International Journal of Scientific Research in Multidisciplinary Studies
Vol.10, Issue.5, pp.19-25, May 2024
E-ISSN: 2454-9312 P-ISSN: 2454-6143
Available online at: www.isroset.org

Research Article

Rainfall Analysis of Guwahati City Using the Method of L-Moments, Tl-Moments & Maximum Likelihood Estimation

Ruhiteswar Choudhury[1], Tanusree Deb Roy[2*]

[1,2]Dept. of Statistics, Assam University, Silchar, Assam, India

*Corresponding Author: tanusree.deb.roy@gmail.com

Received: 23/Mar/2024; **Accepted**: 25/Apr/2024; **Published**: 31/May/2024. | **DOI**: https://doi.org/10.26438/ijsrms/v10i5.1925

Abstract— This paper consists of three types of estimation of parameter methods which are namely Maximum Likelihood Estimation (MLE), L-Moments, TL-Moments for estimating parameters of four types of probability distribution including Generalized Extreme Value (GEV), Generalized Pareto (GPA), Generalized Logistic (GLO) & Pearson Type III (PEIII) distribution. Furthermore, Root Mean Square Error (RMSE) values have been calculated to assess the performance of the mentioned distributions for the respective seasons. Finally, 37 years of rainfall data for each month has been analysed, where for winter & Pre-Monsoon season the Pearson Type III distribution is computed as the best fitted probability distribution, for Monsoon and Post-Monsoon season GEV and GPA are found to be the best fitted probability distributions respectively. Interpreting the RMSE values, out of the three kinds of parameter estimation method, L-Moments turns out to be the most efficient method.
.

Keywords— Parameter, Generalized Extreme Value, Generalized Logistic, Generalized Pareto, Pearson Type III, Root Mean Square Error.

1. Introduction

Guwahati, the largest city of Assam and North-East India, a major riverine port city. Here, total annual average precipitation is 1722 mm. Most of the times, 2-3 hours of constant heavy rainfall causes flood in the city[1]. Rainfall is an important and essential phenomena to understand the socio-economic fabric of the Guwahati City, which is located at 26.148^0 N latitude and 91.731^0 E longitude in the state Assam, India, which is situated at the southern bank of the Brahmaputra river[2], and the border of the state Assam. Effective handling of water supplies, prevention of flooding, and sustainable growth planning all depend on an understanding of Guwahati's rainfall patterns. The city has experienced several extreme rainfall events and associated challenges such as flash floods, landslides, and waterlogging. So, analysing rainfall trends and behaviour based on the probability distribution is an essential phenomenon. Long term rainfall patterns may get influenced by the global climatic changes and this may result with the danger of increasing the occurrences of droughts and floods. South-West monsoons have a significant impact on water resources for every sector of India as it brings nearly 80% of total rainfall in a year[3]. Accessing rainfall pattern in Guwahati can aid in climate change impact assessments and adaptation planning. With climate change projections indicating potential shifts in precipitation patterns, it becomes imperative to study the historical rainfall trends to better anticipate future changes and their consequences. The findings of such an analysis will assist policymakers, urban planners, and water resource managers in formulating strategies to mitigate risks associated with changing rainfall patterns and ensure sustainable water management practices[4].

2. Related Work

Hydrological trend analysis involves studying precipitation, rainfall, floods, and extreme events. This study focuses on seasonal rainfall analysis of Guwahati City.

[5] have shown the importance of Probability Distribution in rainfall analysis is an essential task to understand the behaviour of the rainfall patterns of a region to accurately forecast the extreme events and take preventive steps. [6] described various probability distribution and different methods of parameter estimation technique among them method of moments is one of the most basic method of estimating distribution parameters among which several researchers have computed the parameters using the method of moments like [7], also the method of Maximum Likelihood Estimation(MLE) is a superior approach to the method of moments as described by [8] is one of the most important and widely used methods for estimation of parameters in

