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Research Article

Statistical Inference for Rayleigh Distribution Using Ranked Set Sampling: A Bayesian Approach

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Abstract— Working on symmetrical or asymmetrical data is complicated since each requires a different probability density function. Many statistical distributions can be used for these data types, where choosing one should be satisfied with the correct data type. So, we apply the ranked set sampling technique, which is essential in gaining data when dealing with units in a population is expensive. However, their classification is simple according to the variable of interest. The Rayleigh distribution has recently played a crucial role in analyzing symmetrical or asymmetrical complex data sets specifically in modeling claim and risk data used in actuarial and financial studies, and its density can take different symmetric and asymmetric possible shapes. It is proposed in various areas, such as reliability, survival, economics, actuarial science, and insurance. In this paper, we provide Bayesian estimation for a parameter of the Rayleigh distribution based on a simple random sample (SRS) and ranked set sampling (RSS) using two loss functions; the squared error loss function, and the linex loss function. The results of the simulation study showed that the Bayes estimates based on RSS are more efficient than the estimates based on SRS for different sample sizes of (n) and different values of the parameter α.

Keywords— Bayesian analysis; Linex loss function; Ranked set sampling; Rayleigh distribution; Squared error loss function; Simulation Study.

1. Introduction

Lord Rayleigh introduced the Rayleigh distribution [1], and it is now widely used in many research fields, including acoustics, communication engineering, survival analysis and reliability theory, economics, actuarial science, insurance, and life testing of electro-vacuum devices. This distribution's failure rate is a linear function of time, which is one of its key characteristics.

Compared to the exponential distribution, the Rayleigh distribution's dependability function drops at a far faster pace. This distribution's relevance in real-life scenarios is noteworthy since it has relationships with several distributions, including the Chi-square, Weibull, and generalized extreme value distributions [2]. Recent years have seen much research on the estimate, prediction, and several other conclusions regarding the Rayleigh distribution; see e.g. [3-5].

On the other hand, the well-known data combining method is the simple random sampling (SRS) design. Under these conditions, we may use ranked sampling approaches to analyze several representative samples from the attached population and restore the statistical inference's efficacy. Cost-effective sampling is essential in many contexts, including fisheries and medical research, especially when measuring the feature of interest is costly [6].

Ranking set sampling (RSS) planning was introduced by McIntyre [7] as a contemporary method for SRS in subjects where the benefit variable is expensive or difficult to manage. Numerous analyses have determined that the RSS approach is a better fit than the SRS. To learn more about these two approaches, see [8].

In this paper[1], we investigate the SRS and RSS, using Bayes estimation when the underlying distribution is Rayleigh. We recall that a random variable X has a Rayleigh distribution with parameter $\alpha > 0$; the probability density function (PDF) of Rayleigh distribution is;

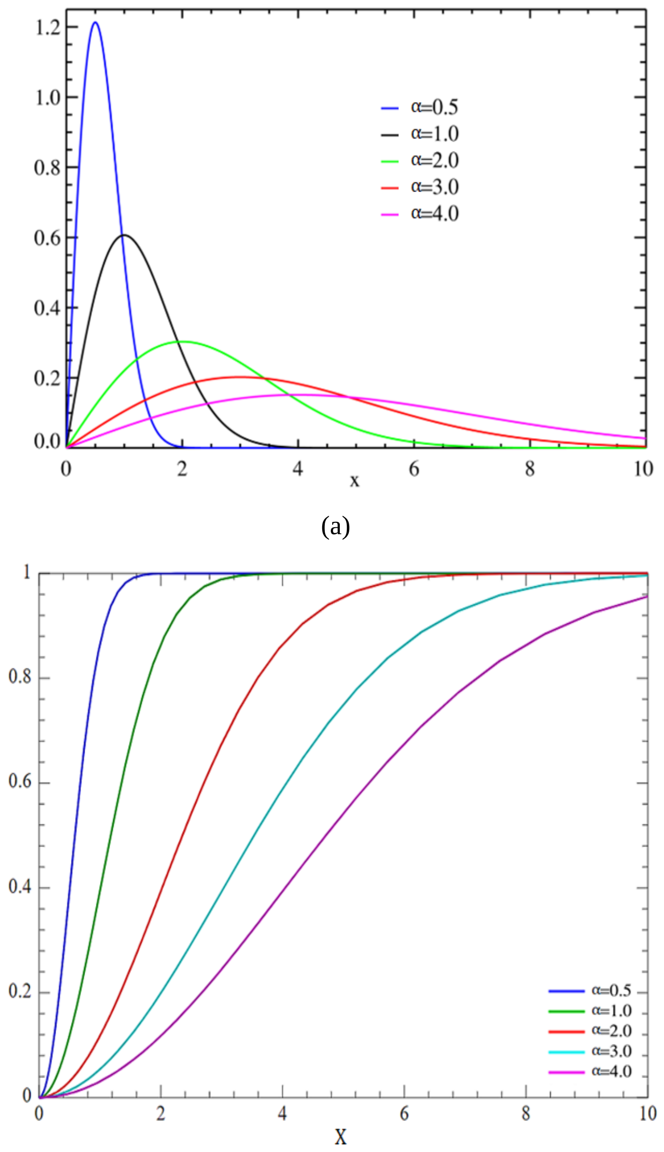
$$f(x) = \alpha x \exp\left[\frac{-\alpha x^2}{2}\right],\ \alpha > 0, x \geq 0, \tag{1}$$

and the corresponding cumulative distribution function (CDF) of Rayleigh is;

[1] This paper is extracted from a master's thesis entitled "Bayesian Analysis Using Ranked Set Sampling", in King Saud University, Saudi Arabia.

$$F(x;\alpha,\beta) = 1 - \exp\left[\frac{-\alpha x^2}{2}\right], \ \alpha > 0, x \geq 0. \tag{2}$$

Figure 1, presents graphical representation of PDF and CDF for different values of the parameter (α) for the Rayleigh distribution.



(a)

(b)

Figure 1: PDF (a) and CDF (b) of the Rayleigh distribution.

This distribution is considered as one of continuous distributions. Rayleigh distribution is a special case of Weibull distribution when $\beta = 2$ and it has various applications in different aspects, this distribution is widely used in communication engineering, reliability analysis and statistics.

The remainder of this paper is organized as follows; Section 2 contains the Bayes estimation based on SRS. Section 3 offers the Bayes estimation based on RSS. Section 4, introduces a simulation study and comparisons. In Section 5, the main conclusions obtained from this paper are discussed.

2. Bayes Estimator Based on SRS

In this section, the Bayes estimates based on SRS and RSS are derived for the unknown parameter (α) of the Rayleigh distribution. The Bayes estimates are obtained based on a conjugate prior and a non-informative prior for the parameter of this model. This is done with respect to both symmetric loss function (squared error loss), and asymmetric loss function (Linear-exponential; LINEX), for more details about asymmetric loss functions, see [9, 10].

