International Journal of Scientific Research in Mathematical and Statistical Sciences
Volume-9, Issue-1, pp.01-06, February (2022)
DOI: https://doi.org/10.26438/ijsrmss/v9i1.16

Research Paper

E-ISSN: 2348-4519

Bipolar Double-Framed Uncertainty Ideal Structures

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Available online at: www.isroset.org

Received: 02/Jan/2022, Accepted: 04/Feb/2022, Online: 28/Feb/2022

Abstract: Ideal concepts are involved in many mathematical applications various author have been studied and analytical in different ways. In this paper, the concept of bipolar double-framed fuzzy subalgebra's in terms of R-ideals is proposed. Also the relationship between bipolar double-framed fuzzy soft ideal and bipolar double-framed fuzzy soft R-ideals is studied some interesting results also analyzed.

***Keywords*:** soft set, fuzzy set, bipolar fuzzy soft set, double-framed fuzzy set, R-ideal, subalgebra, soft algebra, approximate function, universe.

I. INTRODUCTION

After the concept of fuzzy sets of Zadeh [23], Lee [12] proposed an extension of fuzzy sets namely Bipolar valued fuzzy sets (BVFS). Their range of membership degree has been extended from the interval [0,1] to [-1,1] and in [5], a comparison is dealt with other fuzzy settings. The concept of bipolar soft sets and operations of bipolar soft sets were introduced by Shabir and Naz [15]. Abdullahet.al[1] defined notion of bipolarfuzzy soft set sbycombining soft sets and bipolarfuzzy sets defined by Zhang [22] and presented set-theoretical operations of bipolar fuzzy soft sets. Naz and Shabir [15] proposed the concept of fuzzy bipolar soft sets and investigated algebraic structures on fuzzy bipolar soft sets. Akram et .al [3] defined the concept of bipolar fuzzy soft subsemigroup and bipolar fuzzy soft-ideals in a semigroup. These Bipolar valued fuzzy sets possess degrees of membership that denote the degree of satisfaction to the property corresponding to a fuzzy set and its counter-property in a bipolar valued fuzzy set. The membership degree 0 refers that the elements are irrelevant to the corresponding property. Further, the membership degrees (0,1] show that the elements somewhat satisfy the property, and the membership degrees [-1,0) denote that the elements somewhat satisfy the implicit counter property.There are two kinds of representations in the definition of bipolar valued fuzzy sets. They are canonical representation and reduced representation. In this work, the canonical representation of a bipolar valued fuzzy sets is utilized. In 2011, Bipolar valued fuzzy K−subalgebras are discussed by Farhat Nisar [5]. Inspired by the concepts recently, the concept of bipolar valued fuzzy subalgebras/ideals of a BF-algebra[6] has been discussed by applying the notion of Bipolar Valued Fuzzy Set (BVFS) in BF-algebras[6]. Group symmetry plays a vital role to analyse molecule structures. Isotope molecules decayswith a certain rate, so the fuzzy sense comes into it.Jun and Ahn [10] studied the applications of double framed soft sets. They also introduced the notations of double framed soft algebras and discussed their properties by giving several examples. Naz [16] introduced some operations on double framed double soft sets. Hadipour [6] applied the concept of double framed soft set in BF-algebras and introduced the notations of double framed BF-algebras. He also investigated its properties. Cho et al. [4] studied the concept of double framed soft near ring. Shabir and Khan[19], worked on double framed soft topological spaces and defined its notation. For further information, we refer to reader papers [[17], [18]] regarding soft algebraic structures.

Set theoretical operations between soft sets were introduced by Maji et.al [14]. Ali et.al [2] defined some new operations on the soft sets and Sezgin and Atagun [21] studied on soft set operations as well. In this paper,Ideal concepts are involved in many mathematical applications various author have been studied and analytical in different ways. In this paper, the concept of bipolar double-framed fuzzy sub algebra's in terms of R-ideals is proposed. Also the relationship between bipolar double-framed fuzzy soft ideal and bipolar double-framed fuzzy soft R-ideals is studied some interesting results also analyzed.

II. PRELIMINARIES

Definition-2.1[K.Lee, 2000]: By a BCI-algebra ,we mean algebra (X, $*$, 0) of type (2, 0) satisfying the following axioms, for all x,y,z ∈X;

$B(I_1)$: $(x*y)*(x*z)*(z*y) = 0$
$B(I_2)$: $(x*(x*y)*y) = 0$
$B(I_3)$: $(x*x) = 0$
$B(I_4)$: $(x*y) = 0$ and $y*x = 0$ imply $x = y$

We can define a partial ordering ≤ by x≤y if and only if $x*y = 0$.
If a BCI-algebra, X satisfying $0*x = 0$ for all x∈X, then we say that X is a BCK-algebra. Any BCK-algebra X satisfying the following axioms for all x,y,z∈X.
(BCK-1): $(x*y)*z = (x*z)*y$
(BCK-2): $(x*z)*(y*z)*(x*y) = 0$
(BCK-3): $(x*x) = 0$
(BCK-4): $(x*y) = 0 = y$, $(x*z)*(y*z) = 0$, $(z*y)*(z*x) = 0$.

