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An Efficient Ratio-Cum-Product Estimator for Finite Population Mean

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Abstract**—** Use of auxiliary information in estimating population parameters with greater precision is a common practice in survey sampling literature. For example, ratio-cum-product estimator is the one which makes use of auxiliary information. Here, in this paper, ratio-cum-product method of estimation is carried forward with the introduction of a new estimator for finite population mean. The estimator, to first order of approximation, is found to be biased, but its mean square error is found to be less than that of the customary ratio-cum-product estimator under feasible condition. The performance of the estimator has been examined in double sampling. Empirical investigations have been carried out to demonstrate the superiority of the estimator.

Keywords**—** Auxiliary variable, Ratio-cum-product estimator, Double sampling, Bias, Mean square error

I. INTRODUCTION

While an auxiliary variable, positively correlated with the study variable, results in the customary ratio estimator, the customary product estimator is arrived at when the auxiliary variable and the study variable are negatively correlated. Singh[7] has utilized two auxiliary variables, one positively and the other negatively correlated with the study variable, to propose the conventional ratio-cum-product estimator for estimating population mean. Invoking the technique due to Singh[7], we have, in this paper, come up with a new ratio-cum-product estimator using a preassigned constant α.

II. REVIEW OF LITERATURE

Let $U = (U_1, \dots\dots, U_N)$ be a finite population of size N. A sample of size n is drawn from the population with simple random sampling without replacement. Let y *be* the study variable and x, z be the auxiliary variables. $\bar{X}, \bar{Z}$ are known population means where as $\bar{y}, \bar{x}$ *and* $\bar{z}$ are the sample means.

It is well-known that the simple mean estimator of the study variable y for estimating the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ is $\bar{y}$, which is unbiased and its variance is given by

$$V(\bar{y}) = f_1 \bar{Y}^2 C_y^2 , \tag{2.1}$$

where $f_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$ and

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2.$$

The classical ratio and product estimators of population mean $\bar{Y}$ are, respectively, defined by

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X} , \tag{2.2}$$

$$\bar{y}_p = \frac{\bar{y}}{\bar{Z}}\bar{z} . \tag{2.3}$$

Assuming $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, i.e., \bar{y} = \bar{Y}(1 + e_0)$,

$e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}, i.e., \bar{x} = \bar{X}(1 + e_1)$

and $e_2 = \frac{\bar{z}-\bar{Z}}{\bar{Z}}, i.e., \bar{z} = \bar{Z}(1 + e_2)$,

such that $E(e_i) = 0$,

$E({e_0}^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_y^2 = f_1 C_y^2$,

$E({e_1}^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_x^2 = f_1 C_x^2$,

$E({e_2}^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_z^2 = f_1 C_z^2$.

We present below the expressions for the Biases and MSEs, up to $o(n^{-1})$:

$$B(\bar{y}_r) = f_1 \bar{Y}^2 \left(C_x^2 - \rho_{yx} C_y C_x\right), \tag{2.4}$$

$$B(\bar{y}_p) = f_1 \bar{Y}^2 \rho_{yz} C_y C_z , \tag{2.5}$$

$$M(\bar{y}_r) = f_1 \bar{Y}^2 \left(C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x\right) , \tag{2.6}$$

$$M(\bar{y}_p) = f_1 \bar{Y}^2 \left(C_y^2 + C_z^2 + 2\rho_{yz} C_y C_z\right) , \tag{2.7}$$

where C_y, C_x and C_z are the coefficients of variation of y, x and z, respectively. While ρ_{yx} is the correlation coefficient between y *and* x assumed to be positive, ρ_{yz} is the correlation coefficient between y *and* z assumed to be negative.

Singh[7], taking into account both positively and negatively correlated variables with the study variable, proposed, for the first time, the ratio-cum-product estimator for estimating the population mean which is given by

$$\bar{y}_{rp} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{Z}}\right), \tag{2.8}$$

whose Bias and MSE, up to $o(n^{-1})$, are given by

$$B(\bar{y}_{rp}) = f_1\bar{Y}(C_x^2 - \rho_{yx}C_yC_x - \rho_{xz}C_xC_z + \rho_{yz}C_yC_z) \quad (2.9)$$

And $M(\bar{y}_{rp}) = f_1\bar{Y}^2(C_y^2 + C_x^2 + C_z^2 - 2\rho_{yx}C_yC_x - 2\rho_{xz}C_xC_z + 2\rho_{yz}C_yC_z)$. (2.10)

Moving a step forward, Panda and Sen[5] proposed a weighted ratio-cum-product estimator for finite population mean. The estimator thus proposed was, under optimal weights, was found to be more efficient than the competing estimators under practical conditions.

