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Combined Effect of Rotation and Hall Currents on Magneto-Thermal Convection of Ferromagnetic Fluids Saturating Porous Medium

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Abstract-In this study, the effect of Hall currents and rotation on the onset of convection in a porous medium layer saturated by an electrically conducting ferromagnetic fluid heated from below using linear stability analysis is investigated. Darcy law for the ferromagnetic fluid is used to model the momentum equations for a porous medium. The employed model incorporates the effects of polarization force and body couple. The coupled partial differential equations governing the physical problem are reduced to a set of ordinary differential equations using normal mode technique. These equations are solved analytically for stress- free boundaries and numerical results are computed by obtaining approximate solutions using Galerkin method using the software Mathematica for the case of stationary convection. It is found that the magnetic field and magnetization have a stabilizing effect as such their effect is to postpone the onset of thermal instability; whereas Hall currents are found to hasten the same. The medium permeability and rotation hastens the onset of convection under certain conditions.

Keywords- Hall currents, rotation parameter, ferromagnetic fluid, medium permeability, medium porosity.

I. INTRODUCTION

Ferrohydrodynamics deals with the mechanics of fluid motions influenced by strongforces of magnetic polarization. Ferromagnetic fluids are electrically non-conducting colloidal suspensions of solid ferromagnetic particles in a non-electrically conducting carrier fluid like water, kerosene, hydrocarbon, *etc.* These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomena. The polarization force and the body couple are the two main features that distinguish ferromagnetic fluid from ordinary fluid. Ferromagnetic fluids are not found in nature but are artificially synthesized. Soon after the method of formation of ferromagnetic fluids in the early or mid 1960s, the importance of ferrohydrodynamics was realized. During the last half century, research on magnetic liquids has been very productive in many fields. Strong efforts have been undertaken to synthesize stable suspensions of magnetic particles with different performances in magnetism, fluid mechanics or physical chemistry. An authoritative introduction to this fascinating subject has been discussed in detail in the celebrated monograph by Rosensweig [1]. This monograph reviews several applications of heat transfer through ferromagnetic fluids. One such phenomenon is enhanced convective cooling having a temperature dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of magnetic field, temperature and density of the fluid. In this analysis, it is assumed that the magnetization is aligned with the magnetic field. Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson [2]. He explained the concept of thermomechanical interaction in ferromagnetic fluids. Thermo-convective stability of ferromagnetic fluids without considering buoyancy effects has been investigated by Lalas and Carmi [3], whereas Shliomis [4] analyzed the linearized relation for magnetized perturbed quantities at the limit of instability.

The medium has been considered to be non-porous in all the above studies. There has been a lot of interest, in recent years, in the study of the breakdown of the stability of a fluid layer subjected to a vertical temperature gradient in a porous medium and the possibility of the convective flow. The stability of flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood [5], Wooding [6], Sunil [7] and many others. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law. A macroscopic equation describing incompressible flow of a fluid of viscosity μ, through a macroscopically homogeneous and isotropic porous medium of permeability k_1, is well-known Darcy's equation, in which the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\left(\frac{\mu}{k_1}\right)\vec{q}$ where $\vec{q}$ is the filter velocity of the fluid. The thermoconvective instability in a ferromagnetic fluid saturating a porous medium of very large permeability subjected to a vertical magnetic field has been studied using the Brinkman model by Vaidyanathan [8], and indicated that only stationary convection can exist. In the presence of strong electric field, the electric conductivity is affected by the magnetic field. Consequently, the conductivity parallel to the electric field is reduced. Hence, the current is reduced in the direction normal to both electric and magnetic field. This phenomenon is known as Hall effect. Sharma and Gupta [9] have studied the effect of rotation on the thermal convection of micropolar fluid in the presence of suspended particles.

In the present paper, the effect of Hall currents and the rotation on thermal stability of ferromagnetic fluid heated from below saturating a porous medium in the presence of horizontal magnetic field has been investigated numerically in the porous medium.