Definition-2.2 [L.A.Zadeh, 1965]:Let 'X' be a non empty set. A fuzzy set A drawn from X is defined as A= {(x: $\mu_A(x)$) / x∈X} where μ_A: A → [0 , 1] is the membership function of the fuzzy set A.

Definition-2.3 [K.Lee, 2000]: Let X be a universe .Then a bipolar fuzzy set A on X is defined by positive membership function μ_A^+, that is, μ_A^+: X → [0 ,1] and a negative membership function μ_A^-, that is, μ_A^- : X → [-1,0] .For the sake of simplicity, we shall use the symbol
A= {(x, μ_A^+, μ_A^-) / x∈X}.

Definition-2.4: A collection (F, A) is called a double-framed fuzzy soft set if and only if $F: A \to P(U)$, when P(U) is the collection of all double-framed fuzzy soft set on the universal set 'U' and 'A' is a non-empty subset of the parameter set E.

Let (F, A) and (G, B) be two double-framed fuzzy soft sets over (U, E). Then the union of two double-framed fuzzy soft set $C = A \cup B,\ \forall e \in C$,

$$H(e) = \begin{cases} F(e), if e \in A - B \\ G(e), if e \in B - A \\ F(e) \cup G(e), if e \in A \cap B \end{cases}$$

$$and\ is\ written\ as\ (F, A) \cup (G, B) = (H, C)$$

Also the intersection of two double-framed fuzzy soft set $C = A \cap B,\ \forall e \in C$, $H(e) = F(c) \cap G(c)$ and is written as $(F, A) \cap (G, B) = (H, C)$.

Definition -2.5: [Moldtsov 1999]: Let U be an initial universe, P (U) be the power set of U, Ebe the set of all parameters and A⊆ E. A soft set (f_A, E) onthe universe U is defined by the set of order pairs(f_A, E) = {(e, f_A (e)): e∈ E, f_A∈ P (U)} where f_A : E → P (U) such that f_A (e) = ϕ ife ∉ A.Here f_A is called an approximate function of the soft set.

Example-2.6:Let U = $\{u_1,u_2,u_3,u_4\}$ be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)}
be a set ofparameters. If A = $\{e_1, e_2\}$ ⊆ E. Let $f_A(e_1)$= $\{u_1, u_2, u_3, u_4\}$ and
$f_A(e_2)$= $\{u_1,u_2,u_3\}$.Then we write thesoft set (f_A , E)= {(e_1, $\{u_1, u_2, u_3, u_4\}$), (e_2,{ u_1,u_2,u_3})}
over U which describe the "colour of the shirts"which Mr. X is going to buy. We may represent the soft set in the following form

U	e_1	e_2	e_3
u_1	0.7	0.5	0
u_2	0.5	0.1	0
u_3	0.4	0.5	0
u_4	0.2	0	0

Definition-2.7[Bipolar double-framed fuzzy soft set**]:** Let X is a non-empty set. A bipolar double-framed fuzzy soft set (BPDFSS).F = {(x, m_F^P, n_F^P, m_F^N, n_F^N) / x∈X} where m_F^P: x → [0, 1], n_F^P: X → [0 , 1], m_F^N: X → [0 ,1], m_F^N: X→ [0 , 1] are the mappings such that 0 ≤ (m_F) + (n_F) ≤ 1 and -1 ≤ (m_F) + (n_F) ≤ 0 and $m_F^P(x)$ denote the positive membership degree, $n_F^P(x)$ denote the positive non-membership degree, $m_F^N(x)$ denote the negative membership degree, $n_F^N(x)$ denote the negative non-membership degree. The degree of inderminancy is
$\prod_F^P(x) = \sqrt{1 - (m_F^P(x)) - (n_F^P(x))}$ and
$\prod_F^N(x) = \sqrt{1 - (m_F^N(x)) - (n_F^N(x))}$

