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Mathematical Modeling Methods for Solving Environmental Problems for Sustainable Development in Ogoniland

Kanee Goodfaith Leyira

Department of Mathematical Science, Abubakar Tafawa Balewa University, PMB 0248 Bauchi, Nigeria
&
Department of Theology, Love of Christ (TLC) Bible College, PO Box 3060, Chino Valley, Arizona AZ 8 6323, USA

Author's Mail Id: kanegoodfaith@gmail.com, Tel. +234 8033940344, 8026208559

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***Abstract* -** The research aim at providing mathematical modeling methods for solving environmental problems for sustainable development in Ogoniland. The step of analysis involves application of standard mathematical techniques and procedures to solve‖ the model to obtain the desired results and the analysis is done according to the rules of mathematics and the system.

***Keywords*-**Degradation, Differential Equation, Environment, Environmental Problem, Mathematical Modeling, Simulation, Sustainable Development

I. INTRODUCTION

1.0 Background

Ogoniland is one of the prominent and indigenous areas in the Niger Delta region, south-south Nigeria, and has a calamitous history of pollution from oil spills, gas flares, and oil well fires of which, have some generational consequences on the environment and the lives of Ogoni people. Ogoniland has a population of close to 832,000 people and Covering around 1,000 km^2 lies on Latitude 4.8550N and Longitude 6.988E of a coastal plain terraces across the four Local Government Areas of Khana, Gokana, Tai and Eleme in Rivers State of Nigeria [8]. From historical antecedent, Ogoniland and its people are synonymous with the state of the struggle against environmental degradation and injustice of which the region have been exposed and subjected to from long years of oil exploitation, exploration, and production activities by Multinational Oil Companies and the Government of Nigeria in the area since late 1950s. It is against this background that, the researcher observed that using mathematics to solve and find some solutions to these environmental problems pays. It necessitates the uses of mathematical formulations and analysis to simulate the dynamics of hazardous Environmental Problems for sustainable development [14]. By Mathematical modeling, comprises the change of the system under study from its natural environment to mathematical environment in terms of intellectual symbols and equations [2]. The researcher employed some differential equations to model these problems through mathematical formulation and analysis for supporting natural resources allocation, flooding prevention and control, environmental pollution control, ecological protection, and sustainable development improving, where a number of innovative perspectives and findings are advanced [1]. It is known that a single differential equation can serve as a mathematical model for many different phenomena especially in solving environmental problems ranging from groundwater to land use [2,20]. The research aim at providing mathematical modeling methods for solving environmental problems for sustainable development, having visited the oil spills, gas flare and oil well fires sites in Ogoniland.

1.1 Conceptual Clarification

Differential Equation (DE): An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a differential equation [

Environment: Environment includes all the factors such as physical, chemical, biological affecting the ecosystem of the unique geographical and biological features like soil, climate, flora and fauna which create a variety of different types of ecosystem such as forest, fields, and Ocean. The abnormal and unusual environmental and ecological disturbances can upset the stability and even destroy the ecosystem [6].

Environmental pollution can be described as any undesirable changes in the physical, chemical or biological characteristics of any component of the environment which can cause harmful effects on various forms of life and properties [4].

Environmental Sustainability

Environmental sustainability is referring to the deliberate efforts by states in international system in relation to

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policy formulation and implementation aimed at reducing the negative impact of human activities towards the eco-system e.g. water, land, air and the aquatic animals living inside them and human being inclusive.

Modeling: Modeling can be defined as the process of request of essential information or define the routine of or involvement to simulate a real system to accomplish assured goals [2]

Mathematical Modeling: The mathematical model or mechanistic approach of environmental systems refers to the mathematical expressions like symbols and equations used to describe factors that results to changes or transformation in environmental Processes with time, space and Condition control [2].

Simulation: A simulation of a method is the procedure of a model of the method. The model can be reconfigured and tested with; frequently, this is difficult, too costly or unrealistic to do in the method it signifies [1,2].