Let $\pi^*(\alpha \mid \underline{x})$ and $\pi^*(\alpha \mid \underline{y})$ is the posterior density of α given simple random sampling SRS$(\underline{X})$ and posterior density of α given ranked set sampling RSS$(\underline{Y})$, respectively.

2.1 Conjugate prior for (α)

In this subsection, Let $X_1, X_2, \cdots, X_n$ be independent and identically distributed (IID) random variables from Rayleigh distribution with parameter α and we assume gamma is the conjugate prior of Rayleigh density.

The density of gamma random variables with parameters a,b can be given as;

$$\pi(\alpha) = \frac{b^a}{\Gamma(a)} \alpha^{(a-1)} \exp[-b\alpha] \ , \qquad \alpha > 0\ , \ a, b > 0. \tag{3}$$

The posterior density of parameter α based on SRS can be given as following;

$$\begin{aligned} g_C(\alpha \mid \underline{x}) &= \frac{\pi(\alpha) \prod_{i=1}^{n} f(x_i)}{\int_\alpha \pi(\alpha) \prod_{i=1}^{n} f(x_i) d\alpha}, \\ &= \frac{\frac{b^a}{\Gamma(a)} \alpha^{(a-1)} \exp(-b\alpha) \alpha^n \prod_{i=1}^{n} (x_i) e^{-\frac{\alpha}{2} \sum_{i=1}^{n} x_i^2}}{\int_0^\infty \frac{b^a}{\Gamma(a)} \alpha^{a+n-1} \prod_{i=1}^{n} (x_i) e^{-\alpha(b + \frac{1}{2} \sum_{i=1}^{n} x^2)} d\alpha}, \\ &= \frac{\alpha^{a+n-1} e^{-\alpha(b+\frac{1}{2}\sum_{i=1}^{n} x_i^2)}}{\int_0^\infty \alpha^{a+n-1} e^{-\alpha(b+\frac{1}{2}\sum_{i=1}^{n} x_i^2)} d\alpha}. \end{aligned} \tag{4}$$

Therefore, the final pattern for the posterior becomes as following;

$$g_C(\alpha \mid \underline{x}) = \frac{\alpha^{a+n-1} e^{-\alpha(b+\frac{1}{2}\sum_{i=1}^{n} x_i^2)} (b + \frac{1}{2}\sum_{i=1}^{n} x_i^2)^{a+n}}{\Gamma(a+n)}. \tag{5}$$

The Bayesian estimation of a using squared error loss function is;

$$\begin{aligned}\alpha^*_{RCS} &= E(\alpha \mid \underline{X}),\\ &= \int_0^\infty \alpha g(\alpha \mid \underline{x}) d\alpha,\\ &= \frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n}\int_0^\infty \alpha^{a+n} e^{-\alpha(b+\frac{1}{2}\sum_{i=1}^n x_i^2)} d\alpha}{\Gamma(a+n)},\\ &= \frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n}}{\Gamma(a+n)}\int_0^\infty \alpha^{(a+n)} e^{-\alpha(b+\frac{1}{2}\sum_{i=1}^n x_i^2)} d\alpha,\\ &= \frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{(a+n)}\Gamma(a+n+1)(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{-(a+n+1)}}{\Gamma(a+n)},\\ &= \frac{(a+n)(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n}}{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n+1}}.\end{aligned} \tag{6}$$

Therefore, the Bayes estimator based on SRS under symmetric (squared error) loss function, when there is information prior for parameter a, of Rayleigh distribution, becomes as following;

$$\alpha^*_{RCS} = \frac{a+n}{b+\frac{1}{2}\sum_{i=1}^n x_i^2}. \tag{7}$$

The Bayesian estimation for the parameter (α) of Rayleigh distribution using linear exponential (LINEX) loss function is;

$$\begin{aligned}\alpha^*_{RCL}(\underline{x}) &= -\frac{1}{c}\ln E(e^{-c\alpha}),\\ &= -\frac{1}{c}\ln\left[\int_0^\infty \frac{e^{-c\alpha}\alpha^{a+n-1}(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n} e^{-\alpha(b+\frac{1}{2}\sum_{i=1}^n x_i^2)} d\alpha}{\Gamma(a+n)}\right],\\ &= -\frac{1}{c}\ln\left[\frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n}}{\Gamma(a+n)}\int_0^\infty \alpha^{a+n-1} e^{-\alpha(b+\frac{1}{2}\sum_{i=1}^n x_i^2+c)} d\alpha\right],\\ &= -\frac{1}{c}\ln\left[\frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n}(\Gamma(a+n))}{(\Gamma(a+n))(b+\frac{1}{2}\sum_{i=1}^n x_i^2+c)^{a+n}}\right],\\ &= -\frac{1}{c}\ln\left[\frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)^{a+n}}{(b+\frac{1}{2}\sum_{i=1}^n x_i^2+c)^{a+n}}\right].\end{aligned}$$

Then;

$$\alpha^*_{RCL}(\underline{x}) = -\frac{1}{c}\ln\left[\frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)}{(b+\frac{1}{2}\sum_{i=1}^n x_i^2+c)}\right]^{a+n}. \tag{8}$$

Thus, the Bayes estimator based on SRS under asymmetric (LINEX) loss function, when there is information prior for parameter (α) of the Rayleigh distribution becomes as following;

$$\alpha^*_{RCL}(\underline{x}) = -\frac{(a+n)}{c}\ln\left[\frac{(b+\frac{1}{2}\sum_{i=1}^n x_i^2)}{(b+\frac{1}{2}\sum_{i=1}^n x_i^2+c)}\right], \quad c \neq 0. \tag{9}$$

2.2 Non-Informative prior for (α)

Now, we are using the non-informative prior distribution of the parameter a under SRS as we used it earlier in the previous chapter. This relation can be obtained by;

$$g(\alpha) \propto \sqrt{I(\alpha)},$$

where I (α) is the Fisher information for Rayleigh distribution, Thus, we obtained that;

$$g(\alpha) \propto \frac{1}{\alpha}, \qquad \alpha > 0.$$

Proof:

It is known that;

$$I(\alpha) = E\left[-\frac{\partial^2 \ln \mathcal{L}f(X;\alpha)}{\partial\alpha^2}\right], \tag{10}$$

where $\mathcal{L}$ denotes the likelihood function.