Definition-2.8: Let F_1 = {(x, m_{F1}^P, n_{F1}^P, m_{F1}^N, n_{F1}^N) / x∈X} and F_2 = {(x, m_{F2}^P, n_{F2}^P, m_{F2}^N, n_{F2}^N)} be BPDFS sets.Then,
(i) $F_1 U F_2$ = {(x, max (m_{F1}^P, m_{F2}^P), min (n_{F1}^P, n_{F2}^P), min (m_{F1}^N, m_{F2}^N), max (m_{F1}^N, m_{F2}^N/ x∈X }
(ii) $F_1 \cap F_2$ = {(x, min (m_{F1}^P, m_{F2}^P), max (n_{F1}^P, n_{F2}^P), max (m_{F1}^N, m_{F2}^N), min (m_{F1}^N, m_{F2}^N/ x∈X }
(iii) F_1^C = {(x, n_F^P, m_F^P, n_F^N, m_F^N)/ x∈X }
(iv) $F_1 \subset F_2$ = if and only if $m_{F1}^P(x) \le m_{F2}^P(x)$, $n_{F1}^P(x) \ge n_{F2}^P(x)$, $m_{F1}^N(x) \ge m_{F2}^N(x)$, $n_{F1}^N(x) \le n_{F2}^N(x)$.

III. BIPOLARDOUBLE-FRAMED FUZZY SOFT ALGEBRA

Definition-3.1: A bipolar double-framed fuzzy soft set F in X called bipolar double-framed fuzzy soft sub algebra of X if it satisfies,
(i) $m_F^P(x*y) \ge T\ \{m_F^P(x), m_F^P(y)\}$
(ii) $n_F^P(x*y) \le S\ \{n_F^P(x), n_F^P(y)\}$
(iii) $m_F^N(x*y) \le S\{m_F^N(x), m_F^N(y)\}$
(iv) $n_F^N(x*y) \ge T\ \{n_F^N(x), n_F^N(y)\}$ for all x, y∈X.

Definition-3.2: A Bipolar double-framed fuzzy soft set 'F' of a BCK-algebra (BPDFSS), X is called a bipolar double-framed fuzzy soft ideal of X if the following conditions are satisfied
(i) $m_F^P(0) \ge m_F^P(x)$ and $n_F^P(0) \le n_F^P(x)$
(ii) $m_F^N(0) \le m_F^N(x)$ and $n_F^N(0) \ge n_F^N(x)$

(iii) $m_F^P(x) \geq T\{m_F^P(x * y), m_F^P(y)\}$
$n_F^P(x) \leq S\{n_F^P(x * y), n_F^P(y)\}$

(iv) $m_F^N(x) \leq S\{m_F^N(x * y), m_F^N(y)\}$
$n_F^N(x) \geq T\{n_F^N(x * y), n_F^N(y)\}$ for all x, y∈X.

***Definition-3.3*:** A bipolar fuzzy soft set F in X is called a bipolar double-framed fuzzy soft R-ideal (BPDFSRI) of X if it satisfies,

(i) $m_F^P(0) \geq m_F^P(x)$ and $n_F^P(0) \leq n_F^P(x)$

(ii) $m_F^N(0) \leq m_F^N(x)$ and $n_F^N(0) \geq n_F^N(x)$

(iii) $m_F^P(y * x) \geq T\{m_F^P(x * z) * (0 * y), m_F^P(z)\}$and
$n_F^P(y * x) \leq S\{n_F^P(x * z) * (0 * y), n_F^P(z)\}$

(iv) $m_F^N(y * x) \leq S\{m_F^N(x * z) * (0 * y), m_F^N(z)\}$
$n_F^N(y * x) \geq T\{n_F^N(x * z) * (0 * y), n_F^N(z)\}$ for all x, y, z∈X.

***Example-3.4*:**Consider a BCK-algebra X = {0,1,2,3} with the following Cayley's table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Define a BPDFSS 'F' in X by

X	0 1 2 3
(m_F^P, n_F^P)	[0.2, 0.5] [0.4, 0.6] [0.5, 0.7] [0.2, 0.9]
(m_F^N, n_F^N)	[-0.7, -0.1] [-0.9, -0] [-0.4, -0] [-0.7, -0]

Then F is BPDFSRI of X.
The following are the standard results with relevant proof.

***Theorem-3.5*:** If 'F' is a BPDFSRI of X, then $m_F^P(x) = m_F^P(0 * x)$, $n_F^P(x) = n_F^P(0 * x)$, $m_F^N(x) = m_F^N(0 * x)$ and $n_F^N(x) = n_F^N(0 * x)$ for all x∈X.