Sustainable Development: Sustainable development is a society project, and a political one, that cannot be defined and implemented without science. Broadly defined, sustainable development is a systems approach to growth and development and to manage natural, produced, and social capital for the welfare of their own and future generations. Issues of sustainability are inherently complex and constantly changing [19].

II. RELATED WORK

Mathematical models in the environmental field can be drawn to back to the 1900s, the original work of Streeter and Phelps on disbanded oxygen being the utmost quoted.

Currently, determined essentially by adjusting forces, environmental studies have to be multidisciplinary, distributing with an inclusive range of pollutants experiencing difficult biotic and abiotic processes in the soil, surface water, groundwater, and atmospheric compartments of the ecosphere [2]. It is of great significance to study the parameters of the established mathematical models in environmental processes, by making necessary adjustments, so that the various compartments can be developed in a more conducive system required by man. Environmental processes are usually based on theoretical deductive approach [2].

Environmental pollution studies conducted to monitor ambient levels and to quantify the concentration of various pollutants entering a given environmental area are of great interest for possible adverse health effects [11].

The United Nations World Commission on Environment and Development (WCED) in its 1987 report 'Our Common Future' defines sustainable development as: "Development that meets the needs of the present without compromising the ability of future generations to meet their own needs." Under the principles of the United Nations Charter 'The Millennium Declaration' identified principles and treaties on sustainable development, including economic development, social development and environmental protection l19].

Contaminant transport in groundwater was postulated in 1987, where Bear offered the fundamental equations. Advertise transport and dispersive transport were the two components which Bear presented for hydrodynamic dispersion of the contaminant concentration [1,2].

Nigeria in this 21st century has witnessed vast industrial revolution, machines with large exhaust fumes used in production of goods contributes deposition of greenhouse gases to a greater extent (Chukwu, 2011) in addition Nigeria has been recorded as the largest gas flaring that flares over 70% of methane [2, 22].
The use of models to solve environmental problems is very important as it simplifies the problem and proffer information that could help in decision making [2, 12].

III. METHODOLOGY

Mathematical models are recognized as feasible tool that proffers strategic solutions to most environmental and ecological impacts of global warming and climatic change [2]. It is known that a single differential equation can serve as a mathematical model for many different phenomena especially in solving environmental problems ranging from groundwater to land use. In this research, some differential equations are employed to model these problems through mathematical formulation. And these equations can be solve using Analytical and Numerical methods [1].
The figure below illustrates the various ways of which mathematical model can be formulated in solving Environmental Problems.

The steps of the modeling process shown in FIGURE 1 [20]

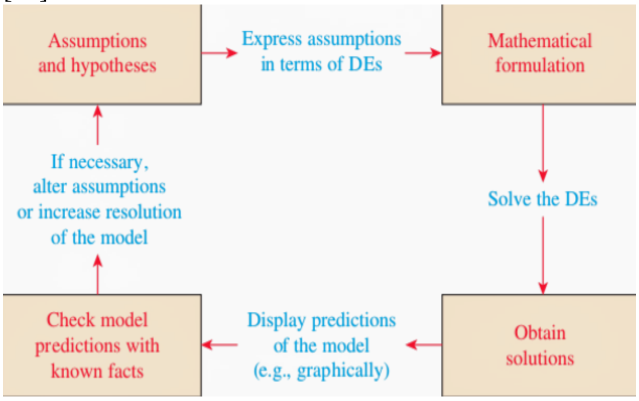

Figure1. Steps in the modeling process

The modeling process above consist; Assumptions and hypotheses, Mathematical formulation, obtain solutions, and Check model predictions with known facts.