Firstly;

$$\ln \mathcal{L}f(X;\alpha) = n\ln\alpha + \sum_{i=1}^n \ln x_i - \frac{1}{2}\alpha\sum_{i=1}^n x_i^2,$$

$$\frac{\partial \ln \mathcal{L}f(X;\alpha)}{\partial\alpha} = \frac{n}{\alpha} - \frac{1}{2}\sum_{i=1}^n x_i^2,$$

$$\frac{\partial^2 \ln \mathcal{L}f(X;\alpha)}{\partial\alpha^2} = -\frac{n}{\alpha^2}.$$

Finally;

$$\begin{aligned}I(\alpha) &= E\left[-\frac{\partial^2 \ln \mathcal{L}f(X;\alpha)}{\partial\alpha^2}\right],\\ &= E\left[\frac{n}{\alpha^2}\right],\\ &= \frac{n}{\alpha^2}.\end{aligned}$$

Therefore;

$$g(\alpha) \propto \sqrt{I(\alpha)},$$

$$g(\alpha) \propto \frac{1}{\alpha} \quad , \qquad \alpha > 0.$$

Using the likelihood function and the prior distribution of the parameter (α), the posterior distribution of (α) can be written as;

$$\begin{aligned} g_J(\alpha \mid \underline{x}) &= \frac{(\frac{1}{\alpha}) \prod_{i=1}^{n} f(x_i \mid \alpha)}{\int_0^\infty (\frac{1}{\alpha}) \prod_{i=1}^{n} f(x_i \mid \alpha) d\alpha} , \\ &= \frac{(\frac{1}{\alpha})(\alpha)^n (\prod_{i=1}^{n} x_i) e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2}}{\int_0^\infty \alpha^{(n-1)} (\prod_{i=1}^{n} x_i) e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} d\alpha} , \end{aligned}$$

Then;

$$= \frac{\alpha^{(n-1)} e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} (\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n}{\Gamma(n)}$$

We can noted that the posterior distribution of (α) is gamma distribution with parameters $\left(n, \frac{1}{2}\sum_{i=1}^{n} x_i^2\right)$.

The Bayesian estimation of the parameter (α), using squared error loss function is;

$$\begin{aligned} \alpha^*_{RNS} &= E(\alpha|\underline{X}) , \\ &= \int_0^\infty \frac{\alpha^n e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} (\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n}{\Gamma(n)} d\alpha , \\ &= \frac{(\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n}{\Gamma(n)} \int_0^\infty \alpha^n e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2} d\alpha , \\ &= \frac{(\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n \Gamma(n+1)}{\Gamma(n)(\frac{1}{2}\sum_{i=1}^{n} x_i^2)^{n+1}} , \\ &= \frac{n}{\frac{1}{2}\sum_{i=1}^{n} x_i^2} . \end{aligned} \tag{11}$$

This is the Bayes estimator based on SRS using symmetric loss function when little or no prior information is available for the parameter (α).

Now, we derive the Bayes estimator of the parameter (α), using LINEX loss function based on SRS;

$$\begin{aligned} \alpha^*{}_{RNL} &= -\frac{1}{c}\ln[E(e^{-c\alpha})] , \\ &= -\frac{1}{c}\ln\left[\int_0^\infty \frac{e^{-c\alpha}\alpha^{n-1}(\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n e^{-\frac{\alpha}{2}\sum_{i=1}^{n} x_i^2}}{\Gamma(n)} d\alpha\right] , \\ &= -\frac{1}{c}\ln\left[\frac{(\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n}{\Gamma(n)} \int_0^\infty \alpha^{n-1} e^{-\frac{\alpha}{2}(\sum_{i=1}^{n} x_i^2 + c)} d\alpha\right] , \\ &= -\frac{1}{c}\ln\left[\frac{(\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n \Gamma(n)}{\Gamma(n)(\frac{1}{2}\sum_{i=1}^{n} x_i^2 + c)^n}\right] , \\ &= -\frac{1}{c}\ln\left[\frac{(\frac{1}{2}\sum_{i=1}^{n} x_i^2)^n}{(\frac{1}{2}\sum_{i=1}^{n} x_i^2 + c)^n}\right] , \\ &= -\frac{1}{c}\ln\left[\frac{\frac{1}{2}\sum_{i=1}^{n} x_i^2}{\frac{1}{2}\sum_{i=1}^{n} x_i^2 + c}\right]^n . \end{aligned}$$

Then, the Bayes estimator based on simple random sampling using asymmetric (LINEX) loss function when there is not information prior for the parameter a of Rayleigh distribution is given by;

$$\alpha^*{}_{RNL} = -\frac{n}{c}\ln\left[\frac{\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n} x_i^2 + 2c}\right] \quad , \quad c \neq 0. \tag{12}$$

3. Bayes Estimator Based on RSS

Assume that the variable of interest has a density function $f(x \mid \alpha)$, given by Equation (1) and a distribution function $F(x \mid \alpha)$, given by Equation (2), and α has a prior density function $\pi(\alpha)$.

Let $X_1, X_2, \cdots, X_n$ be a SRS and $X_{11}, X_{12}, \cdots, X_{1n}; X_{21}, X_{22}, \cdots, X_{2n}; \cdots; X_{n1}, X_{n2}, \cdots, X_{nn}$ be the visual (judgment) order statistics of n sets of simple random samples each set of size (n).

We will assume throughout the study that there is no error in ranking, i.e. the visual ranking is the same as the actual ranking.

Definition: *Let $Y_{i1}, Y_{i2}, \cdots, Y_{in}$ be a set of order statistics, where Y_{i1} is taken from the first set, Y_{i2} is taken from the second set and Y_{in} is taken from the last set. $(Y_{11}, Y_{22}, \cdots, Y_{nn})$ is called a balanced RSS (BRSS). The density function of Y_i is;*

$$g(y_i \mid \alpha) = \frac{n!}{(i-1)!(n-i)!}[F(y_i \mid \alpha)]^{i-1}[1-F(y_i \mid \alpha]^{n-i} f(y_i \mid \alpha). \tag{13}$$

From Equations (1) and (2) in Equations (13), then we have;

$$g(y_i \mid \alpha) = \sum_{k=0}^{i-1} i \binom{n}{i}\binom{i-1}{k}(-1)^k \, \alpha y_i e^{-\frac{1}{2}\alpha y_i^2(k+n-i+1)}. \tag{14}$$

The joint density is given by;

$$\begin{aligned} g(\underline{y} \mid \alpha) &= \prod_{i=1}^{n} g(y_i \mid \alpha)\,, \\ &= \prod_{i=1}^{n}\sum_{k=0}^{i-1}[i\binom{n}{i}\binom{i-1}{k}(-1)^k]\alpha y_i e^{-\frac{1}{2}\alpha y_i^2(k+n-i+1)}\,. \end{aligned} \tag{15}$$

