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Demonstration of Phase Shift of the Universe using an Effective Equation of State Parameter

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***Abstract*—** An effective pressure parameter for the fluid of universe of matter and dark energy is proposed and is shown to be a fine simple technique for demonstrating the shift of our flat universe from its phase of deceleration to acceleration that is believed to have occurred in the past and also the present state of acceleration. The two component fluid model proposed is consistent with the measured red shifts of galaxies and it describes the dynamics of matter era and dark era elegantly using a possible set of data. It is rather about an effective equation of state parameter and how phase shift could be introduced into the evolution of the universe than the accuracy of the set of values used/produced, which might be modified by a better data set.

***Keywords*—** Effective equation of state parameter, Fluid of universe, Dark energy, Flat universe, Redshift, Matter era, Dark era.

I. INTRODUCTION

Einstein's Field equations of General Theory of Relativity: $G_{ik} = \frac{8\pi G}{c^4} T_{ik}$, T_{ik} is the stress-energy tensor and G_{ik} is the Einstein tensor states that gravitational field is manifested as a curved space-time [1,2].

The simplest model of the universe called Friedmann model is based on the assumption that the matter distribution in the universe is homogeneous and isotropic on very large scales called cosmological principle. If cosmological principle is true, then Robertson-Walker metric:

$$ds^2 = c^2dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2 sin^2\theta d\varphi^2\right]$$

can be assumed for our universe and Einstein's field equations produce Friedmann equations, the fundamental equations of standard cosmology given by [1,2]

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi G\rho}{3} = \frac{-kc^2}{R^2}, \quad \frac{\dot{R}}{R} = H \tag{1}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{8\pi G\rho}{c^2} = \frac{-kc^2}{R^2} \tag{2}$$

Combining,

$$\frac{\ddot{R}}{R} = -\frac{4\pi G\rho}{3}(1+3w) \tag{3}$$

ρ is the mass density, P the pressure, R the scale factor, H the Hubble number and k the curvature parameter of the universe. ρ includes density ρ_r of radiation, ρ_m of matter, ρ_d of dark energy (dark energy is assumed to be the effect of expansion of space-time curved by matter in the model and it came into play when stars and galaxies began to form and growing in strength with the expansion of the universe), which are respectively 0%, 30%, and 70% of the total in the present universe [3].

Eq. (3) says that universe should expand with deceleration when $w > \frac{1}{3}$ and with acceleration when $< -\frac{1}{3}$.

Hubble's law is an established fact that universe is expanding and the recessional velocity of a point in space is proportional to its distance from the observer [4]. The brightest microwave background fluctuations that have been measured by WMAP to the accuracy of 0.004 are about one degree across [3] to say that our universe is flat and Friedmann equations become:

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi G\rho}{3} = 0 \tag{4}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{8\pi G\rho}{c^2} = 0 \tag{5}$$

Using the apparent magnitude-redshift data for the distant Type Ia supernovae, Hubble parameter has been fine-tuned close to 70km/s/Mpc by WMAP [5] and other cosmological tests. The results of SCP establishes the fact that $\ddot{R} > 0$ presently; it was decelerating in the past until about 5 billion years back [6,7].The redshift z of a galaxy distant t is related to the present scale factor by [8]:

$$1 + z = \frac{R_p}{R_t} \tag{6}$$

and the redshift of the galaxies distant 13.4Gyr is about 11 [9] and that of galaxies distant 5Gyr is about 0.5 according to the data from HST [10]. And the current diameter of the observable universe is about 8.8×10^{26} [11] . The actual size of the universe may not be of the order of 10^{27}. Inflation [12] says that it is at least 3×10^{23} times the size of the observable universe [13]; Susskind's challenge (https://arxiv.org/pdf/hep-th/0610199v2.pdf) [14] makes it $10^{10^{10^{122}}}$; it can even be smaller than 1×10^{27} due to the curvature of the universe [15]. The equation of state that helps us to solve Friedmann equations is written as $P = \omega\rho c^2$, ω is the equation of state parameter. ω is $\frac{1}{3}$ for radiation-era or when the universe has only radiation, 0 for matter-era or when only matter [16]. And it is -1 when the universe has only the constant cosmological constant [17,18]. Also, by energy conservation:

$$dU + PdV = 0 \qquad (7)$$

U is the energy of the universe and V the volume. This on integration leads to $\rho_r \propto R^{-4}$, $\rho_m \propto R^{-3}$, $\rho_d \propto R^0$ [16]. The rest of the paper is organized as follows, section II discusses our proposed model, section III discusses the dynamics of the universe under the new model and section IV discusses the conclusions and consequences of the model on the evolution of our universe.

II. THE MODEL

C Sivakumar et al. [19] proposed a fluctuating ω in the equation of state, leading to a stochastic evolution of early universe. The fluid of universe comprises radiation, matter and dark energy, effect of radiation lasted for a negligibly small time compared to the present age of the universe, which is 13.77 Gyr [5]. Here we propose a model in which the pressure of the universe having matter and dark energy, inspired by models of changing equation of state parameter like [20] be

$$\mathrm{P} = \omega_{eff}\rho c^2 = \left(a - b\left(\frac{R_0}{R}\right)^n\right)pc^2 \qquad (8)$$

Where R_0 is the scale factor of the universe at the shift from deceleration to acceleration, ω_{eff} is the effective equation of state parameter of the fluid of matter and dark energy a, b and n are dimensionless constants.

And Friedmann equations are:

$$H^2 - \frac{8\pi(\rho_m + \rho_d)G}{3} = 0 \qquad (9)$$

$$2\frac{\ddot{R}}{R} + H^2 + 8\pi G\left(a - b\left(\frac{R_0}{R}\right)^n\right)(\rho_m + \rho_d) = 0 \qquad (10)$$

Giving

$$\ddot{R} = -\frac{4\pi G}{3}R(\rho_m + \rho_d)\left(1 + 3\left(a - b\left(\frac{R_0}{R}\right)^n\right)\right) \qquad (11)$$

with $\ddot{R} < 0$, $R < R_0$ and $\ddot{R} > 0$, $R > R_0$.

Friedmann equations with the equation of state and the law of conservation of energy are quite enough to talk about the dynamics of the universe.

III DYNAMICS OF THE UNIVERSE

We have

$$\frac{R_p}{R_i} = 12 \qquad (12)$$

R_p is the present scale factor and R_i the scale factor when universe is say 400Myr old with $\omega_{eff} = 0$.
And

$$\frac{R_p}{R_0} = 1.5 \qquad (13)$$

to give

$$\frac{R_0}{R_i} = 8 \qquad (14)$$

Integrating, the total density:

$$dU + PdV = 0 = d(R^3\rho) + \left(a - b\left(\frac{R_0}{R}\right)^n\right)\rho dR^3$$

$$\rho = \rho_i\left(\frac{R_i}{R}\right)^{3(1+\alpha)} exp\left(-3\frac{b}{n}\left[\left(\frac{R_0}{R}\right)^n - \left(\frac{R_0}{R_i}\right)^n\right]\right) \qquad (15)$$

and Hubble number:

$$H = \left(\frac{1}{R}\right)^{1.5(1+\alpha)} exp\left(-1.5\frac{b}{n}\left[\left(\frac{R_0}{R}\right)^n - \left(\frac{R_0}{R_i}\right)^n\right]\right) \qquad (16)$$

With

$$\frac{3}{8\pi G} = 1.791 \times 10^9 kg^2N^{-1}m^{-2} = \rho_i R_i^{3(1+\alpha)} \qquad (17)$$

