, "Geometric programming for communication systems", Foundations and Trends® in Communications and Information Theory, Vol.**2**, No.**2**, pp.**100–154, 2005.**

[4] R. Creese, "Geometric programming for design and cost optimization 2nd edition", Springer Nature, **2022.**

[5] R. J. Duffin, E. L. Peterson, C. Zener, "Geometric programming: theory and application", 1st Edition, **1967.**

[6] E. Ernst, M. Volle, "Generalized courant–beltrami penalty functions and zero duality gap for conic convex programs", Journal of Natural Sciences, Vol.**17**, No.**4**, pp.**945–964, 2013.**

[7] M. Hassan, A. Baharum, "Modified courant-beltrami penalty function and a duality gap for invex optimization problem", International Journal for Simulation and Multidisciplinary Design Optimization, Vol.**10**, No.**10**, pp.**12-18, 2019.**

[8] S. Islam, "Multi-objective geometric programming problem and its applications". Yugoslav journal of operations research, Vol.**20**, No.**2, 2016.**

[9] Y. Karimian, A. Mirzazadeh, S. H. Pasandideh, M. Namakshenas,. "A geometric programming approach for a vendor managed inventory of a multiretailer multi-item epq model". RAIRO-Operations Research, Vol.**54**, No.**5**, pp.**1401–1418, 2020.**

[10] A. S. H. Kugele, W. Ahmed, B. Sarkar, "Geometric programming solution of second degree difficulty for carbon ejection controlled reliable smart production system". RAIRO-Operations Research, Vol.**56**, No.**2**, pp.**1013–1029.14, 2022.**

[11] J. Le Ny, G. J. Pappas, "Geometric programming and mechanism design for air traffic conflict resolution". In Proceedings of the American control conference, Vol.**5**, No.**1**, pp.**3069–3074, 2010.**

[12] A. K. Ojha, K. Biswal, "Multi-objective geometric programming problem with weighted mean method", Journal of Operations Research, Vol.**7**, No.**2**, pp.**10-15, 2010.**

[13] V. M. Preciado, M. Zargham, "Optimal resource allocation for network protection: A geometric programming approach", IEEE Transactions on Control of Network 15 Systems, Vol.**1**, No.**1**, pp.**99–108, 2014.**

[14] W. I. Zangwill, "Non-linear programming via penalty functions", Management science, Vol.**13**, No.**5**, pp.**344–358, 1967.**

[15] C. Zener, "A mathematical aid in optimizing engineering designs", Proceedings of the National Academy of Sciences, Vol.**47**, No.**4**, pp.**537–539, 1961.**

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