From Takahasi and Wakimoto[11], we have;

$$f(x \mid \alpha) = \frac{1}{n}\sum_{i=1}^{n} f_i(x \mid \alpha) \tag{16}$$

Where $f_i(x \mid \alpha)$ is the density of the i^th order statistic in a random sample of size n evaluated at x. Thus;

$$f(\underline{x} \mid \alpha) = \prod_{i=1}^{n} f(x_i \mid \alpha) = \frac{1}{n^n}\sum_{i_1=1}^{n}\sum_{i_2=1}^{n}\cdots\sum_{i_n=1}^{n}\prod_{j=1}^{n} f_{i_j}(x_j \mid \alpha). \tag{17}$$

Then;

$$g(\underline{y} \mid \alpha) = \sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}[\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j}](\alpha)^n(\prod_{j=1}^{n} y_j)e^{-\frac{1}{2}\alpha\sum\limits_{j=1}^{n} y_j^2(i_j+n-j+1)}. \tag{18}$$

3.1 Conjugate prior for (α)

We assume gamma with parameters (a,b) is conjugate prior for the parameter (α), which is as follows;

$$\pi(\alpha) = \frac{b^a}{\Gamma(a)}\alpha^{a-1}e^{-b\alpha}\,, \qquad a,b>0\,,\alpha>0. \tag{19}$$

Then, the posterior density of α is;

$$\begin{aligned} g_C(\alpha|\underline{Y}) &= \frac{\pi(\alpha)g(\underline{y} \mid \alpha)}{\int_0^\infty \pi(\alpha)g(\underline{y} \mid \alpha)d\alpha}\,, \\ &= \frac{\alpha^{n+a-1}e^{-b\alpha}\sum\limits_{i_1=0}^{0}\sum\limits_{i_2=0}^{1}\cdots\sum\limits_{i_n=0}^{n-1}(\prod\limits_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})e^{-\frac{1}{2}\alpha\sum\limits_{j=1}^{n} y_j^2(i_j+n-j+1)}}{\int_0^\infty \alpha^{n+a-1}e^{-b\alpha}\sum\limits_{i_1=0}^{0}\sum\limits_{i_2=0}^{1}\cdots\sum\limits_{i_n=0}^{n-1}(\prod\limits_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})e^{-\frac{1}{2}\alpha\sum\limits_{j=1}^{n} y_j^2(i_j+n-j+1)}d\alpha}\,, \\ &= \frac{\alpha^{n+a-1}e^{-b\alpha}\sum\limits_{i_1=0}^{0}\sum\limits_{i_2=0}^{1}\cdots\sum\limits_{i_n=0}^{n-1}(\prod\limits_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})e^{-\frac{1}{2}\alpha\sum\limits_{j=1}^{n} y_j^2(i_j+n-j+1)}}{\sum\limits_{i_1=0}^{0}\sum\limits_{i_2=0}^{1}\cdots\sum\limits_{i_n=0}^{n-1}(\prod\limits_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(a+n)}{[\frac{1}{2}\sum\limits_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{a+n}}}\,. \end{aligned}$$

Since, the Bayesian estimation of α based on squared error loss function is the posterior mean for the parameter α, and which is as follows;

$$\hat{\alpha}_{RCS} = E(\alpha|\underline{Y}).$$

In mathematical optimization and decision theory, a loss function or cost function (sometimes also called an error function) [12] is a function that maps an event or values of one or more variables onto a real number intuitively representing some "cost" associated with the event. An optimization problem seeks to minimize a loss function. An objective function is either a loss function or its opposite (in specific domains, variously called a reward function, a profit function, a utility function, a fitness function, etc.), in which case it is to be maximized. The loss function could include terms from several levels of the hierarchy.

In statistics, typically a loss function is used for parameter estimation, and the event in question is some function of the difference between estimated and true values for an instance of dataset. Moreover, these techniques are ineffective when the data set comprises outlier values. The parameters must be estimated using the robust estimator to get the best estimation. Many papers in several models explore many robust estimators; see e.g. [13-19].

Thus;

$$\begin{aligned}\hat{\alpha}_{RCS} &= E(\alpha|\underline{Y}) \ ,\\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\int_0^\infty \alpha^{n+a} e^{-\alpha[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]}d\alpha}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(a+n)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{a+n}}} \ ,\\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(a+n+1)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{(a+n+1)}}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(a+n)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{a+n}}} \ .\end{aligned}$$

Therefore, the Bayes estimator based on RSS ranked set sampling using squared error loss function when there is informative prior for the parameter of Rayleigh distribution, is;

$$\hat{\alpha}_{RCS} = \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})(a+n)[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{-(a+n+1)}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{-(a+n)}} \ . \tag{20}$$

To obtain the Bayesian estimation of α based on LINEX loss function. **Firstly**, we need to calculate the posterior expectation of $e^{-c\alpha}$ as follows;

$$\begin{aligned}E(e^{-c\alpha}) &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\int_0^\infty \alpha^{n+a-1} e^{-\alpha[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b+c]}d\alpha}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(a+n)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{a+n}}} \ ,\\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(a+n)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b+c]^{a+n}}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(a+n)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{a+n}}} \ ,\\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{1}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b+c]^{a+n}}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{1}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{a+n}}} \ ,\\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})}\frac{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b+c]^{-(a+n)}}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{-(a+n)}} \ .\end{aligned}$$

Finally, the Bayes estimator of α based on (RSS) and using LINEX when there is informative prior for the parameter α of Rayleigh distribution, given by;

$$\hat{\alpha}_{RCL} = \frac{-1}{c}\ln[E(e^{-c\alpha})] \quad , \qquad c \neq 0.$$

$$\hat{\alpha}_{RCL} = \frac{-1}{c}\ln\left[\frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\,[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b+c]^{-(a+n)}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\,[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+b]^{-(a+n)}}\right]. \tag{21}$$

3.2 Non-informative prior for (α)

The non-informative prior distribution of the parameter α by;

$$g(\alpha) \propto \sqrt{I(\alpha)}. \tag{22}$$

Therefore;

$$g(\alpha) \propto \frac{1}{\alpha} \quad , \quad \alpha > 0\,. \tag{23}$$

The posterior density can be written as;

$$\begin{aligned} g_J(\alpha|\underline{Y}) &= \frac{(\frac{1}{\alpha})g(\underline{y}\mid\alpha)}{\int_0^\infty(\frac{1}{\alpha})g(\underline{y}\mid\alpha)d\alpha}\,, \\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\alpha^{n-1}e^{-\alpha[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]}}{\int_0^\infty\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\alpha^{n-1}e^{-\alpha[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]}d\alpha}\,, \\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\alpha^{n-1}e^{-\alpha[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\Gamma(n)[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]^{(-n)}}\,. \end{aligned}$$

Then, The Bayesian estimation of α using squared error loss function is;

$$\begin{aligned} \hat{\alpha}_{RNS} &= E(\alpha|\underline{Y})\,, \\ &= \frac{\int_0^\infty\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\alpha^{n}e^{-\alpha[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]}d\alpha}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\Gamma(n)[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]^{(-n)}}\,, \\ &= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(n+1)}{[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]^{(n+1)}}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(n)}{[\frac{1}{2}\sum_{j=1}^{n}y_j^2(i_j+n-j+1)]^{n}}}\,. \end{aligned}$$

Thus;

$$\hat{\alpha}_{RNS} = \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})n[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)]^{-(n+1)}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)]^{(-n)}}. \tag{24}$$

To obtain the Bayesian estimation of α based on LINEX loss function.