Initially, when universe had only matter,

$$\omega_{eff_i} = 0 = a - b\left(\frac{R_0}{R_i}\right)^n \qquad (18)$$

and at the shift to the accelerating phase, by Eq. (3)

$$\omega_{eff_0} = \frac{1}{3} = a - b \qquad (19)$$

Eqs. (18) and (19) have

$$b = a + \frac{1}{3} \qquad (20)$$

$$a = \frac{8^n}{3(8^n - 1)} \qquad (21)$$

Table 1: $R_p(n)$ using Eqs. (16), (20) and (21)

n	a	b	$Rp \times 10^{26}$
0.500	-0.516	-0.182	0.00424
0.420	-0.572	-0.239	4.02
0.415	-0.577	-0.243	7.17
0.412	**-0.579**	**-0.246**	**10.6**
0.410	-0.581	-0.248	13.8
0.400	-0.590	-0.257	52.3

If we accept the actual size of the universe to be 1×10^{27}, then $n \approx 0.412$.(note that this could be done for the other possible R_ps like 3×10^{23} times 1.00×10^{27} or or $10^{10^{10^{122}}}$ or even smaller than 1.00×10^{27}) and hence

$$\omega_{eff} \approx -0.579 + 0.246\left(\frac{R_0}{R}\right)^{0.412}, R_0 \approx 0.6667 \times 10^{27} \qquad (22)$$

Then,

$$\omega_{eff_0} = -0.579 + 0.246 = -0.333$$

Initially,

$\omega_{eff} = 0$ and Eq. (22) gives $\frac{R_0}{R_i} \approx 8$ which

Means $\frac{R_p}{R_i} = 12$ and $\frac{R_p}{R_0} = 1.5$ to verify the proposed equation of state parameter.

Also,

$\omega_{eff} = -0.579 + 0.246 \times (0.6667)^{0.412} \approx -0.371$ to show that universe is accelerating currently (when $R < R_0$, $\omega_{eff} > -0.333$ and when $R > R_0$, $\omega_{eff} < -0.333$)

Table 2: $R(\omega_{eff})$

ω_{eff}	$R \times 10^{27}$
0	0.08
-0.333	0.67
-0.371	1.00
-0.579	∞

If law of partial pressures is true for cosmic fluid, $c^2\rho\omega_{effp}$ is $c^2\rho_{mp}\omega_{mp} + c^2\rho_{dp}\omega_{dp}$

to give:

$$\omega_{dp} = -0.371\frac{\rho_p}{\rho_{dp}} = -0.523$$

The model allows all the parameters-scale factor, Hubble number, mass density, pressure and acceleration to be expressed as a function of ω_{eff} viz.

$$R = R_0\left(\frac{b}{a - \omega_{eff}}\right)^{\frac{1}{n}} \qquad (23)$$

$$H = \left(\frac{1}{R}\right)^{1.5(1+\alpha)} exp\left(-1.5\frac{b}{n}\left[\left(\frac{R_0}{R}\right)^n - \left(\frac{R_0}{R_i}\right)^n\right]\right) \qquad (24)$$

$$\rho = \frac{3H^2}{8\pi G} \qquad (25)$$

$$\mathrm{P} = \omega_{eff}\rho c^2 \qquad (26)$$

$$\ddot{R} = -4\pi G_\rho R\left(\omega_{eff} + 0.333\right) \qquad (27)$$

Now to find the relation for $R(t)$, we need to integrate, $H = \frac{1}{R^D} exp\left(AB - \frac{A}{R^n}\right)$ where $A = 1.5\left(\frac{b}{n}\right)R_0^n < 0,\ B = \frac{1}{R_i^n} > 0,\ D = 1.5(1+\alpha) > 0$

Using $H = \frac{\dot{R}}{R}$

$$R^{D-1} exp\left(\frac{A}{R^n}\right) dR = Edt, E = exp(AB).$$

Integrating (using wolframalpha computational intelligence) in the interval $(t - t_i)$,

$$\left[R^D(-AR^{-n})\Gamma\left(-\frac{D}{n}, -AR^{-n}\right)\right]$$
$$-\left[R_i^D(-AR_i^{-n})^{\frac{D}{n}}\Gamma\left(-\frac{D}{n}, -AR_i^{n}\right)\right]$$
$$= Et - Et_i \qquad (28)$$