III. THE NEWLY PROPOSED ESTIMATOR USING POSITIVELY AND NEGATIVELY CORRELATED VARIABLES

For estimating the population mean $\bar{Y}$, we propose herewith a new generalised ratio-cum-product estimator as

$$\bar{y}'_{rp} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\left(\frac{\bar{z}}{\bar{Z}}\right)^{2-\alpha} \quad (3.1)$$

Substituting the values of e_0, e_1 in the expression, we get

$$\bar{y}'_{rp} = \bar{Y}(1+e_0)\left(\frac{\bar{X}}{\bar{X}(1+e_1)}\right)^{\alpha}\left(\frac{\bar{Z}(1+e_2)}{\bar{Z}}\right)^{2-\alpha},$$

$$= \bar{Y}(1+e_0)(1+e_1)^{-\alpha}(1+e_2)^{2-\alpha}.$$

Retaining terms only up to 2nd degree, we have

$$\bar{y}'_{rp} = \bar{Y}(1+e_0)\left\{1 - \alpha e_1 + \frac{\alpha(\alpha+1)}{2}e_1^2\right\}\left\{1 + (2-\alpha)e_2 + \frac{(1-\alpha)(2-\alpha)}{2}e_2^2\right\},$$

$$= \bar{Y}\left\{1 + e_0 - \alpha e_1 - \alpha e_0e_1 + (2-\alpha)e_2 - \alpha(2-\alpha)e_1e_2 + (2-\alpha)e_0e_2 + \frac{\alpha(\alpha+1)}{2}e_1^2 + \frac{(1-\alpha)(2-\alpha)}{2}e_2^2\right\}.$$

We proceed to find the bias of the proposed estimator, to the first order of approximation, i.e., up to $o(n^{-1})$, as

$$B(\bar{y}'_{rp}) = E(\bar{y}'_{rp}) - \bar{Y}$$

$$= E\left\{\bar{Y}\left(1 + e_0 - \alpha e_1 - \alpha e_0e_1 + (2-\alpha)e_2 - \alpha(2-\alpha)e_1e_2 + (2-\alpha)e_0e_2 + \frac{\alpha(\alpha+1)}{2}e_1^2 + \frac{(1-\alpha)(2-\alpha)}{2}e_2^2\right)\right\} - \bar{Y}.$$

$$\Rightarrow B(\bar{y}'_{rp}) = f_1\bar{Y}\left(-\alpha\rho_{yx}C_yC_x - \alpha(2-\alpha)\rho_{xz}C_xC_z + (2-\alpha)\rho_{yz}C_yC_z + \frac{\alpha(\alpha+1)}{2}C_x^2 + \frac{(1-\alpha)(2-\alpha)}{2}C_z^2\right). \quad (3.2)$$

Similarly, the mean square error of the estimator, to the first order of approximation, i.e., up to $o(n^{-1})$, is derived as

$$M(\bar{y}'_{rp}) = E(\bar{y}'_{rp} - \bar{Y})^2$$

$$= E\left[\bar{Y}\left(1 + e_0 - \alpha e_1 - \alpha e_0e_1 + (2-\alpha)e_2 - \alpha(2-\alpha)e_1e_2 + (2-\alpha)e_0e_2 + \frac{\alpha(\alpha+1)}{2}e_1^2 + \frac{(1-\alpha)(2-\alpha)}{2}e_2^2\right) - \bar{Y}\right]^2,$$

$$= \bar{Y}^2E[e_0^2 + \alpha^2e_1^2 + (2-\alpha)^2e_2^2 - 2\alpha e_0e_1 + 2(2-\alpha)e_0e_2 - 2\alpha(2-\alpha)e_1e_2],$$

$$= f_1\bar{Y}^2\left[\alpha^2(C_x^2 + C_z^2 + 2\rho_{xz}C_xC_z) - 2\alpha\left(2C_z^2 + \rho_{yx}C_yC_x + 2\rho_{xz}C_xC_z + \rho_{yz}C_yC_z\right) + \left(C_y^2 + 4C_z^2 + 4\rho_{yz}C_yC_z\right)\right]. \quad (3.3)$$