Proof: Let F be a BPDFSRI of X.
Taking y = z = 0 in definition 3.3 and 2.1 (iii) and (ii).We get,

$m_F^N(0 * x) \leq m_F^N(x)$, $n_F^N(0 * x) \geq n_F^N(x)$
$m_F^P(0 * x) \geq m_F^P(x)$, $n_F^P(0 * x) \leq n_F^P(x)$

By setting x = z = o in definition 3.3 and 2.1 (iii) and (ii). We get,

$m_F^N(y) = m_F^N(y * 0) \leq m_F^N(0 * (y * 0)) \leq m_F^N(0 * y)$
$n_F^N(y) = n_F^N(y * 0) \geq n_F^N(0 * (y * 0)) \geq n_F^N(0 * y)$
$m_F^P(y) = m_F^P(y * 0) \geq m_F^P(0 * (y * 0)) \geq m_F^P(0 * y)$
$n_F^P(y) = n_F^P(y * 0) \leq n_F^P(0 * (y * 0)) \leq n_F^P(0 * y)$ for all y∈X

Hence $m_F^P(x) = m_F^P(0 * x)$, $n_F^P(x) = n_F^P(0 * x)$,
$m_F^N(x) = m_F^N(0 * x)$, $n_F^N(x) = n_F^N(0 * x)$ for all x∈X.

***Theorem-3.6*:** Every BPDFSRI of X is both a BPDFSA of X and a BPDFSI of X.

Proof: Let F be BPDFSRI of X. Using set definition 3.3 and theorem- 3.5, we have,

$m_F^N(x) = m_F^N(0 * x)$
$\leq S\{m_F^N(x * z) * (0 * 0), m_F^N(z)\}$
$= S\{m_F^N(x * z), m_F^N(z)\}$

$n_F^N(x) = n_F^N(0 * x)$
$\geq T\{n_F^N(x * z) * (0 * 0), n_F^N(z)\}$
$= T\{n_F^N(x * z), n_F^N(z)\}$

$m_F^P(x) = m_F^P(0 * x)$
$\geq T\{m_F^P(x * z) * (0 * 0), m_F^P(z)\}$
$= T\{m_F^P(x * z), m_F^P(z)\}$

$n_F^P(x) = n_F^P(0 * x)$
$\leq S\{n_F^P(x * z) * (0 * 0), n_F^P(z)\}$
$= S\{n_F^P(x * z), n_F^P(z)\}$ for all x, y, z∈X.

Hence A is BPDFSI of X.
Now for any x, y∈X, we have

$m_F^N(x * y) \leq S\{m_F^N((x * y) * x), m_F^N(x)\}$
$= S\{m_F^N(0 * y), m_F^N(x)\}$
$= S\{m_F^N(x), m_F^N(y)\}$

$n_F^N(x * y) \geq T\{n_F^N((x * y) * x), n_F^N(x)\}$
$= T\{n_F^N(0 * y), n_F^N(x)\}$
$= T\{n_F^N(x), n_F^N(y)\}$

$m_F^P(x * y) \geq T\{m_F^P((x * y) * x), m_F^P(x)\}$
$= T\{m_F^P(0 * y), m_F^P(x)\}$
$= T\{m_F^P(x), m_F^P(y)\}$

$n_F^P(x * y) \leq S\{n_F^P((x * y) * x), n_F^P(x)\}$
$= S\{n_F^P(0 * y), n_F^P(x)\}$
$= S\{n_F^P(x), n_F^P(y)\}$

Therefore 'A' is BPDFSA of X.
The following example shows that the reverse of theorem –3.6need not be true.

***Example-3.7*:** Let X = {0,1,2} be a BCK-algebra with the following Cayley's table.

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Define a BPDFS 'F' in X by

x	0 1 2
(m_F^P, n_F^P)	[0.6, 0.9] [0.2, 0.6] [0.2, 0.6]
(m_F^N, n_F^N)	[-0.2, -0.4] [-0.3, -0.5] [-0.5, -0.8]

Then F is both a BPDFSI and a BPDFSA of X, but not BPDFSRI of X.

Theorem-3.8: Let F be a BPDFSI of X. If the inequality $x{*}y \le z$ holds in X, then,

(i) $m_F^N(x) \le S\{m_F^N(y), m_F^N(z)\}$ and $n_F^N(x) \ge T\{n_F^N(y), n_F^N(z)\}$

(ii) $m_F^P(x)) \ge T\{m_F^P(y), m_F^P(z)\}$ and $n_F^P(x) \le S\{n_F^P(y), n_F^P(z)\}$

Proof: Let x,y,z∈X be such that $x*y \le z$. Then $(x*y)*z = 0$ and so

(i) $m_F^N(x) \le S\{m_F^N(x * y), m_F^N(y)\}$
$\le S\{\max\{m_F^N(x * y) * z, m_F^N(z), m_F^N(y)\}\}$
$= S\{S\{m_F^N(0), m_F^N(z), m_F^N(y)\}\} = S\{m_F^N(y), m_F^N(z)\}$