distribution theory. [9], [10] used maximum likelihood estimation for estimating various probability distributions which are significantly better than other estimates using certain goodness of fit results. [5]found that Gamma and Weibull were competing, but Weibull fit well by plotting, Chi-square, and Kolmogorov-Smirnov tests. [11] discovered that L-moments offers a unified approach for statistical inference, making it more robust and advantageous than maximum likelihood estimation. [12]in their study, observed and compared the L-moment parameter estimation with the classical moments and maximum likelihood, where they observe the L-moment approach offers significant benefits with respect to all significant constraints placed on distribution parameter estimates. [13] found that the Generalized Logistic distribution is more consistent than the Generalized Extreme Value distribution in describing the maximum rainfall year-wise in North East India using several methods like L-Moment parameter estimation. [14] found that the best fitted probability distribution based on L-moments is GEV, GLO, GPA, Generalized Normal, and Pearson type III. [15] introduced TL-moments to improve the reliability of L-moment when the outliers are present in the data. [16] found TL-moments more efficient for lower quantile estimation compared to L-moments considering the Generalized Logistic Distribution. [17] found the generalized Pareto distribution as the best fit among the three generalized extreme value, generalized Pareto, and generalized logistic distributions using the method of TL-Moments. [18] developed TL-Moments & LQ Moments of exponentiated generalized extreme value distribution. [19]computed the parameters of four parameter generalized lamda distribution using the methods of L-Moments and TL-Moments correspondingly. [20]applied methods of L-Moments and TL-Moments in ´ the construction of wage distribution.

3. Methods and Materials

3.1 Data Source and Description

The monthly rainfall dataset for the region Guwahati, Assam, was obtained from the India Meteorological Department (IMD) for the period 1985 – 2022.The data is further segmented into four meteorological seasons which is described by IMD, they are winter (January-February), Pre-monsoon (March-May), Monsoon (June-September), Post-monsoon (October-December) as defined by Indian Meteorological Department.

3.2 Fitting of Probability Distribution

In rainfall and hydrological analysis plenty of probability distributions were evaluated the best fitted probability distribution, in this study, extreme events were considered as a result some of the distribution is not considered whereas the distributions under consideration are Generalized Extreme Value (GEV), Gumbel, Generalized Pareto (GPA), Generalized Logistic (GLO), Pearson type III Distribution. Each distribution offers unique characteristics and applications, making them suitable for different types of data and research objectives. The Generalized Logistic Distribution is known for its flexibility in modeling skewed data, while the Generalized Pareto Distribution is often used for modeling extreme values and tail behavior. The Generalized Extreme Value Distribution is particularly useful in analyzing maxima and minima of datasets, crucial for fields like meteorology and finance. Lastly, the Pearson Type III Distribution, with its ability to handle skewed distributions and various shapes, provides a versatile option for modeling diverse data sets. This comparative analysis helps in identifying the most appropriate distribution for accurate data representation and prediction. The description of the distributions along with their density functions, range and parameter considered are described as follows:

The density function of GEV Distribution is –

$$f(x;\xi,\alpha,k)=\frac{1}{\alpha}\left[1+k\left(\frac{x-\xi}{\alpha}\right)\right]^{k}$$

$$\exp\left[-\left(1+k\left(\frac{x-\xi}{\alpha}\right)^{-\frac{1}{k}}\right)\right];\ -\infty<x,\xi,k<\infty,\alpha>0$$

The density function of GPA Distribution is -

$$f(x;\xi,\alpha,k)=\tfrac{1}{\alpha}\left[1+k\left(\tfrac{x-\xi}{\alpha}\right)\right]^{-\left(1+\frac{1}{k}\right)};$$

$$-\infty<x,\xi,k<\infty,\alpha>0$$

The density function of GLO Distribution is

$$f(x;\xi,\alpha,k)=\frac{k}{\alpha}\frac{e^{-(x-\xi)/\alpha}}{\left\{1+e^{-(x-\xi)/\alpha}\right\}^{\alpha+1}},$$

$$-\infty<x,\xi<\infty,\alpha,k>0$$

The density function of PEIII Distribution is -

$$f(x;\xi,\alpha,k)=\frac{1}{\alpha^{k}\Gamma(k)}\left[\left(\frac{x-\xi}{\alpha}\right)\right]^{(k-1)}\exp\left[-\left(\frac{x-\xi}{\alpha}\right)\right],$$

$$-\infty<x,\xi<\infty,\alpha,k>0$$

Maximum Likelihood Estimation, L-Moments, and TL-Moments are the three methods for estimating distribution parameters that were taken into consideration in this study.

3.2 Maximum Likelihood Estimation

The Maximum Likelihood Estimation (MLE) technique is a frequently used and dependable way to estimate the parameters of particular probability distributions. By optimizing the likelihood function, this method guarantees that the observed data is most likely to occur given the predicted parameters. The idea behind the approach was first presented by Carl Friedrich Gauss, who did so in relation to the normal distribution. Nonetheless, MLE was formalized and expanded upon by Sir Ronald A. Fisher in 1912, enabling its application to a wide variety of statistical models and distributions.