To find the optimum value of α, we minimize the MSE with respect to α,
i.e.,

$$\frac{\partial MSE(\bar{y}'_{rp})}{\partial\alpha} = 0$$

$$\Rightarrow \alpha_{opt} = \frac{2C_z^2 + \rho_{yx}C_yC_x + 2\rho_{xz}C_xC_z + \rho_{yz}C_yC_z}{C_x^2 + C_z^2 + 2\rho_{xz}C_xC_z}.$$

Replacing α by α_{opt} in (3.3), we obtain

$$MSE(\bar{y}'_{rp})_{min} = f_1\bar{Y}^2\left[\left(C_y^2 + 4C_z^2 + 4\rho_{yz}C_yC_z\right) - \left\{\frac{\left(2C_z^2 + \rho_{yx}C_yC_x + 2\rho_{xz}C_xC_z + \rho_{yz}C_yC_z\right)^2}{\left(C_x^2 + C_z^2 + 2\rho_{xz}C_xC_z\right)}\right\}\right] \quad (3.4)$$

IV. EFFICIENCY COMPARISON

The newly proposed estimator $\bar{y}'_{rp}$ performs better than the usual ratio-cum-product estimator $\bar{y}_{rp}$ due to Singh[7] iff

$$M(\bar{y}'_{rp}) - M(\bar{y}_{rp}) < 0$$

$$\Rightarrow \left[f_1\bar{Y}^2\left\{\left(C_y^2 + 4C_z^2 + 4\rho_{yz}C_yC_z\right) - \frac{\left(2C_z^2 + \rho_{yx}C_yC_x + 2\rho_{xz}C_xC_z + \rho_{yz}C_yC_z\right)^2}{\left(C_x^2 + C_z^2 + 2\rho_{xz}C_xC_z\right)}\right\} - f_1\bar{Y}^2\left(C_y^2 + C_x^2 + C_z^2 - 2\rho_{yx}C_yC_x - 2\rho_{xz}C_xC_z + 2\rho_{yz}C_yC_z\right)\right] < 0$$

$$\Rightarrow 2C_x^2\rho_{yz}C_yC_z - 2C_z^2\rho_{yz}C_yC_z - 2\rho_{yx}C_yC_x\rho_{yz}C_yC_z < C_x^4 + C_z^4 + {\rho_{yx}}^2{C_y}^2{C_x}^2 + {\rho_{yz}}^2{C_y}^2{C_z}^2 + 2C_z^2\rho_{yx}C_yC_x - 2C_x^2{C_z}^2 - 2C_x^2\rho_{yx}C_yC_x .$$

$$\Rightarrow 2C_x^2C_{yz} - 2C_z^2C_{yz} - 2C_{yx}C_{yz} < C_x^4 + C_z^4 + {C_{yx}}^2 + {C_{yz}}^2 + 2C_z^2C_{yx} - 2C_x^2{C_z}^2 - 2C_x^2C_{yx} . \quad (4.1)$$

Between $\bar{y}'_{rp}$ and $\bar{y}_r$, the former is found to be more efficient than the latter iff

$$M(\bar{y}'_{rp}) - M(\bar{y}_r) < 0$$

$$\Rightarrow \left[f_1\bar{Y}^2\left\{\left(C_y^2 + 4C_z^2 + 4\rho_{yz}C_yC_z\right) - \frac{\left(2C_z^2 + \rho_{yx}C_yC_x + 2\rho_{xz}C_xC_z + \rho_{yz}C_yC_z\right)^2}{\left(C_x^2 + C_z^2 + 2\rho_{xz}C_xC_z\right)}\right\} - f_1\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x\right)\right] < 0.$$