Firstly, we need to calculate the posterior expectation of $e^{-c\alpha}$ as follows;

$$E(e^{-c\alpha}) = \frac{\int_0^\infty \sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\alpha^{n-1}e^{-\alpha[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+c]}d\alpha}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\Gamma(n)[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)]^{(-n)}},$$

$$= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(n)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+c]^{(n)}}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{\Gamma(n)}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)]^{n}}},$$

$$= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{1}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+c]^{(n)}}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})\frac{1}{[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)]^{n}}},$$

$$= \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+c]^{-n}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)]^{-n}}.$$

Finally, the Bayes estimator of α based on ranked set sampling and using LINEX loss function and also when there is not informative prior for the parameter α of Rayleigh distribution is given by;

$$\hat{\alpha}_{RNL} = \frac{-1}{c}\ln\left[\frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)+c]^{-n}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_n=0}^{n-1}(\prod_{j=1}^{n} j\binom{n}{j}\binom{j-1}{i_j}(-1)^{i_j})[\frac{1}{2}\sum_{j=1}^{n} y_j^2(i_j+n-j+1)]^{-n}}\right], \quad c \neq 0. \tag{25}$$

4. Simulation Study and Comparisons

Since we are not aware of any the estimation results concerning of the Bayes estimations based on the ranked set sampling values and comparison that with results of Bayes estimations based on simple random sampling, so a simulation study was conducted in order to compare results of the Bayes estimates. Simulations using the following algorithm, by using MATLAB program.

4.1 Algorithm for simulation study

1. Generate random sample of size *n* from Rayleigh distribution using Equation (2) by inverse transform technique when α = 0.5, 1 and 1.5.
2. Calculate the Bayes estimators that we have obtained in Equations (7), (9), (11) and (12).
3. Now generate random sample of size n^2 from Rayleigh distribution using Equation (2) by inverse transform technique when α = 0.5, 1 and 1.5.
   - These items are the randomly divided into *n* sets of *n* units each.
   - Arrange the items from the smaller to the larger for each set.
   - We select the item with the smallest ranking, $X_{[1]}$ for measurement from the first set.
   - From the second set we select the item with the smallest ranking, $X_{[2]}$, we continue in this manner until we have ranked the items in the nth set and selected the item with the largest ranking, $X_{[n]}$, this means that we take the elements of the diagonal.
   - Calculate the Bayes estimators based on ranked set sampling which is Equations (20), (21), (24) and (25).

4. Using 1000 iterations to study the estimates of the scale parameter; mean squared error (MSE) of the estimates, variance of the estimates and bias of the estimates.
5. Final, we calculate bias, variance and MSE of Bayes estimators based on SRS and RSS.

4.2 Simulation Results and Discussion

- Tables 1-3 show numerical comparisons between the Bayes estimates based on the simple random sampling and the ranked set sampling, when [α = 0.5 in Table 1], [α = 1 in Table 2], and [α = 1.5 in Table 3].
- Tables 4-6 show numerical comparisons between the bias of the Bayes estimates based on the simple random sampling and the ranked set sampling, when [α = 0.5 in Table 4], [α = 1 in Table 5], and [α = 1.5 in Table 6], where the bias is defined by the difference between the true value of the parameter and the Bayes estimator.

The bias of the estimator α^* is noted by *bias** and the bias of the estimator $\hat{\alpha}$ is noted by $bi\hat{a}s$

- Tables 7-9 show numerical comparisons between the variance of the Bayes estimates based on the ranked set sampling and the simple random sampling, when [α = 0.5 in Table 7], [α = 1 in Table 8], and [α = 1.5 in Table 9].

The variance of the estimator α^* is noted by Var^* and the variance of the estimator $\hat{\alpha}$ is noted by $V\hat{a}r$.

Table 1. Estimates of α for the Rayleigh distribution when α = 0.5

Based on SRS							
n	α^*_{RNS}	c	α^*_{RNL}	(a,b)	α^*_{RCS}	c	α^*_{RCL}
3	0.736	0.1	0.721	(1,0.5)	0.862	0.1	0.850
		-0.1	0.753			-0.1	0.876
				(1.5,1)	0.804	0.1	0.794
						-0.1	0.813
4	0.647	0.1	0.640	(1,0.5)	0.737	0.1	0.730
		-0.1	0.654			-0.1	0.744
				(1.5,1)	0.752	0.1	0.746
						-0.1	0.759
5	0.609	0.1	0.604	(1,0.5)	0.678	0.1	0.674
		-0.1	0.614			-0.1	0.684
				(1.5,1)	0.687	0.1	0.683
						-0.1	0.691
Based on RSS							
n	$\hat{\alpha}_{RNS}$	c	$\hat{\alpha}_{RNL}$	(a,b)	$\hat{\alpha}_{RCS}$	c	$\hat{\alpha}_{RCL}$
3	0.604	0.1	0.600	(1,0.5)	0.659	0.1	0.655
		-0.1	0.608			-0.1	0.663
				(1.5,1)	0.680	0.1	0.676
						-0.1	0.684
4	0.552	0.1	0.550	(1,0.5)	0.605	0.1	0.603
		-0.1	0.554			-0.1	0.607
				(1.5,1)	0.622	0.1	0.620
						-0.1	0.624
5	0.531	0.1	0.530	(1,0.5)	0.569	0.1	0.568
		-0.1	0.532			-0.1	0.571
				(1.5,1)	0.567	0.1	0.566
						-0.1	0.568

- Tables 10-12 show numerical comparisons between the MSE of the Bayes estimates based on the ranked set sampling and the simple random sampling, when [α = 0.5 in Table 10], [α = 1 in Table 11], and [α = 1.5 in Table 12], where the MSE is given by;

$$\text{MSE} = [\text{bias}]^2 + \text{variance}. \quad (26)$$

The MSE of the estimator α^* is noted by Mse^* and the MSE of the estimator $\hat{\alpha}$ is noted by $M\hat{S}E$

- Tables 13-15 show the relative efficiency of MSE of Bayes estimates based on ranked set sampling with respect to MSE of Bayes estimates based on SRS, when α = 0.5, α = 1 and α = 1.5 since eff.= MSE^* / $M\hat{S}E$.