MLE is widely used because of its favorable statistical characteristics. It yields consistent parameter estimates, which, as sample size grows, converge to the real parameter values. MLE estimates are also the most efficient among unbiased estimators, obtaining the lowest possible variance. MLE is especially useful in practice because of its efficiency,

which produces parameter estimates that are more precise and reliable than those produced by other techniques.
The likelihood function, which shows the likelihood of witnessing the supplied data under various parameter values, is maximized by choosing parameter values in the MLE procedure. In mathematical terms, this means determining the parameter values that maximize the likelihood of the observed data. This approach is flexible enough to be used with a wide range of data and distributions, even ones with intricate likelihood functions.

The likelihood function is represented

$$L = \prod_{i=1}^{n} f(x_i)$$

The log-likelihood function of GEV Distribution is -

$$\log L = -n \log \alpha - \left(1 + \frac{1}{k}\right) \sum_{i=1}^{n} \log\left(1 + k\left(\frac{x_i - \xi}{\alpha}\right)\right) - \sum_{i=1}^{n} \left(1 + k\left(\frac{x_i - \xi}{\alpha}\right)\right)^{-\frac{1}{k}}$$

The log-likelihood function of GPA Distribution is -

$$\log L = -n \log \alpha - \left(1 + \frac{1}{k}\right) \sum_{i=1}^{n} \log\left(1 + k\left(\frac{x_i - \xi}{\alpha}\right)\right)$$

The log-likelihood function of GLO Distribution is –

$$\log L = n \log\left(\frac{k}{\alpha}\right) - \sum_{i=1}^{n} \frac{(x_i - \xi)}{\alpha} - (\alpha + 1) \sum_{i=1}^{n} \log\left\{1 + \exp\left[-\frac{(x_i - \xi)}{\alpha}\right]\right\}$$

The log-likelihood function of PEIII Distribution is

$$\log L = -n \log \alpha \Gamma(k) + (k - 1) \sum_{i=1}^{n} \left(\frac{x_i - \xi}{\alpha}\right) - \sum_{i=1}^{n} \left(\frac{x_i - \xi}{\alpha}\right)$$

The estimates of the corresponding parameters can be obtained by optimizing the log-likelihood with respect to the parameters.
For each of the distribution, $\hat{\xi}$, $\hat{\alpha}$ & $\hat{\kappa}$ are the location, scale and shape parameter respectively.

The GEV, GLO, GPA, and PEIII distributions' seasonal investigation of parameter estimates provide insight on how extreme values vary in behaviour throughout each season of the year.

Table 1 Parameter Estimates using MLE

Distribution		Seasonal			
		Winter	Pre-Mon	Monsoon	Post-Mon
GLO	$\hat{\alpha}$	10.92	91.03	82.327	36.317
	$\hat{\xi}$	-62.82	-375.01	139.69	-195.6
	$\hat{k}$	684.77	192.99	2.84	354.26
GEV	$\hat{\alpha}$	6.6713	84.58	100.27	29.18
	$\hat{\xi}$	4.5768	96.20	207.73	5.11
	$\hat{k}$	0.8511	0.16	-0.08	5.71
GPA	$\hat{\alpha}$	15.2810	158.8453	137.24	108.26
	$\hat{\xi}$	-	-	-	-
	$\hat{k}$	0.0949	-0.2855	-0.2822	-0.23
PEIII	$\hat{\alpha}$	-	111.0469	48.3917	-
	$\hat{\xi}$	-	1.9873	-34.03	-
	$\hat{k}$	-	1.3506	6.0359	-

From Table 2, it is noted that the parameter estimates for each of the selected distribution using the method of L-Moment have been computed for each of the seasons.

3.3 L-Moments
A different method for using order statistics to estimate a distribution's parameters was carried out by Hosking 1990. L-moments have been calculated using this method that is based on certain linear combinations of order statistics. Because they account both the order statistics and Gini's mean difference, L-moments are thought to be more reliable to sample variability and outliers when estimating parameters.

Probability Weighted Moments (PWM) are used to define L-moments, providing a more stable and dependable estimating technique than standard moments. The L-moment of the rth order is represented as λ_r, which stands for a linear combination of PWM values. Since severe occurrences are frequently included in data in disciplines like hydrology and meteorology, L-moments are especially helpful due to their robustness and reduced sensitivity to extreme values. L-moments offer a method to parameter estimation that is more robust and reliable by taking into account the order of data values.

$$\lambda_r = r^{-1} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} E\left(X_{(i)}\right), \quad r = 1,2,\ldots \qquad \ldots(1)$$

The pdf of the r^{th} order statistic is –

$$f_r(x) = \frac{n!}{(r-1)!\,(n-r)!} [F(x)]^{r-1} [1 - F(x)]^{n-r} \qquad \ldots(2)$$