II. RELATED WORK

An infinite, incompressible, electrically non-conducting thin ferromagnetic fluid, bounded by the planes $z=0$ and $z=d$ saturating a porous medium is considered to include the effect of Hall currents. This layer is heated from below so that uniform temperature gradient $\beta = \left(\left|\frac{dT}{dz}\right|\right)$ is maintained. A uniform horizontal magnetic field $\vec{H} = H(0,0,0)$, vertical rotation $\vec{\Omega}(0,0,\Omega)$ and gravity force $\vec{g} = g(0,0,-g)$ pervade the system. This fluid layer is flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

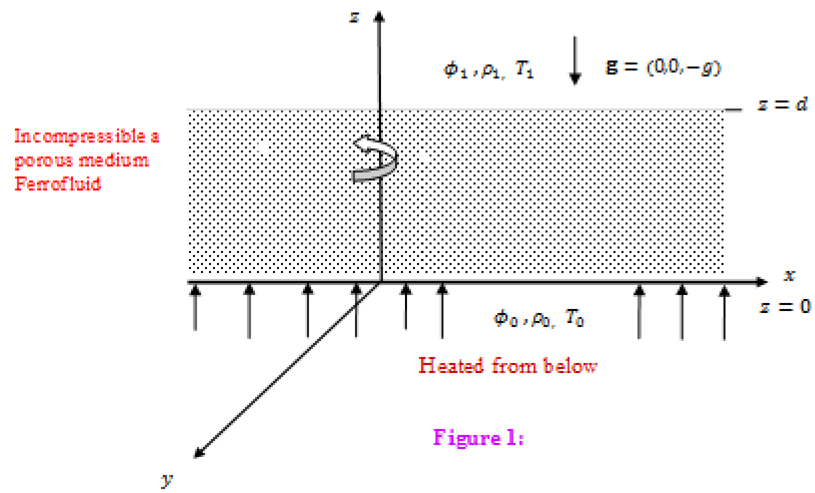

Figure 1:

Let $p, \rho, T, \alpha, \vec{g}, \eta, \mu_e, e, t, \vec{B}$ and $\vec{q}(u,v,w)$ denote the fluid pressure, density, temperature, thermal coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron, time, magnetic induction and fluid (filter) velocity, respectively. The equations expressing the conservation of momentum, mass, temperature, and equation of state of ferromagnetic fluids through saturating porous medium are

$$\frac{1}{\varepsilon}\left[\frac{\partial q}{\partial t}+\frac{1}{\varepsilon}(\vec{q}\cdot\nabla)\vec{q}\right] = -\frac{1}{\rho_0}\nabla\left(p-\frac{\rho_0}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)-\vec{g}\rho+\vec{M}\nabla\cdot\left(H\vec{B}\right)-\frac{\vec{\upsilon}}{k_1}\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times 2\rho_0\left(\vec{v}\times\vec{\Omega}\right) \tag{1}$$

$$\nabla\cdot\vec{q}=0 \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T = \kappa\nabla^2 T \tag{3}$$

$$\rho=\rho_0\left[1-\alpha(T-T_0)\right] \tag{4}$$

Where $E=\varepsilon+(1-\varepsilon)\left(\rho_s c_s \big/ \rho_0 c_i\right)$. The additional term $\nabla\cdot\left(H\vec{B}\right)$ pertinent to a ferromagnetic fluid is the magnetic stress. In the equation (1), the term $2\rho_0\left(\vec{v}\times\vec{\Omega}\right)$ represent the Coriolis acceleration and the term $\frac{1}{2}\left(grad\left|\vec{\Omega}\times\vec{r}\right|^2\right)$ represents the centrifugal force (which is of very small magnitude).The Gauss divergence equation and the magnetic induction equation in the presence of Hall currents yield

$$\nabla\cdot\vec{H}=0 \tag{5}$$

$$\varepsilon\frac{\partial\vec{H}}{\partial t}=\nabla\times\left(\vec{q}\times\vec{H}\right)+\varepsilon\eta\nabla^2 H-\frac{\varepsilon}{4\pi N'e}\nabla\times\left(\nabla\times\vec{H}\right) \tag{6}$$

The density equation of state is

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right] \tag{7}$$

Here α is the thermal expansion coefficient, T_0 is the temperature at reference point. The magnetic field, magnetization, and magnetic induction are related by

$$\vec{B}=\mu_0\left(\vec{H}+\vec{M}\right) \tag{8}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field and temperature so that

$$\vec{M}=\frac{\vec{H}}{H}M(H,T) \tag{9}$$

The equation of state specifying $\vec{M}$ by two thermodynamic variables only $\vec{H}$ *and* T is necessary to complete the system. In the present study, we consider magnetization to be independent of the magnetic field intensity so that $\vec{M}=M(T)$ only. As a first approximation, it is assumed that

$$\vec{M}=M_0\left[1-\gamma\left(T-T_0\right)\right] \tag{10}$$

The basic state is given by

$$\vec{q}=(0,0,0),p=p(z),T=T_0-\beta z,\rho=\rho_0(1+\alpha\beta z)=\rho(z),\vec{M}=M(z) \tag{11}$$