$$\Rightarrow 2C_x^2C_{yx} + 2C_z^2C_{yx} - 4{C_z}^2C_{yx} - 2C_x^2C_{xz} - 2C_{yx}C_{yz} + 4C_x^2C_{yz} + 4C_{yz}C_{xz} < C_x^4 + {C_{yz}}^2 + {C_{yx}}^2 + 4{C_{xz}}^2 - 3C_x^2C_z^2, \quad (4.2)$$

and when compared with product estimator $\bar{y}_p$, the proposed estimator $\bar{y}'_{rp}$ can fare better iff

$$M(\bar{y}'_{rp}) - M(\bar{y}_p) < 0$$

$$\Rightarrow \left[f_1 \bar{Y}^2 \left\{ (C_y^2 + 4C_z^2 + 4\rho_{yz}C_yC_z) - \frac{(2C_z^2 + \rho_{yx}C_yC_x + 2\rho_{xz}C_xC_z + \rho_{yz}C_yC_z)^2}{(C_x^2 + C_z^2 + 2\rho_{xz}C_xC_z)} \right\} - f_1 \bar{Y}^2 (C_y^2 + C_x^2 + 2\rho_{yx}C_yC_x) \right] < 0,$$

$$\Rightarrow 4C_x^2C_{yz} - 2C_x^2C_{yx} - 2C_x^2C_{xz} - 6C_z^2C_{yx} + 4C_{yz}C_{xz} - 2C_{yx}C_{yz} - 8C_{yx}C_{xz} < C_x^4 + {C_{yz}}^2 + {C_{yx}}^2 + 4{C_{xz}}^2 - 3C_x^2C_z^2 . \quad (4.3)$$

V. PERFORMANCE OF THE ESTIMATOR IN DOUBLE SAMPLING.

The population means of the auxiliary variables used in ratio, product and ratio-cum-product estimators are known in advance. But, in most of the situations, they are not known in advance. In the circumstances, the above methods of estimation cannot be used. A natural course of action is to invoke double sampling. This method deals with a large preliminary sample of size n' drawn from the parent population by SRSWOR with a view to estimating the population mean $\bar{X}$ and a subsample of size n is drawn from the large preliminary sample to observe the characteristic under study so as to find $\bar{y}$.

In double sampling, the corresponding estimators become

$$\bar{y}_{rd} = \frac{\bar{y}}{\bar{x}}\bar{x}', \quad (5.1)$$

$$\bar{y}_{pd} = \frac{\bar{y}}{\bar{z}'}\bar{z}, \quad (5.2)$$

$$\bar{y}_{rpd} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{z}'}\right) \quad (5.3)$$

and $$\bar{y}'_{rpd} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)^{\alpha}\left(\frac{\bar{z}}{\bar{z}'}\right)^{2-\alpha}. \quad (5.4)$$

The expression of the biases of the estimators $\bar{y}_{rd}, \bar{y}_{pd}, \bar{y}_{rpd}$ *and* $\bar{y}'_{rpd}$, to the first order of approximation, are, respectively, written as

$$B(\bar{y}_{rd}) = \bar{Y}\rho_{yx}C_yC_x(\theta' - \theta) , \quad (5.5)$$

$$B(\bar{y}_{pd}) = \bar{Y}\rho_{yz}C_yC_z(\theta' + \theta) , \quad (5.6)$$

$$B(\bar{y}_{rpd}) = \bar{Y}\,(\theta - \theta')(C_x^2 - \rho_{yx}C_yC_x - \rho_{xz}C_xC_z + \rho_{yz}C_yC_z) , \quad (5.7)$$

$$\mathrm{B}(\bar{y}'_{rpd}) = \bar{Y}\left[\theta\frac{\alpha(\alpha+1)}{2}C_x^2 - \theta'\{3(\alpha-1)C_x^2\} - \alpha(2-\alpha)\rho_{xz}C_xC_z(\theta-\theta') + C_z^2\left\{\frac{(1-\alpha)(2-\alpha)}{2}\theta - (2-\alpha)^2\theta'\right\} - \alpha\rho_{yx}C_yC_x(\theta-\theta') + (2-\alpha)\rho_{yz}C_yC_z(\theta-\theta')\right]. \quad (5.8)$$