The expectation of the rth order statistic is,

$$E(X_r) = \frac{n!}{(r-1)!\,(n-r)!} \int x [F(x)]^{r-1} [1 - F(x)]^{n-r} dF(x) \qquad \ldots(3)$$

Substituting the value of (2) & (3) in (1), we simplify the r^{th} L-Moments as,

$$\lambda_r = \int_0^1 x(F) P_{r-1}(F(x)) dF(x), \quad r = 1,2,\ldots \qquad \ldots(4)$$

Where, $x(F)$ is the quantile function

$$P_r(F(x)) = \sum p_{r,k} [F(x)]^k$$

$$p_{r,k} = (-1)^{r-k} \binom{r}{k} \binom{r+k}{k}$$

With respect to these equations the first few L-Moments can be computed using the following:

$$\lambda_1 = \int_0^1 x(F)dF(x)$$

$$\lambda_2 = \int_0^1 x(F)[2F-1]dF(x)$$

$$\lambda_3 = \int_0^1 x(F)[6\{F(x)\}^2 - 6F(x) + 1]dF(x)$$

$$\lambda_4 = \int_0^1 x(F)[20\{F(x)\}^3 - 30\{F(x)\}^2 + 12F(x) - 1]dF(x)$$

The L-Moment ratios are defined by –

$$\tau_i = {}^{\lambda_i}/_{\lambda_2}$$

And, $\tau = {}^{\lambda_2}/_{\lambda_1}$ is the coefficient of variation (L-CV).

$\tau_3 = {}^{\lambda_3}/_{\lambda_2}$, is the Skewness (L-Skewness).

$\tau_4 = {}^{\lambda_4}/_{\lambda_2}$, is the Kurtosis (L-Kurtosis).

Estimation of L-Moments is obtained by its unbiased estimator i.e., the sample L-moments. The sample r[th] L-moment (l_r) is defined by –

$$l_r = r^{-1}\binom{n}{r}^{-1}\sum_{i=1}^{n}\left[\sum_{j=0}^{r-1}(-1)^j\binom{r-1}{j}\binom{i-1}{r-1-j}\binom{n-i}{j}\right]X_{(i)}$$

Then the first four sample l-moments are –

$$l_1 = \frac{\sum x_i}{n}$$

$$l_2 = \frac{1}{2}\binom{n}{2}^{-1}\sum_{i>j=1}^{n}\left(X_{(i)} - X_{(j)}\right)$$

$$l_3 = \frac{1}{3}\binom{n}{3}^{-1}\sum_{i>j>k=1}^{n}\left(X_{(i)} - 2X_{(j)} + X_{(k)}\right)$$

$$l_4 = \frac{1}{4}\binom{n}{4}^{-1}\sum_{i>j>k>l=1}^{n}\left(X_{(i)} - 3X_{(j)} + 3X_{(k)} - X_{(l)}\right)$$

Correspondingly, the sample L-moment ratios are calculated using

$$t_i = {}^{l_i}/_{l_2}$$

By solving the following linear equations, the parameter estimates of certain probability distributions can be calculated as –

$$\lambda_i = l_i, i = 1,2, \ldots$$

Table 2 Parameter Estimates using L-Moments

Distribution		Seasonal			
		Winter	Pre-Mon	Monsoon	Post-Mon
GLO	$\hat{\alpha}$	7.2599	63.2856	64.6759	19.087
	$\hat{\xi}$	10.8232	136.4559	245.9317	19.866
	$\hat{k}$	-0.3465	-0.2076	-0.1137	-0.531
GEV	$\hat{\alpha}$	9.5141	92.7428	102.8702	22.018
	$\hat{\xi}$	6.9125	100.0577	207.2555	10.535
	$\hat{k}$	-0.2575	-0.0578	0.0895	-0.495
GPA	$\hat{\alpha}$	17.0632	206.3726	272.5697	31.626
	$\hat{\xi}$	-1.9642	1.9371	86.9671	-7.431
	$\hat{k}$	-0.0293	0.3124	0.5917	-0.387
PEIII	$\hat{\alpha}$	18.0044	126.5776	258.2066	44.145
	$\hat{\xi}$	15.6145	159.1895	118.8824	75.926
	$\hat{k}$	2.0795	1.2549	0.6939	3.3065

From Table 2, it is noted that the parameter estimates for each of the selected distribution using the method of L-Moment have been computed for each of the seasons.