The stability of the basic state is analyzed by superimposing infinitesimal perturbations to the physical quantities describing the system and let $\delta\rho,\delta p,\delta M,\theta,\vec{h}\left(h_x,h_y h_z\right)$ and $\vec{q}(u,v,w)$ denote the perturbations in density, pressure p, magnetization $\vec{M}$, temperature T, magnetic field $\vec{H}=H(0,0,0)$ and filter velocity $\vec{q}$ (zero initially), respectively. The change in magnetization δM and density $\delta\rho$ caused by the perturbations θ and γ in temperature and concentration, are given by

$$\delta M=-\gamma M_0\theta \tag{12}$$

$$\delta\rho=-\rho_0\alpha\theta \tag{13}$$

Then the linearized perturbation equations for ferromagnetic fluid under Boussinesq approximation are

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{k_1}v\right)\nabla^2 w=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(g\alpha-\frac{\gamma M_0\nabla H}{\rho_0}\right)\theta+\frac{\mu_e H}{4\pi\rho_0}\left(\nabla^2\frac{\partial h_z}{\partial x}\right)+2\left(\vec{v}\times\vec{\Omega}\right) \tag{14}$$

$$\left(\frac{1}{\varepsilon}\frac{\partial}{\partial t}+\frac{1}{k_1}v\right)\varsigma=\frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial x} \tag{15}$$

where $\zeta=\left(\frac{\partial v}{\partial x}\right)-\left(\frac{\partial u}{\partial y}\right)$ is the z-component of vortices , where $\xi=\left(\frac{\partial h_y}{\partial x}\right)-\left(\frac{\partial h_x}{\partial y}\right)$ is the z-component of current density.

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)\xi=\frac{H}{\varepsilon}\frac{\partial\zeta}{\partial x}+\frac{H}{4\pi N'e}\left(\nabla^2 h_z\right) \tag{16}$$

$$\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z=\frac{H}{\varepsilon}\frac{\partial w}{\partial x}-\frac{H}{4\pi N'e}\frac{\partial\xi}{\partial x} \tag{17}$$

$$E\left(\frac{\partial}{\partial t}-\kappa\nabla^2\right)\theta=\beta w \tag{18}$$

III. METHODOLOGY

Analyzing the disturbance into normal modes, we assume that perturbation quantities are of the form:

$$[w,\theta,\xi,\zeta,h_z,\gamma]=[W(z),\Theta(z),X(z),Z(z),K(z),\Gamma(z)]\exp\left(ik_x+ik_y+nt\right), \tag{19}$$

where k_x and k_y are wave numbers along x-and y-directions, respectively, $k=\left(k_x{}^2+k_y{}^2\right)^{-1/2}$ is the resultant wave number of the disturbance and n is the growth rate (in general, a complex constant). For functions with this dependence on x, y and t, $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=-k^2\right)$ and $\nabla^2=\frac{\partial^2}{\partial z^2}-k^2$ .Using equation (19), equations (14) - (18) in non –dimensional form become:

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{p_1}\right)\left(D^2-a^2\right)W=-\frac{\alpha a^2 d^2}{v}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)\Theta+\frac{ik_x\mu_e H d^2}{4\pi\rho_0 v}\left(D^2-a^2\right)K \tag{20}$$

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{p_1}\right)Z=\frac{ik_x\mu_e H d^2}{4\pi\rho_0 v}X \tag{21}$$

$$\left(D^2-a^2-p_2\sigma\right)K=-\frac{ik_x H d^2}{\varepsilon\eta}W+\frac{ik_x H d^2}{4\pi N'e\eta}X \tag{22}$$

$$\left(D^2-a^2-p_2\sigma\right)X=-\frac{ik_x H d^2}{\varepsilon\eta}Z-\frac{ik_x H}{4\pi N'e\eta}(D^2-a^2)X \tag{23}$$

$$\left(D^2-a^2-p_2\sigma\right)\Theta=-\frac{\beta d^2}{\kappa}W\ . \tag{24}$$

Where $a=kd,\sigma=\frac{nd^2}{v},p_1=\frac{v}{\kappa}$ is the Prandtl number, $p_2=\frac{v}{\eta}$ is the magnetic Prandtl number, $p_1=\frac{k_1}{d^2}$ is the dimensionless medium permeability.