Table 2. Estimates of α for the Rayleigh distribution when α = 1

Based on SRS							
n	α^*_{RJS}	c	α^*_{RJL}	(a,b)	α^*_{RCS}	c	α^*_{RCL}
3	1.526	0.1	1.446	(1, 0.5)	1.428	0.1	1.397
		-0.1	1.547			-0.1	1.462
				(1.5, 1)	1.320	0.1	1.297
						-0.1	1.343
4	1.342	0.1	1.310	(1, 0.5)	1.384	0.1	1.361
		-0.1	1.378			-0.1	1.409
				(1.5, 1)	1.296	0.1	1.278
						-0.1	1.314
5	1.271	0.1	0.604	(1, 0.5)	1.267	0.1	1.251
		-0.1	0.614			-0.1	1.283
				(1.5, 1)	1.224	0.1	1.211
						-0.1	1.238
Based on RSS							
n	$\hat{\alpha}_{RJS}$	c	$\hat{\alpha}_{RJL}$	(a,b)	$\hat{\alpha}_{RCS}$	c	$\hat{\alpha}_{RCL}$
3	1.237	0.1	1.219	(1, 0.5)	1.253	0.1	1.239
		-0.1	1.257			-0.1	1.268
				(1.5, 1)	1.212	0.1	1.200
						-0.1	1.224
4	1.143	0.1	1.134	(1, 0.5)	1.166	0.1	1.158
		-0.1	1.151			-0.1	1.174
				(1.5, 1)	1.136	0.1	1.129
						-0.1	1.143
5	1.077	0.1	1.072	(1, 0.5)	1.113	0.1	1.109
		-0.1	1.081			-0.1	1.118
				(1.5, 1)	1.107	0.1	1.103
						-0.1	1.112

Table 3. Estimates of α for the Rayleigh distribution when α = 1.5

Based on SRS							
n	α^*_{RJS}	c	α^*_{RJL}	(a,b)	α^*_{RCS}	c	α^*_{RCL}
3	2.251	0.1	2.114	(1, 0.5)	1.926	0.1	1.872
		-0.1	2.470			-0.1	1.986
				(1.5, 1)	1.689	0.1	1.654
						-0.1	1.726
4	1.954	0.1	1.890	(1, 0.5)	1.943	0.1	1.899
		-0.1	2.029			-0.1	1.990
				(1.5, 1)	1.691	0.1	1.663
						-0.1	1.721
5	1.828	0.1	1.786	(1, 0.5)	1.793	0.1	1.763
		-0.1	1.874			-0.1	1.990
				(1.5, 1)	1.656	0.1	1.633
						-0.1	1.681
Based on RSS							
n	$\hat{\alpha}_{RJS}$	c	$\hat{\alpha}_{RJL}$	(a,b)	$\hat{\alpha}_{RCS}$	c	$\hat{\alpha}_{RCL}$
3	1.237	0.1	1.219	(1, 0.5)	1.253	0.1	1.761
		-0.1	1.257			-0.1	1.820
				(1.5, 1)	1.212	0.1	1.643
						-0.1	1.688
4	1.143	0.1	1.134	(1, 0.5)	1.166	0.1	1.707
		-0.1	1.151			-0.1	1.741
				(1.5, 1)	1.136	0.1	1.604
						-0.1	1.631
5	1.077	0.1	1.072	(1, 0.5)	1.113	0.1	1.627
		-0.1	1.081			-0.1	1.648
				(1.5, 1)	1.107	0.1	1.561
						-0.1	1.579

5. Conclusion and Future Scope

1. The Bayes estimates of the parameter α of the Rayleigh distribution based on ranked set sampling (RSS) using

LINEX or squared error loss functions with Jeffrey's' prior are better than The Bayes estimates of the parameter α of the Rayleigh distribution based on simple random sampling (SRS) using LINEX or squared error loss functions with Jeffrey's', to different values of α

Table 4. Bias of the estimates of α for the Rayleigh distribution when α = 0.5

			Based on SRS				
n	$bias^*_{RJS}$	c	$bias^*_{RJL}$	(a,b)	$bias^*_{RCS}$	c	$bias^*_{RCL}$
3	-0.236	0.1	-0.221	(1, 0.5)	-0.362	0.1	-0.350
		-0.1	-0.253			-0.1	-0.376
				(1.5, 1)	-0.304	0.1	-0.294
						-0.1	-0.313
4	-0.147	0.1	-0.140	(1, 0.5)	-0.237	0.1	-0.230
		-0.1	-0.154			-0.1	-0.244
				(1.5, 1)	-0.252	0.1	-0.246
						-0.1	-0.259
5	-0.109	0.1	-0.104	(1, 0.5)	-0.178	0.1	-0.174
		-0.1	-0.114			-0.1	-0.184
				(1.5, 1)	-0.187	0.1	-0.183
						-0.1	-0.191
			Based on RSS				
n	$\hat{bias}_{RJS}$	c	$\hat{bias}_{RJL}$	(a,b)	$\hat{bias}_{RCS}$	c	$\hat{bias}_{RCL}$
3	-0.104	0.1	-0.100	(1, 0.5)	-0.159	0.1	-0.155
		-0.1	-0.108			-0.1	-0.163
				(1.5, 1)	-0.108	0.1	-0.176
						-0.1	-0.184
4	-0.052	0.1	-0.050	(1, 0.5)	-0.105	0.1	-0.103
		-0.1	-0.054			-0.1	-0.107
				(1.5, 1)	-0.122	0.1	-0.120
						-0.1	-0.124
5	-0.031	0.1	-0.030	(1, 0.5)	-0.069	0.1	-0.068
		-0.1	-0.032			-0.1	-0.712
				(1.5, 1)	-0.067	0.1	-0.066
						-0.1	-0.068

Table 5. Bias of the estimates of α for the Rayleigh distribution when α = 1

			Based on SRS				
n	$bias^*_{RJS}$	c	$bias^*_{RJL}$	(a,b)	$bias^*_{RCS}$	c	$bias^*_{RCL}$
3	-0.526	0.1	-0.446	(1, 0.5)	-0.428	0.1	-0.397
		-0.1	-0.547			-0.1	-0.462
				(1.5, 1)	-0.320	0.1	-0.297
						-0.1	-0.343
4	-0.342	0.1	-0.310	(1, 0.5)	-0.384	0.1	-0.361
		-0.1	-0.378			-0.1	-0.409
				(1.5, 1)	-0.296	0.1	-0.278
						-0.1	-0.314
5	-0.271	0.1	-0.251	(1, 0.5)	-0.267	0.1	-0.251
		-0.1	-0.292			-0.1	-0.283
				(1.5, 1)	-0.224	0.1	-0.211
						-0.1	-0.238
			Based on RSS				
n	$\hat{bias}_{RJS}$	c	$\hat{bias}_{RJL}$	(a,b)	$\hat{bias}_{RCS}$	c	$\hat{bias}_{RCL}$
3	-0.237	0.1	-0.219	(1, 0.5)	-0.253	0.1	-0.239
		-0.1	-0.257			-0.1	-0.268
				(1.5, 1)	-0.212	0.1	-0.200
						-0.1	-0.224
4	-0.143	0.1	-0.134	(1, 0.5)	-0.166	0.1	-0.158
		-0.1	-0.151			-0.1	-0.174
				(1.5, 1)	-0.136	0.1	-0.129
						-0.1	-0.143
5	-0.077	0.1	-0.077	(1, 0.5)	-0.113	0.1	-0.109
		-0.1	-0.081			-0.1	-0.118
				(1.5, 1)	-0.107	0.1	-0.103
						-0.1	-0.112