3.4 TL-Moments

[15] introduced Trimmed L-Moments (TL-Moments), a new and more robust statistical approach. This technique builds on the basis of classic L-Moments while improving its resilience, especially in datasets with outliers. TL-Moments enhance by selectively cutting the lowest and biggest values in a dataset, especially t_1.

t_1 represents the smallest values, whereas t_2 represents the largest values. This trimming method effectively minimizes the impact of major outliers, which can bias the study and lead to inaccurate results.

The idea of TL-Moments are the expected value of certain order statistic $E\left(X_{(r-k)}\right)$ where $X_{(r-k)}$ is the $(r-k)^{th}$ order statistic is replaced with expected value $E\left(X_{(r+t_1-k:r+t_1+t_2)}\right)$ where t_1 represents the t_1-smallest & t_2 represents the t_2-largest trimming, where if $t_1 = t_2 = 0$, the TL-Moments estimates has been transformed to L-Moments if $t_1 = t_2 = t$, such trimming are called symmetrical trimming.

$$\lambda_r^{(t_1,t_2)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}E\left(X_{(r+t_1-k:r+t_1+t_2)}\right) \qquad \ldots(1)$$

The expectation of the r^{th} order statistics is given by –

$$E(X_r) = \frac{n!}{(r-1)!\,(n-r)!}\int x[F]^{r-1}[1-F]^{n-r}dF \qquad \ldots(2)$$

In this study, we consider symmetric trimming that is $t_1 = t_2 = 1$ which means largest and smallest values will be trimmed. Then the TL-Moment is defined as

$$\lambda_r^{(1)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k\binom{r-1}{k}\int_0^1 x(F)F^{r-1}(1-F)^{n-1}dF$$

Then the first four TL-Moments are -

$$\lambda_1^{(1)} = 6\int_0^1 x(F)\,F(1-F)dF$$

$$\lambda_2^{(1)} = 6\int_0^1 x(F)\,F(1-F)\,(2F-1)dF$$

$$\lambda_3^{(1)} = \frac{20}{3}\int_0^1 x(F)\,F(1-F)\,(5F^2-5F+1)dF(x)$$

$$\lambda_4^{(1)} = \frac{15}{2}\int_0^1 x(F)\,F(1-F)\,(14F^3-21F^2+9F-1)dF$$

The TL-Moment ratios are defined by –

$$\tau_i^{(1)} = \lambda_i^{(1)} / \lambda_2^{(1)}$$

And, $\tau^{(1)} = \lambda_2^{(1)} / \lambda_1^{(1)}$, is the coefficient of variation (L-CV).

$\tau_3^{(1)} = \lambda_3^{(1)} / \lambda_2^{(1)}$, is the Skewness (L-Skewness).

$\tau_4^{(1)} = \lambda_4^{(1)} / \lambda_2^{(1)}$, is the Kurtosis (L-Kurtosis).

Estimation of TL-Moments is obtained by its unbiased estimator i.e., the sample TL-moments. The sample TL-moment is defined by –

$$l_r^{(t)} = r^{-1}\binom{n}{r+2t}^{-1}$$

$$\sum_{i=1}^{n}\left[\sum_{j=0}^{r-1}(-1)^j\binom{r-1}{j}\binom{i-1}{r+t-1-j}\binom{n-i}{i+j}\right]X_{(i)}$$

Correspondingly, we can get the values of $l_1^{(1)}, l_2^{(1)}, l_3^{(1)}$ & $l_4^{(1)}$ and the respective ratios by using the relation $t_i^{(1)} = l_i^{(1)} / l_2^{(1)}$

Using the TL-Moments we can find the parameter estimates, for that we need to solve the linear equations –

$$\lambda_i^{(1)} = l_i^{(1)}, i = 1,2, \ldots$$

Table 3 Parameter Estimates using TL-Moments

Distribution		Seasonal			
		Winter	Pre-Mon	Monsoon	Post-Mon
GLO	$\hat{\alpha}$	0.1195	0.7190	0.3209	1.3118
	$\hat{\xi}$	0.2027	0.4075	0.1239	0.7868
	$\hat{k}$	-1.5611	-1.6469	-1.7908	-1.233
GEV	$\hat{\alpha}$	0.0981	0.5881	0.2643	1.1376
	$\hat{\xi}$	0.1627	0.1719	0.0211	0.3040
	$\hat{k}$	-1.6040	-1.6847	-1.2163	-1.279
GPA	$\hat{\alpha}$	0.0859	0.5096	0.2273	1.0891
	$\hat{\xi}$	0.1046	-0.1792	-0.1395	-0.387
	$\hat{k}$	-1.6300	-1.7093	-1.8345	-1.288
PEIII	$\hat{\alpha}$	11.9312	137.9015	279.7020	23.429
	$\hat{\xi}$	2.9175	27.9589	41.5638	8.2449
	$\hat{k}$	8.6797	9.9483	13.4830	5.8064

From Table 3, it is noted that the parameter estimates for each of the selected distribution using the method of TL-Moment have been computed for each of the seasons.