Galerkin procedure

A single term Galerkin technique is applied to solve system of equations (20)-(24).The trial solutions satisfying the dimensionless boundary conditions,

$$W=D^2W=0,X=DX=0,DZ=0,\theta=0,K=0\ \text{at}\ z=0\ \text{and}\ z=1. \tag{25}$$

$$W=t-2t^3+t^4,X=t^2-2t^3+t^4,Z=-2t^3+t^4,\Theta=t^2-t^3,K=t-t^2\ . \tag{26}$$

The trial solutions given in equation (26) are the minimal polynomials satisfying the boundary conditions given by equation (25).Now performing the standard Galerkin procedure (Finlayson [10]) for system of differential equations (20)-(24). Substituting the trial solution (26) in differential equations (20)-(24) and calculating the residual by using the boundary conditions (25), we have

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right]\left[-\int_0^1\left(1+36t^4-12t^2+16t^4+8t^3-48t^5\right)dt-a^2\int_0^1\left(t-2t^3+t^4\right)^2dt\right]=\frac{-\alpha a^2d^2}{\nu}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)$$

$$\int_0^1\left(t^3-2t^5+3t^6-t^4-t^7\right)dt+\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(\int_0^1\left(2-6t-24t^2+60t^3-30t^4\right)dt\right)-a^2\left(\int_0^1\left(t^2-t^3-2t^4+3t^5-t^6\right)dt\right)$$

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right]\left[-\frac{17}{35}-a^2\frac{68}{315}\right]=\frac{-\alpha a^2d^2}{\nu}\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right)\frac{17}{840}+\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(-a^2\frac{17}{420}\right) \tag{27}$$

$$\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right]\left(\int_0^1\left(-4t^7+t^8+4t^6\right)dt\right)=\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(\int_0^1\left(-2t^5+5t^6-4t^7+t^8\right)dt\right) \tag{28}$$

$$\left(\int_0^1\left(2-6t-24t^2+60t^3-30t^4\right)dt\right)-\left(a^2+p_2\sigma\right)\left(\int_0^1\left(t^2-t^3-2t^4+3t^5-t^6\right)dt\right)=\frac{ik_xHd^2}{\varepsilon n}\left(\int_0^1\left(t-2t^3+t^4\right)^2dt\right)\frac{ik_xHd^2}{4\pi N'e\eta}$$

$$\left(\int_0^1\left(t^3-t^5+5t^6-4t^7-2t^4+t^8\right)dt\right)$$

$$\Rightarrow\left[-\left(\frac{17}{420}\right)\left(a^2+p_2\sigma\right)\right]=\left(\frac{-68}{315}\right)\left(\frac{ik_xHd^2}{\varepsilon\eta}\right)+\left(\frac{ik_xHd^2}{4\pi N'e\eta}\right)\left(\frac{11}{1260}\right) \tag{29}$$

$$\left(\int_0^1\left(-40t^3+150t^4-168t^5-56t^6\right)dt\right)-\left(a^2+p_2\sigma\right)\left(\int_0^1\left(-2t^5+5t^6-4t^7+t^8\right)dt\right)=-\frac{ik_xHd^2}{\varepsilon n}\left(\int_0^1\left(-4t^7+t^8+4t^6\right)dt\right)-\frac{ik_xHd^2}{4\pi N'e\eta}$$

$$\left(\int_0^1\left(60t^3-24t^2-30t^4\right)\right)-a^2\left(\int_0^1\left(-2t^4+3t^5-t^6\right)dt\right)$$

$$\Rightarrow\left[\left(\frac{41}{126}\right)\left(a^2+p_2\sigma\right)\right]=\left(\frac{-23}{126}\right)\left(\frac{ik_xHd^2}{\varepsilon\eta}\right)+\left(\frac{ik_xHd^2}{4\pi N'e\eta}\right)\left(1+a^2\frac{3}{70}\right) \tag{30}$$

$$\left(\int_0^1\left(6t-40t^3+90t^4-12t^2-42t^5\right)dt\right)-\left(a^2+p_2\sigma\right)\left(\int_0^1\left(t^3-2t^5+3t^6-t^4-t^7\right)dt\right)=-\frac{\beta d^2}{\kappa}\left(\int_0^1\left(t^2+4t^6-4t^4+t^8+2t^5-4t^7\right)dt\right)$$

$$\Rightarrow\left(-\frac{17}{840}\left(a^2+EP_1\sigma\right)\right)=-\frac{\beta d^2}{\kappa}\frac{68}{315} \tag{31}$$