Again, the expressions for the MSE of the estimators $\bar{y}_{rd}, \bar{y}_{pd}, \bar{y}_{rpd}$ *and* $\bar{y}'_{rpd}$, to the first order of approximation, are, respectively, given by

$$M(\bar{y}_{rd}) = \bar{Y}^2[\theta C_y^2 + \theta^*(C_x^2 - 2\rho_{yx}C_yC_x)] , \quad (5.9)$$

$$M(\bar{y}_{pd}) = \bar{Y}^2[\theta C_y^2 + \theta^*(C_x^2 - 2\rho_{yx}C_yC_x)], \quad (5.10)$$

$$M(\bar{y}_{rpd}) = \bar{Y}^2[\theta C_y^2 + \theta^*(C_x^2 + C_z^2 - 2\rho_{yx}C_yC_x + 2\rho_{yz}C_yC_z - 2\rho_{xz}C_xC_z)] , \quad (5.11)$$

$$M(\bar{y}'_{rpd}) = \bar{Y}^2[\theta C_y^2 + \theta^*\{\alpha^2C_x^2 + (2-\alpha)^2C_z^2 - 2\alpha\rho_{yx}C_yC_x - 2\alpha(2-\alpha)\rho_{xz}C_xC_z + 2(2-\alpha)\rho_{yz}C_yC_z\}] , \quad (5.12)$$

where $\theta = \left(\frac{1}{n} - \frac{1}{N}\right), \theta' = \left(\frac{1}{n'} - \frac{1}{N}\right)$

and $\theta^* = (\theta - \theta') = \left(\frac{1}{n} - \frac{1}{n'}\right).$

From the above expressions, it is evident that the performance of the proposed estimator remains unaltered in double sampling.

Furthermore, the percentage gain in efficiency of $\bar{y}'_{rp}$ with respect to $\bar{y}, \bar{y}_r, \bar{y}_p$ and $\bar{y}_{rp}$, when α is optimally determine, is given by

$$G_i = \left[\frac{M(\bar{y}_i)}{M(\bar{y}'_{rp})} - 1\right] \times 100,$$

where i denotes $\bar{y}$, $\bar{y}_r$, $\bar{y}_p$ and $\bar{y}_{rp}$ respectively.

VI. EMPIRICAL STUDY

For the purpose of numerical illustrations, we have considered real data sets from various sources. In all the examples, the conditions for better performance of the newly proposed estimator vis-à-vis the competing estimators hold good.

Population 1

Source: Johnston[4](p.171)

Y: Percentage of hives affected by disease

X: Mean January Temp, ^{0}F

Z: Date of flowering of a particular summer-flowering species(days from January 1)

$N = 10$, $\bar{Y}$ =52, $\bar{X}$ =42, $\bar{Z}$ =200 ,

$\rho_{yx} = 0.7965547$, $\rho_{yz} = -0.9363887$, $\rho_{xz} = -0.7333333$, $\alpha = 0.4814971$

Table 6.1: MSE and PRE of Competing Estimators

Sl. No.	Estimator	$MSE/(\theta\bar{Y}^2)$	Percentage Gain in Efficiency of $\bar{y}'_{rp}$
1	$\bar{y}$	0.024	572.337
2	$\bar{y}_r$	0.008	146.719
3	$\bar{y}_p$	0.013	261.8622
4	$\bar{y}_{rp}$	0.006	76.720
5	$\bar{y}'_{rp}$	**0.003**	0.000

Population 2

Source: Fisher [10](p.180)

Data shows measurements of the flowers(in centimetres) of fifty plants each of species Iris setosa and the variable's given as-

Y : Petal length
X : Sepal length
Z : Sepal width
$N = 150$, $\bar{Y} = 3.758$, $\bar{X} = 5.843333$, $\bar{Z} = 3.057333$, $\rho_{yx} = 0.8717538$, $\rho_{yz} = -0.4284401$, $\rho_{xz} = 0.1175698$, $\alpha = 1.829628$