3.5 Root Mean Square Error(RMSE)

The Root Mean Square Error (RMSE) is a measure of statistical significance that evaluates how well various probability distributions fit observed data in order to determine how well they fit. To be more precise, RMSE measures the variations between observed and estimated values, yielding a single number that encapsulates a model's prediction accuracy. The model fits the data better when the RMSE is less, meaning that the projected values are more in line with the actual observations. Furthermore, the root mean square error (RMSE) shows the standard deviation of the prediction mistakes, providing information on how variable these errors are[21].

The RMSE can be mathematically expressed using the following formula:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{x_i})^2}$$

Where, x_i indicates the observed and $\hat{x_i}$ indicates the estimated value. With the help of which the best method of parameter estimation is also obtained.

Furthermore, the optimal technique for parameter estimate may be found with the use of RMSE. Practitioners and researchers can select the approach that yields the best accurate predictions by contrasting the root mean square error (RMSE) values of many models or estimate methodologies. This guarantees the chosen model is strong and has good generalization capabilities to fresh data, improving the model's output dependability.

4. Results and Discussion

In meteorological and hydrological study, fitting probability distributions to precipitation data is a standard and crucial technique. Planning for agriculture, managing water resources, and assessing flood risk all depend on knowing and forecasting rainfall patterns. The best-fitting probability distribution for precipitation data might differ greatly depending on the season and the parameter estimate approach, as previous research [22], [23] have shown.

The India Meteorological Department's (IMD) division system is used in this study to split the year into four main seasonal categories:

- Winter Season (January-February): Characterized by cooler and drier conditions.
- Pre-Monsoon Season (March-May): Typically marked by increasing temperatures and occasional thunderstorms.
- Monsoon Season (June-September): Dominated by heavy and sustained rainfall.
- Post-Monsoon Season (October-December): A transitional period with receding monsoon rains and gradually cooling temperatures.

The RMSE values for the GEV, GPA, GLO, and PEIII distributions are computed using different parameter estimation methods. The distribution with the lowest RMSE value is considered the best fit for each season and each type of parameter estimation methods.

Table 4 RMSE values for each season for each of the distribution

Distribution	Winter			Pre-Monsoon		
	MLE	LM	TLM	MLE	LM	TLM
GEV	0.000016	0.053914	0.25026	4.0966	0.3262825	21.89358
GPA	0.00012	0.077001	0.25170	23.125	0.7454182	21.84313
GLO	0.0632	0.004482	0.24778	4.7501	1.155568	21.92835
PEIII	-	0.0023	0.02025	18.159	0.3143181	20.86337

Distribution	Monsoon			Post-Monsoon		
	MLE	LM	MLE	LM	MLE	LM
GEV	2.283012	0.3565001	2.283012	0.3565001	2.283012	0.3565001
GPA	61.95482	4.484213	61.95482	4.484213	61.95482	4.484213
GLO	36.12722	6.237043	36.12722	6.237043	36.12722	6.237043
PEIII	60.96398	33.4898	60.96398	33.4898	60.96398	33.4898

The analysis in Table 4 provides a detailed comparison of parameter estimation methods for various probability distributions across different seasons. The focus is on understanding which methods yield the most accurate estimates, as indicated by the Root Mean Square Error (RMSE) values.

For Winter and Post-Monsoon Season:

- Maximum Likelihood Estimation: The study shows that during the winter and post-monsoon seasons, MLE is not suitable for the PEIII distribution. The nature of the parameter space of the distribution during certain seasons or convergence problems might be the reason for this.
- L-Moments(LM) & TL-Moments(TLM): In the winter, both methods work effectively for estimating parameters for all distributions. Lower RMSE results indicate that the PEIII distribution, in particular, fits the data well using these techniques. This suggests that in these situations, LM and TLM are strong substitutes for MLE.

For Pre-Monsoon Season:

- L-Moments(LM): This season, the L-Moments approach is the most reliable method for parameter estimate since it frequently provides the lowest RMSE value for the PEIII distribution. This implies that the pre-monsoon rainfall variability is effectively represented by the L-Moments method.

For Monsoon Season:

- The L-Moments method usually provides the best method of estimation of parameters during the monsoon season. This is particularly relevant for the GEV distribution, which excels at simulating the extreme values typical of the monsoon rainy season. The low RMSE values provide more evidence of the L-Moments method's reliability throughout this season.