IV CONCLUSIONS AND DISCUSSION

What is discussed is a two component-cosmic fluid that evolves during matter era and dark era according to $\omega_{eff} \approx -0.579 + 0.246\left(\frac{R_0}{R}\right)^{0.412}$, $0 = \omega_{effi} \geq \omega_{eff} \geq -0.579 = \omega_{efff}$, if $R_p \approx 1 \times 10^{27}$ is accepted. ω_{efff} is the restriction on ω_{eff} to protect the measured redshifts and size of the universe. It is a flat expanding fluid as required by the microwave background fluctuations measured by WMAP and Hubble's law and the rate of expansion is consistent with the presently accepted value. Currently the expansion is accelerated since $R > R_0$, $\omega_{eff} < -0.333$, as established by SCP under the negative dark energy parameter and shift from the phase of deceleration to acceleration occurred in the past at the effective pressure parameter −0.333 as required by Friedmann equations. The size of the universe at the shift is 0.667 times the current value while the ΛCDM has an incorrect factor of 0.54 [21].

The scale factor $R(t)$ of the universe according to our model is given by integral

$$\int_{R_i}^{R} R^J \exp\left(\frac{A}{R^n}\right) dR = E(t - t_i), \; J = 0.5 + 1.5\alpha,$$
$$A = 1.5\left(\frac{b}{n}\right) R_0^n,$$
$$E = exp\left(1.5\left(\frac{b}{n}\right)\left(\frac{R_0}{R_i}\right)^n\right),$$

which is a little complicated even though $R(\omega_{eff})$ is simple.

An important feature of our model is that the acceleration of the universe which is proportional to

$$exp\left(\frac{3b}{n}\left(\frac{R_0}{R}\right)^n - \left(\frac{R_0}{R}\right)^n\right)\left(a - \omega_{eff}\right)^{\frac{2}{n}+\frac{3a}{n}}\left(\omega_{eff} + 0.333\right),$$

in the late time is decreasing-it decreases to zero as the scale factor tends to infinite, unlike the acceleration of the standard model which increase linearly with the scale factor. This saves the universe from spending an infinite amount of energy to produce infinite acceleration. Density and pressure are also decreasing along with Hubble number. And finally, the present equation of state parameter for dark energy is −0.523 which is less than −0.55 [22].

When R_p is 4.5×10^{29}, around 250 times the size of the observable universe, we have $\omega_{effp} = -0.373$ and $\omega_{dp} = -0.525$ and the evolution is $\omega eff = -0.618 + 0.285\left(\frac{R_0}{R}\right)^{0.373}$.If it is near 3×10^{23} times the size of the observable, say 4.7×10^{50}, $\omega_{eff} = -0.780 + 0.447\left(\frac{R_0}{R}\right)^{0.268}$, $\omega_{effp} = -0.379$ and $\omega_{dp} = -0.534$.

The lowest allowed n is very close to 0.195 which corresponds to $1+a = 0$ and $8^n = 1.50$ and infinite size for the actual universe. This has

$$\omega_{effp} = -1.0 + 0.667\left(\frac{R_0}{R_p}\right)^{0.268} = -0.384 \; and \; \omega_{dp} = -0.541$$

Table 3: Different possible models and their equation of state parameters

$Rp \times 10^{28}$	ω_{effp}	ω_{dp}	n	a	b
0.1	-0.371	-0.523	0.412	-0.579	-0.246
45	-0.373	-0.525	0.373	-0.618	-0.285
∞	-0.384	-0.541	0.195	-1.000	-0.667

Effective equation of state parameter that has not been addressed seriously in literature (notable works are a few like [20,23]) can be a better alternative for the description of the dynamics of the universe we are living in. The one that mentioned is truly beautiful in the light of its strength of predicting cosmological parameters in a simple way with an accurate knowledge of the actual size of the universe.

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