Population 2(A)
Source: Fisher [10](p.180)
Data shows measurements of the flowers(in centimetres) of fifty plants each of species Iris setosa and the variable's given as-
Y: Petal width
X: Sepal length
Z: Sepal width
$N = 150$, $\bar{Y}$ =1.199333, $\bar{Y}$ =5.843333, $\bar{Z}$ =3.057333 , $\rho_{yx} = 0.8179411$, $\rho_{yz} = -0.3661259$, $\rho_{xz} = -0.1175698$, $\alpha = 2.142486$

Table 6.2: MSE and PRE of Competing Estimators

		Population 2		Population 2(A)	
Sl.no.	Estimator	$MSE/(\theta\bar{Y}^2)$	Percentage Gain in Efficiency of $\bar{y}'_{rp}$	$MSE/(\theta\bar{Y}^2)$	Percentage Gain in Efficiency of $\bar{y}'_{rp}$
1	$\bar{y}$	0.220	225.31	0.403	113.87
2	$\bar{y}_r$	0.124	83.81	0.276	46.49
3	$\bar{y}_p$	0.184	170.68	0.358	89.50
4	$\bar{y}_{rp}$	0.092	36.18	0.235	24.64
5	$\bar{y}'_{rp}$	**0.067**	0.00	**0.189**	0.00

Population 3
Source: Gujarati[1](p.277)
Y: Annual sales in MPF, million paired feet
X: Gross National Product (GNP), $, billion
Z: Unemployment rate, %
$N = 16$, $\bar{Y} = 7543.125$, $\bar{X} = 1287.044$, $\bar{Z} = 6.4125$, $\rho_{yx} = 0.2102864$, $\rho_{yz} = -0.2581712$, $\rho_{xz} = 0.7259624$, $\alpha = 1.35302$.

Population 3(A)
Source: Gujarati[1](p.277)
Y: Annual sales in MPF, million paired feet
X: Gross National Product (GNP), $, billion
Z: Prime rate lagged 6 months
$N = 16$, $\bar{Y} = 7543.125$, $\bar{X} = 1287.044$, $\bar{Z} = 9.6375$, $\rho_{yx} = 0.2102864$, $\rho_{yz} = -0.05035153$, $\rho_{xz} = 0.8098949$, $\alpha = 1.579791$.

Table 6.3: MSE and PRE of Competing Estimators

		Population 3		Population 3(A)	
Sl. No.	Estimator	$MSE/(\theta\bar{Y}^2)$	Percentage Gain in Efficiency of $\bar{y}'_{rp}$	$MSE/(\theta\bar{Y}^2)$	Percentage Gain in Efficiency of $\bar{y}'_{rp}$
1	$\bar{y}$	0.026	57.81	0.026	8.20
2	$\bar{y}_r$	0.034	108.74	0.034	43.13
3	$\bar{y}_p$	0.079	378.24	0.211	776.45
4	$\bar{y}_{rp}$	0.034	110.11	0.125	422.98
5	$\bar{y}'_{rp}$	**0.016**	0.00	**0.024**	0.00

Population 4
Source: Henderson & Velleman[3](p.396)
Y: Miles per gallon (mpg)
X: Rear axle ratio (drat)
Z: Displacement(cu.in.) (disp)
$N = 32$, $\bar{Y} = 20.09062$, $\bar{X} = 3.596563$, $\bar{Z} = 230.7219$, $\rho_{yx} = 0.6811719$, $\rho_{yz} = -0.8475514$, $\rho_{xz} = -0.7102139$, $\alpha = 1.812552$.

Population 4(A)
Source: Henderson & Velleman[3](p.396)
Y: Miles/(US) gallon (mpg)
X: Rear axle ratio (drat)
Z: Gross horsepower(hp)
$N = 32$, $\bar{Y} = 20.09062$, $\bar{X} = 3.596563$, $\bar{Z} = 146.6875$, $\rho_{yx} = 0.6811719$, $\rho_{yz} = -0.7761684$, $\rho_{xz} = -0.4487591$, $\alpha = 1.661684$.