2. The Bayes estimates of the parameter α of the Rayleigh distribution based on ranked set sampling (RSS) using LINEX or squared error loss functions with conjugate prior are better than the Bayes estimates of the parameter α of the Rayleigh distribution based on simple random sampling (SRS) using LINEX or squared error loss functions with conjugate prior, to different values of α.

3. The MSE of the Bayes estimates based ranked set sampling using LINEX loss function when e is positive is less than the MSE of the Bayes estimates based ranked set sampling using LINEX loss function when c is negative, using Jeffrey's' prior, as well as the bias.

Table 6. Bias of the estimates of α for the Rayleigh distribution when α = 1.5

			Based on SRS				
n	$bias^*_{RJS}$	c	$bias^*_{RJL}$	(a,b)	$bias^*_{RCS}$	c	$bias^*_{RCL}$
3	-0.751	0.1	-0.614	(1, 0.5)	-0.426	0.1	-0.372
		-0.1	-0.970			-0.1	-0.486
				(1.5, 1)	-0.189	0.1	-0.154
						-0.1	-0.226
4	-0.454	0.1	-0.390	(1, 0.5)	-0.443	0.1	-0.399
		-0.1	-0.529			-0.1	-0.490
				(1.5, 1)	-0.191	0.1	-0.163
						-0.1	-0.221
5	-0.328	0.1	-0.286	(1, 0.5)	-0.293	0.1	-0.263
		-0.1	-0.374			-0.1	-0.325
				(1.5, 1)	-0.156	0.1	-0.133
						-0.1	-0.181
			Based on RSS				
n	$\hat{bias}_{RJS}$	c	$\hat{bias}_{RJL}$	(a,b)	$\hat{bias}_{RCS}$	c	$\hat{bias}_{RCL}$
3	-0.271	0.1	-0.236	(1, 0.5)	-0.290	0.1	-0.261
		-0.1	-0.308			-0.1	-0.320
				(1.5, 1)	-0.165	0.1	-0.143
						-0.1	-0.188
4	-0.183	0.1	-0.165	(1, 0.5)	-0.224	0.1	-0.207
		-0.1	-0.201			-0.1	-0.241
				(1.5, 1)	-0.117	0.1	-0.104
						-0.1	-0.131
5	-0.093	0.1	-0.083	(1, 0.5)	-0.137	0.1	-0.127
		-0.1	-0.104			-0.1	-0.148
				(1.5, 1)	-0.070	0.1	-0.061
						-0.1	-0.079

Table 7. Variance of the estimates of α for the Rayleigh distribution when α = 0.5

			Based on SRS				
n	Var^*_{RJS}	c	Var^*_{RJL}	(a,b)	Var^*_{RCS}	c	Var^*_{RCL}
3	0.396	0.1	0.347	(1,0.5)	0.320	0.1	0.297
		-0.1	0.466			-0.1	0.346
				(1.5,1)	0.192	0.1	0.182
						-0.1	0.203
4	0.166	0.1	0.156	(1,0.5)	0.174	0.1	0.166
		-0.1	0.176			-0.1	0.182
				(1.5,1)	0.146	0.1	0.140
						-0.1	0.152
5	0.116	0.1	0.111	(1,0.5)	0.106	0.1	0.103
		-0.1	0.121			-0.1	0.110
				(1.5,1)	0.097	0.1	0.094
						-0.1	0.100
			Based on RSS				
n	$\hat{Var}_{RJS}$	c	$\hat{Var}_{RJL}$	(a,b)	$\hat{Var}_{RCS}$	c	$\hat{Var}_{RCL}$
3	0.088	0.1	0.085	(1,0.5)	0.098	0.1	0.095
		-0.1	0.091			-0.1	0.102
				(1.5,1)	0.086	0.1	0.084
						-0.1	0.089
4	0.040	0.1	0.039	(1,0.5)	0.051	0.1	0.050
		-0.1	0.041			-0.1	0.052
				(1.5,1)	0.046	0.1	0.045
						-0.1	0.046
5	0.023	0.1	0.022	(1,0.5)	0.025	0.1	0.025
		-0.1	0.023			-0.1	0.026
				(1.5,1)	0.023	0.1	0.023
						-0.1	0.024

4. The results of the simulation showed that the MSE of the Bayes estimates based on ranked set sampling (RSS) using LINEX or squared error loss functions with conjugate prior or with Jeffrey's' prior are always less than the MSE of the Bayes estimates based on simple random sampling (SRS) using LINEX or squared error loss functions with conjugate prior, or with Jeffrey's'

prior, for different sample size of n and for different values of the parameter α, and so, would the bias and the variance of the Bayes estimates based on ranked set sampling smaller than the bias and the variance of the Bayes estimates based on simple random sampling, for all of Bayes estimates.

Table 8. Variance of the estimates of α for the Rayleigh distribution when $\alpha = 1$

n	Var^*_{RJS}	c	Var^*_{RJL}	(a,b)	Var^*_{RCS}	c	Var^*_{RCL}
			Based on SRS				
3	4.397	0.1	2.130	(1,0.5)	0.562	0.1	0.508
		-0.1	2.122			-0.1	0.626
				(1.5,1)	0.307	0.1	0.286
						-0.1	0.330
4	0.897	0.1	0.780	(1,0.5)	0.467	0.1	0.431
		-0.1	1.059			-0.1	0.509
				(1.5,1)	0.303	0.1	0.286
						-0.1	0.321
5	0.462	0.1	0.429	(1,0.5)	0.296	0.1	0.281
		-0.1	0.499			-0.1	0.313
				(1.5,1)	0.222	0.1	0.212
						-0.1	0.233
			Based on RSS				
n	$\hat{Var}_{RJS}$	c	$\hat{Var}_{RJL}$	(a,b)	$\hat{Var}_{RCS}$	c	$\hat{Var}_{RCL}$
3	0.508	0.1	0.467	(1,0.5)	0.302	0.1	0.287
		-0.1	0.559			-0.1	0.320
				(1.5,1)	0.217	0.1	0.208
						-0.1	0.226
4	0.185	0.1	0.179	(1,0.5)	0.159	0.1	0.155
		-0.1	0.191			-0.1	0.164
				(1.5,1)	0.130	0.1	0.127
						-0.1	0.133
5	0.100	0.1	0.098	(1,0.5)	0.107	0.1	0.105
		-0.1	0.102			-0.1	0.109
				(1.5,1)	0.083	0.1	0.081
						-0.1	0.084