The Stationary Convection

When instability sets in stationary convection, the marginal state will be characterized by $\sigma=0$, Equations (27)-(31) reduced and constitute a system of linear algebraic equations. Following the procedure adopted by Nield and Kuznetsov [11] we have the following equation for the monotonic instability boundary,

$$\begin{bmatrix} \left(\frac{-17}{35}-a^2\frac{68}{315}\right)\frac{1}{p_1} & \frac{17}{840}\left(\frac{\alpha a^2d^2}{\nu}\right)\left(g-\frac{\gamma M_0\nabla H}{\rho_0\alpha}\right) & \left(a^2\frac{17}{420}\right)\left(\frac{ik_x\mu_eHd^2}{4\pi\rho_0\nu}\right) & 0 & 0 \\ \frac{\beta d^2}{\kappa}\frac{68}{315} & -\frac{17}{840}a^2 & 0 & 0 & 0 \\ \left(\frac{68}{315}\right)\left(\frac{ik_xHd^2}{\varepsilon\eta}\right) & 0 & -\left(\frac{17}{420}a^2\right) & 0 & \left(\frac{ik_xHd^2}{4\pi N'e\eta}\right)\left(-\frac{11}{1260}\right) \\ 0 & 0 & \left(\frac{\mathrm{ik}_xH}{4\pi N'e\eta}\right)\left(1+a^2\frac{3}{70}\right) & \frac{23}{126}\frac{ik_xHd^2}{\varepsilon\eta} & \frac{41}{126}a^2 \\ 0 & 0 & 0 & \left(-\frac{41}{126}\right)\left(\frac{ik_x\mu_eHd^2}{4\pi\rho_0\nu}\right) & 0 \end{bmatrix}\begin{bmatrix} W \\ \Theta \\ X \\ Z \\ K \end{bmatrix}=0 \tag{32}$$

$$R_f = \frac{\frac{4632959}{441082908000}\left(\frac{a^8\cos^4\theta Q}{\varepsilon}\right)+\frac{4632959}{5292000}\left(\frac{a^8\cos^4\theta Q}{\varepsilon p_1}\right)+\frac{4632959}{160030080}\left(\frac{a^6\cos^2\theta Q}{\varepsilon p_1}\right)-\frac{4632959}{10080}\left(\frac{a^6}{p_1^{\,2}}\right)-\frac{4632959}{280}\left(\frac{a^8}{p_1^{\,2}}\right)-\frac{73117}{240}\left(\frac{a^4\cos^2\theta M}{p_1^2}\right)-\frac{61491397}{5600}\left(\frac{a^6\cos^2\theta M}{p_1^2}\right)-\frac{658053}{1400}\left(\frac{a^8\cos^2\theta M}{p_1^2}\right)}{\frac{4632959}{441082908000}\left(\frac{a^6\cos^2\theta Q}{\varepsilon}\right)-\frac{4632959}{27783000}\left(\frac{a^6}{p_1}\right)-\frac{73117}{661500}\left(\frac{a^4\cos^2\theta M}{p_1}\right)-\frac{73117}{15435000}\left(\frac{a^6\cos^2\theta M}{p_1}\right)} \tag{33}$$

$$\frac{dR_f}{dQ} = \frac{\left[\frac{4632959}{441082908000}\left(\frac{a^8\cos^4\theta}{\varepsilon}\right)+\frac{4632959}{5292000}\left(\frac{a^8\cos^4\theta}{\varepsilon p_1}\right)+\frac{4632959}{160030080}\left(\frac{a^6\cos^2\theta}{\varepsilon p_1}\right)\right]A_1-\frac{4632959}{10080}\left(\frac{a^6}{p_1^{\,2}}\right)-\frac{4632959}{27783000}\left(\frac{a^6}{p_1}\right)}{\mathrm{A}_1^2} \tag{34}$$