$n_F^N(x) \ge T\{n_F^N(x * y), n_F^N(y)\}$
$\ge T\{T\{n_F^N(x * y) * z, n_F^N(z), n_F^N(y)\}\}$
$= T\{T\{n_F^N(0), n_F^N(z), n_F^N(y)\}\} = T\{n_F^N(y), n_F^N(z)\}$.Also

(ii) $m_F^P(x)) \ge T\{m_F^P(x * y), m_F^P(y)\}$
$\ge T\{T\{m_F^P(x * y) * z, m_F^P(z), m_F^P(y)\}\}$
$= T\{T\{m_F^P(0), m_F^P(z), m_F^P(y)\}\} = T\{m_F^P(y), m_F^P(z)\}$

$n_F^P(x) \le S\{n_F^P(x * y), n_F^P(y)\}$
$\le S\{S\{n_F^P(x * y) * z, n_F^P(z), n_F^P(y)\}\}$
$= S\{S\{n_F^P(0), n_F^P(z), n_F^P(y)\}\} = S\{n_F^P(y), n_F^P(z)\}$

Hence the proof.

Theorem-3.9: Let F be a BPDFSI of X. Then the following are equivalent.

(i) F is a BPDFSRI of X

(ii) F satisfies the following conditions,

$m_F^N(y * (x * z)) \le m_F^N((x * z) * (0 * y))$
$n_F^N(y * (x * z)) \ge n_F^N((x * z) * (0 * y))$
$m_F^P(y * (x * z)) \ge m_F^P((x * z) * (0 * y))$
$n_F^P(y * (x * z)) \le n_F^P((x * z) * (0 * y))$ for all x, y, z∈X

(iii) F satisfies the following conditions.

$m_F^N(y * x) \le m_F^N((x * (0 * y)), n_F^N(y * x) \ge n_F^N((x * (0 * y))$
$m_F^P(y * x) \ge m_F^P((x * (0 * y)), n_F^P(y * x) \le n_F^P((x * (0 * y))$

Proof: (i) → (ii)

Assume that 'A' is a BPDFSRI of X and let x,y,z∈X using definition 3.3, we get,

$m_F^N(y * (x * z)) \le S\{m_F^N((x * z) * 0) * (0 * y)), m_F^N(0)\}$
$= m_F^N((x * z) * (0 * y))$ and
$n_F^N(y * (x * z)) \ge T\{n_F^N((x * z) * 0) * (0 * y)), n_F^N(0)\}$
$= n_F^N((x * z) * (0 * y))$
$m_F^P(y * (x * z)) \ge T\{m_F^P((x * z) * 0) * (0 * y)), m_F^P(0)\}$
$= m_F^P((x * z) * (0 * y))$
$n_F^P(y * (x * z)) \le S\{n_F^P((x * z) * 0) * (0 * y)), n_F^P(0)\}$
$= n_F^P((x * z) * (0 * y))$

(ii) →(iii) taking z=0 in (ii) using (i) induce (iii).

(iii)→(i) Note that $(x * (0 * y)) * ((x * z) * (0 * y)) \le z$

For all x,y,z∈X.

It follows from (iii) and theorem – 3.8 that,

$m_F^N(y * x) \le m_F^N(x * (0 * y))$
$\le S\{m_F^N((x * z) * (0 * y)), m_F^N(z)\}$
$n_F^N(y * x) \ge n_F^N(x * (0 * y))$
$\ge T\{n_F^N((x * z) * (0 * y)), n_F^N(z)\}$
$m_F^P(y * x) \ge m_F^P(x * (0 * y))$
$\ge T\{m_F^P((x * z) * (0 * y)), m_F^P(z)\}$
$n_F^P(y * x) \le n_F^P(x * (0 * y))$
$\le S\{n_F^P((x * z) * (0 * y)), n_F^P(z)\}$

Hence 'F' is a BPDFSI of X.

Theorem-3.10: Every BPDFSI of X is a BPDFSRI of X if X is associative.

Proof: Let 'F' be a BPDFSI of X since $0{*}x=x$ for all x∈X, that is,

$y * x = (0 * y) * x$
$= (0 * x) * y$
$= x * y$
$= x * (0 * y)$ for all x, y∈X.

Therefore,

$m_F^N(y * x) = m_F^N(x * (0 * y))$,
$n_F^N(y * x) = n_F^N(x * (0 * y))$
$m_F^P(y * x) = m_F^P(x * (0 * y))$
$n_F^P(y * x) = n_F^P(x * (0 * y))$ by theorem – 3.9.

We conclude that 'A' is BPDFSRI of X.

The following section implemented the bipolar characteristics of R-ideal structures.