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Groundwater Flow and Solute Transport Modeling
General governing equation for groundwater flow for three dimensional unconfined, transient, heterogeneous, and anisotropic groundwater flow reads [1]:

$$\frac{\partial}{\partial x}(Kx\frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(Ky\frac{\partial h}{\partial y}) + \frac{\partial}{\partial z}(Kz\frac{\partial h}{\partial z}) = Ss\frac{\partial h}{\partial t} - R \quad \text{........ 1}$$

where: K_x, K_y, K_z hydraulic conductivity tensor, h: hydraulic head, $S_{s:}$ storage coefficient;
R: source or sink

Contaminant transport in groundwater
Advertise transport and dispersive transport were the two components which Bear presented for hydrodynamic dispersion of the contaminant concentration [2]. The outcome of microscopic variation of velocity is an expression of dispersive flux which is a microscopic flux. Bear (1987), postulated the fundamental equations:

$$\partial\frac{(nc^k)}{\partial t} = \frac{\partial}{\partial x_i}\left(nD_{ij}\frac{\partial c^k}{\partial x_j}\right) - \frac{\partial}{\partial x_i}(nv_{si}c^k) + q_s sc^k s + \Sigma R_n \quad \text{.......... 2}$$

C^k = Dissolved concentration of species k, kgm^{-3}, n = Porosity of the subsurface medium, dimensionless, t = Time s, x_i = Distance along the respective Cartesian coordinate axis m,

D_{ij} = Hydrodynamic dispersion coefficient tensor, $m^2\ s^{-1}$, v_{si} = Seepage or linear pore water velocity, ms^{-1}, q_s = Volumetric flow rate per unit volume of aquifer representing fluid sources (positive) and sinks (negative), s^{-1}, Seepage or linear pore water velocity relates to specific discharge or Darcy flux through the relationship $v_{si} = \frac{q_i}{n}$, $C^k s$ = Concentration of the source or sink flux for species k, kgm^{-3}

Surface Waters
In surface waters area the mathematical models are used for solving wastewater treatment, industrial pollution, agricultural pollution, protection of potable water sources and others [3].

The basic equations of the model developed [1]:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + g\frac{\partial H}{\partial x} = g\frac{dh_0}{dx} \quad \text{........ 3}$$

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x}[A(x,t)u] = 0 \quad \text{........ 4}$$

Where u(x, t) - the average speed of the water flow, A(x, t) - cross-sectional area of the flow, H(x, t) - full depth of the basin, $h_0(x)$ - undisturbed value, g – acceleration of free fall, x – coordinate in the direction of the river flow, t – time.
Cross sectional area of flow is calculated as follows:

$$A(x,t) = \int z(y;x,t)dy \quad \text{........ 5}$$

Analytical solutions have been obtained, which describe the transformation of a steady flow in a channel with variable parameters [3]. The results enable performing hydrological analysis for small rivers like one of it in Duburo River in Ogoniland.

Uncertainty of river water quality
A methodology for analyzing the model uncertainty of river water quality was developed, aiming to assess the ecological status of small rivers [21]. The methodology was applied to Oreto river in Italy. Thomann, Mueller (1987) and Chapra (1997) developed the mathematical model, based on the advection-dispersion equation for one-dimensional flow:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D_L\frac{\partial^2 C}{\partial t^2} - f(C) \quad \text{........ 6}$$

[10].

Where C − concentration of a generic pollutant, t – time, x – longitudinal displacement, u – velocity, D_L – diffusion coefficient, f(C) – a generic term for reactions involving the pollutant C.

System of equations of Sen - Venan
A mathematical model based on the system of equations of Sen - Venan is developed in [1]. A river discharge of Ural and Samara river basins in Russia was modeled. It was used the one dimensional continuity equation and the equation of motion for river channels, with lateral inflows [3]:

$$\omega\frac{\partial V}{\partial x} + V\frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial t} = q \quad \text{........ 7}$$

Where Q – water consumption (m^3/s), ω – cross sectional area (m^2), V – the average flow velocity (m/s), q – side flow per unit of length (m^2/s), x- spatial coordinate (m), t - coordinated time (s)

IV. RESULTS AND DISCUSSION

Interpretation and evaluation of results
During the iterative process, performance of the model is compared against the real system to ensure that the objectives are satisfactorily met. This process consists of two main tasks; calibration and validation [1].

Task 1: Calibrating the model: In the calibration process, observed data from the real system are used. An efficient way to calibrate a model is to perform preliminary sensitivity analysis on model outputs to each parameter one by one. If the model cannot be calibrated to be within acceptable limits, the modeler should backtrack and reevaluate the system characterization and/or the model formulation steps [1].