$$\frac{dR_f}{dM} = \frac{\left[-\frac{73117}{240}\left(\frac{a^4\cos^2\theta}{p_1^2}\right)-\frac{61491397}{5600}\left(\frac{a^6\cos^2\theta}{p_1^2}\right)-\frac{658053}{1400}\left(\frac{a^8\cos^2\theta}{p_1^2}\right)\right]A_1+\left[\frac{73117}{661500}\left(\frac{a^4\cos^2\theta}{p_1}\right)+\frac{73117}{15435000}\left(\frac{a^6\cos^2\theta}{p_1}\right)\right]A_2}{\mathrm{A}_1^2}$$

$$\frac{dR_f}{dp_1} = \frac{A_1A_3-A_2A_4}{\mathrm{A}_1^2} \tag{35}$$

where $\mathrm{A}_1 = \frac{4632959}{441082908000}\left(\frac{a^6\cos^2\theta Q}{\varepsilon}\right)-\frac{4632959}{27783000}\left(\frac{a^6}{p_1}\right)-\frac{73117}{661500}\left(\frac{a^4\cos^2\theta M}{p_1}\right)-\frac{73117}{15435000}\left(\frac{a^6\cos^2\theta M}{p_1}\right)$

$$\mathrm{A}_2 = \frac{4632959}{441082908000}\left(\frac{a^8\cos^4\theta Q}{\varepsilon}\right)+\frac{4632959}{5292000}\left(\frac{a^8\cos^2\theta Q}{\varepsilon p_1}\right)+\frac{4632959}{160030080}\left(\frac{a^6\cos^2\theta Q}{\varepsilon p_1}\right)-\frac{4632959}{10080}\left(\frac{a^6}{p_1^{\,2}}\right)-\frac{4632959}{280}\left(\frac{a^8}{p_1^{\,2}}\right)-\frac{73117}{240}\left(\frac{a^4\cos^2\theta M}{p_1^{\,2}}\right)-\frac{61491397}{5600}\left(\frac{a^6\cos^2\theta M}{p_1^{\,2}}\right)-\frac{658053}{1400}\left(\frac{a^8\cos^2\theta M}{p_1^{\,2}}\right)$$

$$\mathrm{A}_3 = -\frac{4632959}{5292000}\left(\frac{a^8\cos^2\theta Q}{\varepsilon p_1^2}\right)-\frac{4632959}{160030080}\left(\frac{a^6\cos^2\theta Q}{\varepsilon p_1^2}\right)+\frac{4632959}{10080}\left(\frac{2a^6}{p_1^{\,3}}\right)-\frac{4632959}{280}\left(\frac{2a^8}{p_1^{\,3}}\right)+\frac{73117}{240}\left(\frac{2a^4\cos^2\theta M}{p_1}\right)+\frac{73117}{240}\left(\frac{2a^4\cos^2\theta M}{p_1^{\,3}}\right)+\frac{61491397}{5600}\left(\frac{2a^6\cos^2\theta M}{p_1^{\,3}}\right)+\frac{658053}{1400}\left(\frac{2a^8\cos^2\theta M}{p_1^{\,3}}\right)$$

$$\mathrm{A}_4 = \frac{4632959}{27783000}\left(\frac{a^6}{p_1^2}\right)+\frac{73117}{661500}\left(\frac{a^4\cos^2\theta M}{p_1^{\,2}}\right)+\frac{73117}{15435000}\left(\frac{a^6\cos^2\theta M}{p_1^{\,2}}\right)$$

IV. RESULTS AND DISCUSSIONS

Graphs have been plotted between the modified Rayleigh number R and magnetic field parameter Q, Hall current parameter M and medium permeability parameter P for various values of wave number $a = 2,4,8.$ In figure (2), Rayleigh number R is plotted against medium permeability P and it is found that the medium permeability always hastens the onset of convection for all wave numbers as the Rayleigh number decreases with an increase in medium permeability parameter. In figure (3), Rayleigh number R is plotted against magnetic field Q depicting thereby the stabilizing effect of the magnetic field. In figure (4), Rayleigh number R is plotted against the wave number a depicting that medium permeability hastens the onset of convection for small wave numbers near $x = 1$ as the Rayleigh number decreases with an increase in medium permeability parameter and postpones the onset of convection for higher wave numbers as the Rayleigh number increases with an increase in medium permeability parameter, whereas in figure (5) Rayleigh number R is plotted against the Hall current showing thereby that Hall currents have a destabilizing effect on the thermal convection.