Table 6.4: MSE and PRE of Competing Estimators

		Population 4		Population 4(A)	
Sl. No.	Estimator	$MSE/(\theta\bar{Y}^2)$	Percentage Gain in Efficiency of $\bar{y}'_{rp}$	$MSE/(\theta\bar{Y}^2)$	Percentage Gain in Efficiency of $\bar{y}'_{rp}$
1	$\bar{y}$	0.089	80.16	0.089	146.65
2	$\bar{y}_r$	0.051	2.77	0.051	40.70
3	$\bar{y}_p$	0.105	110.98	0.091	148.86
4	$\bar{y}_{rp}$	0.180	260.68	0.114	213.845
5	$\bar{y}'_{rp}$	**0.050**	0.00	**0.036**	0.00

Population 5
Source: Gujarati[1](p.354)
Y: New passenger cars sold in U.S.

Z:Consumer price index(CPI), all items, all urban consumers.
X: Personal disposal income(PDI), billions of dollars.
$N = 16, \quad \bar{Y} = 10005.12, \quad \bar{X} = 1745.544, \quad \bar{Z} = 219.15, \rho_{yx} = 0.00485119, \quad \rho_{yz} = -0.1035994, \quad \rho_{xz} = 0.9912736, \quad \alpha = 0.8760091.$

Population 5(A)
Source: Gujarati[1](p.354)
Y: New passenger cars sold in U.S.
X: Personal disposal income(PDI), billions of dollars.
Z: New cars, consumer price index.
$N = 16, \quad \bar{Y} = 10005.12, \quad \bar{X} = 1745.544, \quad \bar{Z} = 162.2125, \quad \rho_{yx} = 0.00485119, \quad \rho_{yz} = -0.06618338, \rho_{xz} = 0.9913537, \quad \alpha = 0.7378249.$

Table 6.5: MSE and PRE of Competing Estimators

		Population 5		**Population 5(A)**	
Sl. No.	**Estimator**	$MSE/(\theta\bar{Y}^2)$	**Percentage Gain in Efficiency of $\bar{y}'_{rp}$**	$MSE/(\theta\bar{Y}^2)$	**Percentage Gain in Efficiency of $\bar{y}'_{rp}$**
1	$\bar{y}$	0.013	108.56	0.013	34.95
2	$\bar{y}_r$	0.193	2889.21	0.193	1834.18
3	$\bar{y}_p$	0.118	1726.73	0.073	625.56
4	$\bar{y}_{rp}$	0.015	136.85	0.041	312.44
5	$\bar{y}'_{rp}$	**0.006**	0.00	**0.010**	0.00

Population 6
Source: Singh[8]
Y: Number of females employed
X: Number of females in service
Z: Number of educated females
$N = 61, \quad \bar{Y} = 7.46, \quad \bar{X} = 5.31, \quad \bar{Z} = 179, C_y^2 = 0.5046, C_x^2 = 0.5737, \quad C_z^2 = 0.0633, \quad \rho_{yx} = 0.7737, \quad \rho_{yz} = -0.2070, \quad \rho_{xz} = -0.0033, \quad \alpha = 0.7937652.$

Table 6.6: MSE and PRE of Competing Estimators

Sl. No.	**Estimator**	$MSE/(\theta\bar{Y}^2)$	**Percentage Gain in Efficiency of $\bar{y}'_{rp}$**
1	$\bar{y}$	0.504	141.13
2	$\bar{y}_r$	0.245	17.43
3	$\bar{y}_p$	0.494	136.02
4	$\bar{y}_{rp}$	0.236	12.92
5	$\bar{y}'_{rp}$	**0.209**	0.00

VII. CONCLUSION

The importance of survey sampling in statistical analysis cannot be overemphasized. Researches are being continuously carried out with a view to improving upon the existing findings. Use of auxiliary information has been a shot in the arm to enhance the efficiency of an estimator over the last few decades. Ratio and regression methods of estimation are the best examples where auxiliary information has proved its worth.

We have, in this paper, attempted to go a step forward by proposing a new ratio-cum-product estimator for estimating population mean. The estimator fares better than other four competing estimators, i.e., mean per unit estimator, the usual ratio estimator, product estimator and ratio-cum-product estimator under certain conditions that hold good in practice. The performance remains unchanged when double sampling or two-phase sampling is carried out in the absence of knowledge on population means of auxiliary variables. Empirical investigations, involving data from real populations, have revealed that the proposed estimator is far more superior to the competing estimators in terms of efficiency, prompting us to use the estimator in practice.

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