The L-Moments approach consistently performs better than alternative parameter estimate strategies in all seasons and distributions. Its lower Root mean square error (RMSE) values in different circumstances imply that it offers more precise and reliable parameter estimation method for seasonal rainfall analysis.

The results underscore the importance of choosing the right parameter estimation method for accurate precipitation modeling. The L-Moments method, in particular, proves to be superior across various distributions and seasons, making it a preferred choice for researchers and practitioners in hydrology and meteorology. By providing more precise estimates, it enhances the reliability of rainfall models, which is crucial for effective water resource management and planning.

5. Conclusion and Future Scope

In conclusion, this work has carefully used a variety of parameter estimation approaches to fit probability distributions to rainfall data in order to examine seasonal trends and offer a thorough comprehension of the data. The techniques used include TL-Moments, L-Moments, and Maximum Likelihood Estimation (MLE). When it comes to managing outliers and determining distribution parameters, each technique has unique benefits. Even though the MLE approach appears to be effective, there are instances in which it is unable to produce parameter estimates because the likelihood function does not converge. On the other hand, the robustness and dependability of the L-Moments and TL-Moments approaches are demonstrated by the way they reliably produce parameter estimations across each season.

Despite the existence of outliers, the L-Moments and TL-Moments approaches produce higher parameter estimates because they make use of Probability Weighted Moments (PWM) and order statistics. Especially in severe situations, this technique offers more accurate rainfall data analysis. For most seasons, the L-Moments approach is the most effective out of the three methods that were examined based on its goodness of fit, as determined by the Root Mean Square Error (RMSE).

In particular, the best-fit probability distribution for the Winter and Pre-Monsoon seasons is shown to be the Pearson Type III (PEIII) Distribution. The Generalized Extreme Value (GEV) Distribution is most suitable for the Monsoon season, whereas the Generalized Pareto (GPA) Distribution is most suitable for the Post-Monsoon season. These results emphasize the need of carefully choosing the distribution model and estimation technique to appropriately represent the distinctive features of seasonal rainfall data. Insights for hydrological modelling and climate analysis are gained from this work, which highlights the L-Moments method's resilience and effectiveness in fitting probability distributions to rainfall data.

5.1 Future Scope of the work:

Long-term climate forecasts may become more accurate if these probability distributions and estimate techniques are included with climate models. This might be especially helpful in forecasting severe weather and comprehending how rainfall patterns are affected by climate change.

Data Availability

Datasets that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of Interest

The authors declare that they do not have any conflict of interest.

Funding Source

There is no funding source.

Authors' Contributions

Ruhiteswar Choudhury carried out all calculations and wrote the main manuscript text.

Tanusree Deb Roy provided the main help in polishing this paper and offered constructive comments on the Conceptualization and Methodology used in this paper.

Acknowledgements

The authors are thankful to India Meteorological Department (IMD) New Delhi, India for providing the monthly rainfall data from January 1985 to December 2022 for the Guwahati District of Assam, India.

References

[1] A.N. Patowary, "A STUDY ON LONG TERM RAINFALL PATTERN OF DHUBRI AND GUWAHATI IN ASSAM, INDIA: A TIME SERIES APPROACH," *Int. J. Adv. Res. Comput. Sci.*, Aug., Vol.**8**, No.**7**, pp.**708–713**, **2017.**

[2] "Guwahati Geography, Guwahati Climate, Best Time to Visit Guwahati." Accessed: Apr. **29**, **2024**.

[3] S.K. Jain and V. kumar, "Trend analysis of rainfall and temperature data for India," **2021.**

[4] B.J. Gharphalia, R. L. Deka, A. N. Islam, P. Dutta, and K. Medhi, "Variability and Trends of Rainfall Events in the Brahmaputra Valley of Assam," *Int. J. Curr. Microbiol. Appl. Sci.*, Nov., Vol.**7**, No.**11**, pp.**1902–1912**, **2018**.