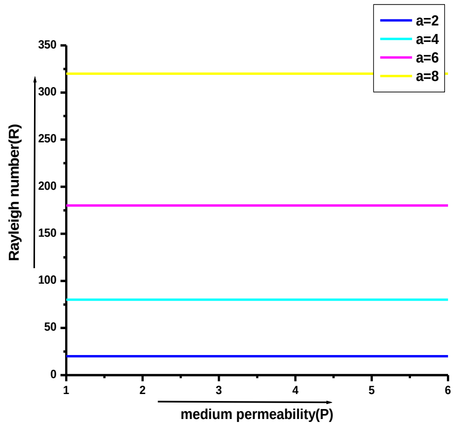

Figure 2.Variation of *R1 and P* for fixed
$P = 50, \theta = 45°, M = 10, Q1\ (= 10, 20, \ldots, 60)$

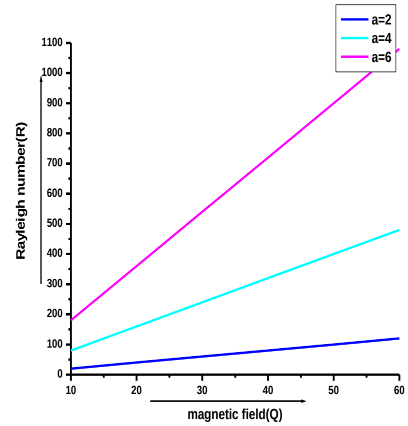

Figure 3.Variation of *R1 with Q1* for fixed
$M = 0.1, Q1 = 10, \theta = 45°, \text{and } P\ (= 1, 2, \ldots, 6)$

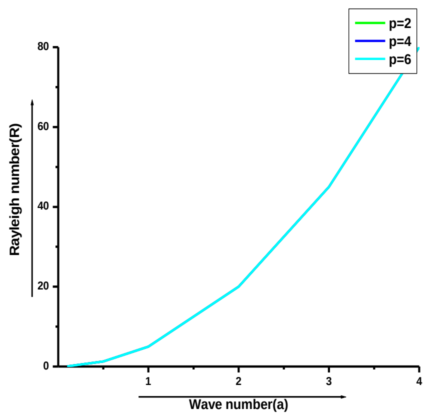

Figure 4.Variation of *R1 and a* for a fixed
$M = 100, Q1 = 10, \theta = 45°, \text{for } a\ (= 0.1, 0.5, 1, \ldots, 4)$

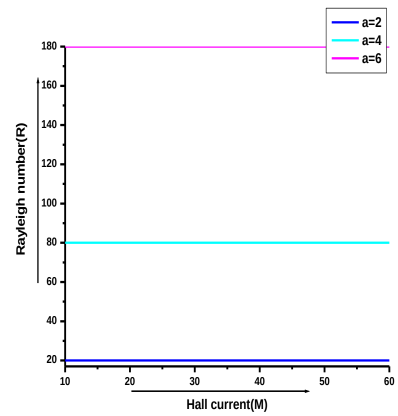

Figure 5.Variation of *R1 with Q1* for fixed
$P = 50, \theta = 45°, M = 10, Q1\ (= 10, 20, \ldots, 60)$

V. CONCLUSION AND FUTURE SCOPE

The effect of various parameters such as magnetic field, Hall currents, magnetization, medium permeability and rotation has been investigated analytically as well as numerically. The main results from the analysis of the paper are as follows.

● In order to investigate the effects of magnetic field, Hall currents, magnetization and medium permeability, we examine the behavior of ${dR}/{dQ}$, ${dR}/{dM}$ and ${dR}/{dp}$ analytically.

● It is found that Hall currents have a destabilizing effect whereas magnetic field and magnetization have a stabilizing effect on the system. Figures 3 and figure 5 support the analytic results graphically. The reasons for stabilizing effect of magnetic field and destabilizing effect of Hall currents are accounted by Chandrasekhar [12].

● For M , the medium permeability always hastens the onset of convection for all wave numbers as the Rayleigh number decreases with an increase in medium permeability parameter whereas for $M > 1$ the medium permeability hastens the onset of convection for small wave numbers as the Rayleigh number decreases with an increase in medium permeability parameter and postpones the onset of convection for higher wave numbers as the Rayleigh number increases with an increase in medium permeability parameter. The effect of various parameters such as magnetic field, Hall currents, magnetization, medium permeability and rotation could be extended for the salinity gradient i.e solute parameter even in the porous medium.

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