Task 2: Validating the model: A model can be considered valid if the agreement between the two under various conditions meets the goal and performance criteria. Most environmental systems can be approximated in a satisfactory manner by linear and time variant descriptions in a lumped or distributed manner, at least for specified and restricted conditions. Analytical solutions are possible for limited types of systems; while computer

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based mathematical modeling using numerical solutions provide solutions for problems of complex geometry and properties [1].

Table 1: Typical Procedures of Mathematical Models used in solving problems in different environmental matrices [1, 2 17].

Environmental media	Issues/ concerns	Use of models
Atmosphere	Hazardous air pollutants, air emissions, toxic releases, acid rain; particulates, smog, health concerns	Concentration profiles; exposure; design and analysis of control processes and equipment; evaluation of management actions; environmental impact assessment of new projects; compliance with regulations [7].
Surface water	Wastewater treatment plant discharge ;industrial discharges; agricultural /urban runoff; storm water discharge; potable water source; food chain [7]	Fate and transport of pollutants; concentration plumes; design and analysis of control processes and equipment; waste load allocations; evaluation of management actions; environmental impact assessment of new projects; compliance with regulations [5].
Groundwater	Leaking underground storage tanks; leachates from landfills and agriculture; injection; potable water source [5].	Fate and transport of pollutants; design and analysis of remedial actions; drawdowns; compliance with regulations [5].
Subsurface	Land application of solid and hazardous wastes; spills; leachates from landfills; contamination of potable aquifers [5].	Fate and transport of pollutants; concentration plumes; design and analysis of control processes; evaluation of management actions [5].
Ocean	Sludge disposal; spills; outfalls; food chain [5],	Fate and transport of pollutants; concentration plumes; design and analysis of control processes; evaluation of management actions {5}.

Table 1

Mathematical analysis; This step of analysis involves application of standard mathematical techniques and procedures to solve‖ the model to obtain the desired results. The analysis is done according to the rules of mathematics and the system.

V. CONCLUSION AND FUTURE SCOPE

The use of mathematical modeling to solve environmental problems is a powerful tool, that simplifies problems ranging from groundwater to land use, oil spillage, gas flaring and oil well fire in Ogoniland. In this research, some differential equations are employed to model these problems through mathematical formulation. And these equations can be solve using Analytical and Numerical methods The Mathematical models in this research describe complex environmental processes and interactions, characterize the spatial and temporal variations, and predict the fate and transport of the contaminants; thereby assess potential risks existing in various resources-related activists and the associated socioeconomic and environmental impacts under a variety of system conditions. This work can be modified and expand upon in the future.

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AUTHOR PROFILE

Kanee, Goodfaith Leyira is a Nigerian Soldier, Mathematician and Theologian with a **Ph.D in Theology** from Northwestern Christian University, Florida, USA, **B.Th /Th.M in Theology** from Love of Christ (TLC) Bible College, Arizona, USA, **M.Sc Mathematics** from Abubakar Tafawa Balewa University, Bauchi, Nigeria**, B.Sc[Ed] Mathematics** from University of Jos, Jos, Nigeria, **Diploma in Statistics** from Rivers State College of Arts and Science, Port Harcourt and **I) Diploma in Mathematics II) Diploma in Journalism** from ALISON, Ireland respectively.

HOBBY: Reading, Teaching, Swimming and Listening to News. **MEMBERSHIP**: Mathematical Association of Nigeria (MAN), Nigeria Mathematics Society (NMS), Bible Society of Nigeria (BSN) and Teacher's Registration Council of Nigeria (TRCN). **AREA OF SPECIALIZATION**: Statistics, Complex Analysis, Applied and Computational Mathematics, Expository Preaching, Systematic and Exegetical Theology.

Currently, a serving personnel of the Nigerian Army and student of both Abubakar Tafawa Balewa University, Bauchi, Nigeria, And Love of Christ (TLC) Bible College, Arizona, USA.

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