[5] S. Deka, S. Kakaty, and M. Borah, "USE OF PROBABILITY DISTRIBUTIONS FOR THE ANALYSIS OF DAILY RAINFALL DATA OF NORTH EAST INDIA," *MAUSAM*, Oct, Vol.**59**, No.**4**, pp.**518–527**, **2008.**

[6] H. Cramer, *Random Variables and Probability Distributions*. Cambridge University Press, **2004.**

[7] C.S. and A.A. Rather, "Transmuted Generalized Uniform Distribution," *Int. J. Sci. Res. Math. Stat. Sci.*, Oct., Vol.**5**, No.**5**, pp.**25–32**, **2018.**

[8] A.A.R. and C. Subramanian, "Characterization and Estimation of Length Biased Weighted Generalized Uniform Distribution," *Int. J. Sci. Res. Math. Stat. Sci.*, Oct., Vol.**5**, No.**5**, pp.**72–76, 2018.**

[9] Q. Shao, "MAXIMUM LIKELIHOOD ESTIMATION FOR GENERALISED LOGISTIC DISTRIBUTIONS," *Commun. Stat. - Theory Methods*, Jan., Vol.**31**, No.**10**, pp.**1687–1700, 2002.**

[10] J. Simpson, "USE OF THE GAMMA DISTRIBUTION IN SINGLE-CLOUD RAINFALL ANALYSIS," *Mon. Weather Rev.*, Apr., Vol.**100**, No.**4**, pp.**309–312**, **1972.**

[11] J.R.M. Hosking, "L-Moments: Analysis and Estimation of Distributions Using Linear Combinations of Order Statistics," *J. R. Stat. Soc. Ser. B Methodol.*, Sep., Vol.**52**, No.1, pp**.105–124, 1990.**

[12] T.S. Gubareva and B. I. Gartsman, "Estimating distribution parameters of extreme hydrometeorological characteristics by L-moments method," *Water Resour.*, Jul., Vol.**37**, No.**4**, pp.**437–445**, **2010.**

[13] S. Deka, "Distributions of Annual Maximum Rainfall Series of North-East India," *Eur. Water*, Vol.**27**, No.**28**, pp.**3–14, 2009.**

[14] R. Bora, A. Bhuyn, and B. K. Dutta, "Regional Analysis of Seasonal maximum monthly rainfall Using L-moments: A case study for Brahmaputra Valley region of Assam, India," In Review, preprint, Jul. **2021.**

[15] E.A.H. Elamir and A. H. Seheult, "Trimmed L-moments," *Comput. Stat. Data Anal.*, Jul., Vol.**43**, No.**3**, pp.**299–314, 2003.**

[16] A.B. Shabri, Z. M. Daud, and N. M. Ariff, "Regional analysis of annual maximum rainfall using TL-moments method," *Theor. Appl. Climatol.*, Jun., Vol.**104**, No.**3–4**, pp. **561–570, 2011.**

[17] I. Ahmad, A. Abbas, M. Aslam, and I. Ahmad, "Total annual rainfall frequency analysis in Pakistan using methods of L-moments and TL-moments," *Sci IntLahore*, Vol.**27**, pp.**2331, 2015.**

[18] N.A.T. Abu El-Magd, "TL-moments of the exponentiated generalized extreme value distribution," *J. Adv. Res.*, Oct., Vol.**1**, No.**4**, pp.**351–359, 2010.**

[19] W.H. Asquith, "L-moments and TL-moments of the generalized lambda distribution," *Comput. Stat. Data Anal.*, May, Vol.**51**, No.**9**, pp.**4484–4496, 2007.**

[20] D. Bílková, "Robust Parameter Estimations Using L-Moments, TL-Moments and the Order Statistics," *Am. J. Appl. Math.*, Vol.**2**, No.**2**, pp.**36, 2014.**

[21] T. Chai and R. R. Draxler, "Root mean square error (RMSE) or mean absolute error (MAE)," *Geosci. Model Dev.*, Vol.**7**, No.**3**, pp.**1247–1250, 2014.**

[22] M.A. Sharma and J. B. Singh, "Use of Probability Distribution in Rainfall Analysis," *N. Y. Sci. J.*, Vol.**3**, No **9**, pp.**40–49, 2010.**

[23] V. Golroudbary, Y. Zeng, C. M. Mannaerts, and Z. (Bob) Su, "Attributing seasonal variation of daily extreme precipitation events across The Netherlands," *Weather Clim. Extrem.*, Dec., Vol.**14**, pp.**56–66**, **2016.**

AUTHORS PROFILE

Mr. Ruhiteswar Choudhury completed his B.Sc. in Statistics from Srikishan Sarda College Hailakandi in the year 2021, following which he completed his M.Sc. in Statistics from the Department of Statistics, Assam University in 2023 and is currently working as a Research Scholar at the Department of Statistics, Assam University Silchar.

Dr. Tanusree Deb Roy is working in the field of Time Series Analysis and Distribution Theory. She has contributed to the development of statistical models in the field of distribution theory. She has completed her Masters and Ph.D. in Statistics from Gauhati University. Her recent research has been covered in many reputed National and International Journals.