IV. BIPOLARDOUBLE-FRAMED R-IDEAL STRUCTURES

Theorem-4.1: Let 'F' be a BPDFRI of X. Then the set $\Delta = \{x \in X \,/\, m_F^N((x) = m_F^N(0), n_F^N(x) = n_F^N(0), m_F^P(x) = m_F^P(0), n_F^P(x) = n_F^P(0)\}$ is an R- ideal of X.

Proof: Obviously 0∈Δ. Let x,y,z∈X be such that $((x * z) * (0 * y)) \in \Delta$ and $z \in \Delta$. Then,

$m_F^N(0) \le m_F^N(y * x)$
$\le S\{m_F^N(x * (0 * y), m_F^N(z)\}$
$= m_F^N(0)$
$n_F^N(0) \ge n_F^N(y * x)$
$\ge T\{n_F^N(x * (0 * y), n_F^N(z)\}$
$= n_F^N(0)$
$m_F^P(0) \ge m_F^P(y * x)$
$\ge T\{m_F^P(x * (0 * y), m_F^P(z)\}$
$= m_F^P(0)$
$n_F^P(0) \le n_F^P(y * x)$
$\le S\{n_F^P(x * (0 * y), n_F^P(z)\}$
$= n_F^P(0)$

By using definition 2.1 it follows that,

$m_F^N(y * x) = m_F^N(0), n_F^N(y * x) = n_F^N(0)$. That is $y * x \in \Delta$. Therefore Δ is an R-ideal of X.

Theorem-4.2:If F_1 and $F_2$are BPDFSRI's of X, then $F_1 \cap F_2$ is also BPDFSRI of X.

Proof: Now,

$m_{F1}^N(0) \le m_{F1}^N(x), n_{F1}^N(0) \ge n_{F1}^N(x)$ and
$m_{F2}^N(0) \le m_{F2}^N(x), n_{F2}^N(0) \ge n_{F2}^N(x)$ for all x ∈ X.
$S\{m_{F1}^N(0), m_{F2}^N(0)\} \le S\{m_{F1}^N(x), n_{F2}^N(x)\} = m_{F1\cap F2}^N(0) \le m_{F1\cap F2}^N(0)$ and
$T\{m_{F1}^N(0), m_{F2}^N(0)\} \ge T\{m_{F1}^N(x), n_{F2}^N(x)\} = n_{F1\cap F2}^N(0) \ge n_{F1\cap F2}^N(0)$ for all x ∈ X.

Also,
$m_{F1}{}^N(y * x) \leq S\ \{m_{F1}{}^N(x * z) * (0 * y), m_{F1}{}^N(x)\ \}$
$m_{F2}{}^N(y * x) \leq S\ \{m_{F2}{}^N(x * z) * (0 * y), m_{F2}{}^N(x)\ \}$
$n_{F1}{}^N(y * x) \geq T\ \{n_{F1}{}^N(x * z) * (0 * y), n_{F1}{}^N(x)\}$
$n_{F2}{}^N(y * x) \geq T\ \{n_{F2}{}^N(x * z) * (0 * y), n_{F2}{}^N(x)\ \}$
$S\{\ m_{F1}{}^N(y * x), m_{F2}{}^N(y * x)\} \leq S\ \{\ S\{m_{F1}{}^N(x * z) * (0 * y), m_{F1}{}^N(z)\}, S\ \{m_{F2}{}^N(x * z) * (0 * y), m_{F2}{}^N(z)\}\}$
$T\{\ n_{F1}{}^N(y * x), n_{F2}{}^N(y * x)\} \geq T\ \{\ T\{n_{F1}{}^N(x * z) * (0 * y), n_{F1}{}^N(z)\}, T\ \{n_{F2}{}^N(x * z) * (0 * y), n_{F2}{}^N(z)\}\}$
$m_{F1\cap F2}{}^N \leq S\ \{m_{F1\cap F2}{}^N(x * z) * (0 * y), m_{F1\cap F2}{}^N(z)\}$,
$n_{F1\cap F2}{}^N \geq T\ \{n_{F1\cap F2}{}^N(x * z) * (0 * y), n_{F1\cap F2}{}^N(z)\ \}$ and
$m_{F1}{}^P(0) \geq m_{F1}{}^P(x), n_{F1}{}^P(0) \leq\ n_{F1}{}^P(x)$ and
$m_{F2}{}^P(0) \geq m_{F2}{}^P(x), n_{F2}{}^P(0) \leq\ n_{F2}{}^P(x)\ V\ x \in X$.
$T\{m_{F1}{}^P(0), m_{F2}{}^P(0)\} \geq T\ m_{F1}{}^P(x), m_{F2}{}^P(x)\}$
$= m_{F1\cap F2}{}^P(0) \geq m_{F1\cap F2}{}^P(\bar{x})$ and
$S\{n_{F1}{}^P(0), n_{F2}{}^P(0)\} \leq S\ n_{F1}{}^P(x), n_{F2}{}^P(x)\}$
$= n_{F1\cap F2}{}^P(0) \leq n_{F1\cap F2}{}^P(x)\ V\ x \in X$
Again,
$m_{F1}{}^P(y * x) \geq T\ \{m_{F1}{}^P(x * z) * (0 * y), m_{F1}{}^P(z)\}$
$m_{F2}{}^P(y * x) \geq T\ \{m_{F2}{}^P(x * z) * (0 * y), m_{F2}{}^P(z)\}$
$n_{F1}{}^P(y * x) \leq S\ \{n_{F1}{}^P(x * z) * (0 * y), n_{F1}{}^P(z)\}$
$n_{F2}{}^P(y * x) \leq S\ \{n_{F2}{}^P(x * z) * (0 * y), n_{F2}{}^P(z)\}$
$T\{\ m_{F1}{}^N(y * x), m_{F2}{}^N(y * x)\} \geq T\ \{T\{\ m_F{}^P(x * z) * (0 * y), m_F{}^P(z)\}, T\ \{m_{F2}{}^P(x * z) * (0 * y), m_{F2}{}^P(z)\}\}$
$S\{\ n_{F1}{}^N(y * x), n_{F2}{}^N(y * x)\} \leq S\ \{S\{\ n_F{}^P(x * z) * (0 * y), n_F{}^P(z)\}, S\ \{n_{F2}{}^P(x * z) * (0 * y), n_{F2}{}^P(z)\}$
$m_{F1\cap F2}{}^P \geq T\ \{m_{F1\cap F2}{}^P(x * z) * (0 * y), m_{F1\cap F2}{}^P(z)\ \}\}$
$n_{F1\cap F2}{}^P \leq S\ \{n_{F1\cap F2}{}^P(x * z) * (0 * y), n_{F1\cap F2}{}^P(z)\ \}, V\ x \in X.$
Hence $F_1 \cap F_2$ is also BPDFSRI of X.

Definition-4.3**:** For a bipolar double-framed fuzzy soft set 'F' in X and (α, β) ∈ [0, 1] and (γ, σ)∈ [-1, 0],the positive (α, β)- cut and negative (γ, σ)-cutare denoted by $F^P{}_{(\alpha,\beta)}$ and $F^N{}_{(\gamma,\sigma)}$ and are defined as follows:
$F^P{}_{(\alpha,\beta)} = \{\ x \in X\ /\ m_{F1}{}^P(x) \geq \alpha$ and $n_F{}^P(x) \leq \beta\}$ and
$F^N{}_{(\gamma,\delta)} = \{\ x \in X\ /\ m_{F1}{}^N(x) \leq\ \gamma$ and $n_F{}^N(x) \leq \delta\ \}$ with $\alpha + \beta \leq 1$ and $\gamma + \delta \geq -1$ respectively.
The bipolar double-framed fuzzysoft level cut of F denoted by $F_{cut}$is denoted to be the set
$F_{cut} = (F^P{}_{(\alpha,\beta)}\ ,\ F^N{}_{(\gamma,\sigma)})$.

Theorem-4.4**:** A bipolar double-framed fuzzy soft set F in X is a BPDFSRI of X if and only if for all (α, β) ∈[0, 1] and (γ,σ)∈[-1,0], the non-empty positive (α,β)-cut and the non-empty negative (γ,σ)-cut are BPDFSRI's of X.
Proof: Let 'A' be BPDFSRI of X and assume that $F^P{}_{(\alpha,\beta)}$, $F^N{}_{(\gamma,\sigma)}$ are non-empty for (α, β) ∈[0, 1] and (γ,σ)∈[-1,0] obviously $0 \in F^P{}_{(\alpha,\beta)} \cap F^N{}_{(\gamma,\sigma)}$.
Let for all x, y, z ∈ X be such that $m_F{}^N((x * z) * (0 * y)) \in F^N{}_{(\gamma,\sigma)}$ and $m_F{}^N(z) \in F^N{}_{(\gamma,\sigma)}$
$n_F{}^N((x * z) * (0 * y)) \in F^N{}_{(\gamma,\sigma)}$ and $n_F{}^N((z) \in \cap F^N{}_{(\gamma,\sigma)}$.
Then
$m_F{}^N((x * z) * (0 * y)) \leq \gamma, m_F{}^N(z) \leq \gamma$
$n_F{}^N((x * z) * (0 * y)) \geq \sigma, n_F{}^N(z) \geq \sigma$
it follows from definition 2.1 that
$m_F{}^N(y * x) \leq S\ \{m_F{}^N(x * z) * (0 * y), m_F{}^N(z)\} \leq \gamma$ and
$n_F{}^N(y * x) \geq T\ \{n_F{}^N(x * z) * (0 * y), n_F{}^N(z)\} \geq \sigma$
So that $y * x \in F^N{}_{(\gamma,\sigma)}$.
Now assume that ,
$m_F{}^P((x * z) * (0 * y)) \in F^P{}_{(\alpha,\beta)}$ and $m_F{}^P(z) \in F^P{}_{(\alpha,\beta)}$, also
$n_F{}^P((x * z) * (0 * y)) \in F^P{}_{(\alpha,\beta)}$ and $n_F{}^P((z) \in \cap F^P{}_{(\alpha,\beta)}$
Then
$m_F{}^P((x * z) * (0 * y)) \geq \alpha, m_F{}^P(z) \geq \alpha$
$n_F{}^P((x * z) * (0 * y)) \leq\ \beta, n_F{}^P(z) \leq\ \beta$
It follows from the definition 2.1 that
$m_F{}^P(y * x) \geq T\ \{m_F{}^P(x * z) * (0 * y), m_F{}^P(z)\} \geq \alpha$ and
$n_F{}^P(y * x) \leq S\ \{n_F{}^P(x * z) * (0 * y), n_F{}^P(z)\} \leq\ \beta$
So that $y * x \in F^P{}_{(\alpha,\beta)}$.Therefore $F^P{}_{(\alpha,\beta)}$ and $F^P{}_{(\gamma,\sigma)}$ are R-ideal of X.
Conversely, suppose that the non-empty negative (γ, σ)-cut and the non-empty positive (α , β)-cut are R-ideals of X for every (α , β) ∈[0, 1] and (γ, σ) ∈ [-1, 0].
If $m_F{}^N((0) > m_F{}^N((a), n_F{}^N((0) < n_F{}^N((a)$
$m_F{}^P((0) < m_F{}^P((b), n_F{}^P((0) > n_F{}^P((b)$ for some a, b∈ X, then $0 \notin F^N{}_{(mFN(a),nFN(a))}$ or
$0 \notin F^P{}_{(mF^P(a),\ nF^P(a))}$. This is a contradiction that$m_F{}^N(0) \leq m_F{}^N(x), n_F{}^N(0) \geq\ n_F{}^N(x)$ and
$m_F{}^P(0) \geq\ m_F{}^P(x), n_F{}^P(0) \leq\ n_F{}^P(x)$ for all x ∈ X.
Assume that
$m_F{}^N((b * a) > S\ \{m_F{}^N(a * c) * (0 * b), m_F{}^N(c)\} = \gamma$
$n_F{}^N((b * a) < T\ \{n_F{}^N(a * c) * (0 * b), n_F{}^N(c)\} =\sigma$ for some a, b, c ∈ X. Then,
$(a * b) * (0 * b) \in F^N{}_{(\gamma,\sigma)}$ and $c \in F^N{}_{(\gamma,\sigma)}$, but $b * a \notin F^N{}_{(\gamma,\sigma)}$. This is impossible and thus,
$m_F{}^N((y * x) \leq S\ \{m_F{}^N(x * z) * (0 * y), m_F{}^N(z)\} = \gamma$ and
$n_F{}^N((y * x) \geq\ T\{n_F{}^N(x * z) * (0 * y), n_F{}^N(z)\} = \sigma$for all x, y, z ∈ X.
If
$m_F{}^P((b * a) < T\ \{m_F{}^P(a * c) * (0 * b), m_F{}^P(c)\} = \alpha$
$n_F{}^P((b * a) > S\ \{n_F{}^P(a * c) * (0 * b), n_F{}^P(c)\} = \beta$ for some a, b, c ∈ X. then, $(a * c) * (0 * b) \in F^P{}_{(\alpha,\beta)}$ and $c \in F^P{}_{(\alpha,\beta)}$ but $b * a \notin F^P{}_{(\alpha,\beta)}$, this is the impossible and thus
$m_F{}^P(y * x) \geq\ T\ \{m_F{}^P(x * z) * (0 * y), m_F{}^P(z)\} = \alpha$
$n_F{}^P(y * x) \leq S\ \{n_F{}^P(x * z) * (0 * y), n_F{}^P(z)\} = \beta$ for all x, y, z ∈ X.
Consequently F in BPDFSRI of X.

V.CONCLUSION

Here, the notion of bipolar double-framed fuzzy soft R-ideals in terms of BCK-algebra is introduced and their properties are investigated. Also relationships between bipolar double-framed fuzzy soft subalgebra and bipolar double-framed fuzzy soft ideals are analyzed.

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