Table 9. Variance of the estimates of α for the Rayleigh distribution when $\alpha = 1.5$

n	Var^*_{RJS}	c	Var^*_{RJL}	(a,b)	Var^*_{RCS}	c	Var^*_{RCL}
			Based on SRS				
3	4.808	0.1	3.085	(1,0.5)	0.855	0.1	0.757
		-0.1	11.465			-0.1	0.977
				(1.5,1)	0.386	0.1	0.355
						-0.1	0.421
4	1.681	0.1	1.379	(1,0.5)	0.764	0.1	0.692
		-0.1	2.196			-0.1	0.851
				(1.5,1)	0.396	0.1	0.344
						-0.1	0.397
5	1.045	0.1	0.925	(1,0.5)	0.543	0.1	0.504
		-0.1	1.200			-0.1	0.587
				(1.5,1)	0.334	0.1	0.314
						-0.1	0.355
			Based on RSS				
n	$\hat{Var}_{RJS}$	c	$\hat{Var}_{RJL}$	(a,b)	$\hat{Var}_{RCS}$	c	$\hat{Var}_{RCL}$
3	0.752	0.1	0.685	(1,0.5)	0.530	0.1	0.495
		-0.1	0.833			-0.1	0.568
				(1.5,1)	0.333	0.1	0.316
						-0.1	0.352
4	0.362	0.1	0.347	(1,0.5)	0.347	0.1	0.332
		-0.1	0.380			-0.1	0.363
				(1.5,1)	0.239	0.1	0.231
						-0.1	0.248
5	0.206	.1	0.201	(1,0.5)	0.190	0.1	0.185
		-0.1	0.212			-01	0.195
				(1.5,1)	0.154	0.1	0.150
						-0.1	0.157

5. The MSE of the Bayes estimates based ranked set sampling using LINEX loss function when *c* is positive is less than the MSE of the Bayes estimates based ranked set sampling using LINEX loss function when *c* is negative, using conjugate prior, as well as the bias.

6. The MSE of the Bayes estimates based ranked set sampling using LINEX loss function when *c* is positive are less than the MSE of the Bayes estimates based ranked set sampling using mean squared error loss function, using conjugate prior, as well as the bias.

Table 10. MSE of the estimates of α for the Rayleigh distribution when $\alpha = 0.5$

n	MSE^*_{RJS}	c	MSE^*_{RJL}	(a,b)	MSE^*_{RCS}	c	MSE^*_{RCL}
			Based on SRS				
3	0.452	0.1	0.397	(1,0.5)	0.452	0.1	0.420
		-0.1	0.530			-0.1	0.488
				(1.5,1)	0.285	0.1	0.269
						-0.1	0.301
4	0.187	0.1	0.176	(1,0.5)	0.230	0.1	0.219
		-0.1	0.200			-0.1	0.242
				(1.5,1)	0.210	0.1	0.201
						-0.1	0.219
5	0.128	0.1	0.122	(1,0.5)	0.138	0.1	0.133
		-0.1	0.134			-0.1	0.144
				(1.5,1)	0.132	0.1	0.127
						-0.1	0.136
			Based on RSS				
n	$\hat{MSE}_{RJS}$	c	$\hat{MSE}_{RJL}$	(a,b)	$\hat{MSE}_{RCS}$	c	$\hat{MSE}_{RCL}$
3	0.099	0.1	0.095	(1,0.5)	0.124	0.1	0.119
		-0.1	0.103			-0.1	0.129
				(1.5,1)	0.119	0.1	0.115
						-0.1	0.123
4	0.043	0.1	0.042	(1,0.5)	0.062	0.1	0.061
		-0.1	0.044			-0.1	0.063
				(1.5,1)	0.061	0.1	0.060
						-0.1	0.062
5	0.023	0.1	0.023	(1,0.5)	0.030	0.1	0.029
		-0.1	0.024			-0.1	0.030
				(1.5,1)	0.028	0.1	0.028
						-0.1	0.029

Table 11. MSE of the estimates of α for the Rayleigh distribution when $\alpha = 1$

n	MSE^*_{RJS}	c	MSE^*_{RJL}	(a,b)	MSE^*_{RCS}	c	MSE^*_{RCL}
			Based on SRS				
3	4.675	0.1	2.329	(1,0.5)	0.746	0.1	0.667
		-0.1	2.421			-0.1	0.840
				(1.5,1)	0.409	0.1	0.375
						-0.1	0.448
4	1.015	0.1	0.876	(1,0.5)	0.615	0.1	0.561
		-0.1	1.202			-0.1	0.676
				(1.5,1)	0.390	0.1	0.363
						-0.1	0.420
5	0.536	0.1	0.492	(1,0.5)	0.367	0.1	0.344
		-0.1	0.585			-0.1	0.393
				(1.5,1)	0.273	0.1	0.257
						-0.1	0.290
			Based on RSS				
n	$\hat{MSE}_{RJS}$	c	$\hat{MSE}_{RJL}$	(a,b)	$\hat{MSE}_{RCS}$	c	$\hat{MSE}_{RCL}$
3	0.565	0.1	0.515	(1,0.5)	0.367	0.1	0.344
		-0.1	0.625			-0.1	0.392
				(1.5,1)	0.262	0.1	0.248
						-0.1	0.277
4	0.205	0.1	0.197	(1,0.5)	0.187	0.1	0.180
		-0.1	0.214			-0.1	0.195
				(1.5,1)	0.148	0.1	0.143
						-0.1	0.154
5	0.106	0.1	0.103	(1,0.5)	0.120	0.1	0.117
		-0.1	0.109			-0.1	0.123
				(1.5,1)	0.094	0.1	0.092
						-0.1	0.097

7. Tables 13, 14 and 15 show that always the relative efficiency of MSE of Bayes estimates based on ranked set sampling are larger than one therefore, ranked set

sampling (RSS) is more efficient than the estimators based on simple random sampling (SRS).

Table 12. MSE of the estimates of α for the Rayleigh distribution when $\alpha = 1.5$

n	MSE^*_{RJS}	c	MSE^*_{RJL}	(a,b)	MSE^*_{RCS}	c	MSE^*_{RCL}
Based on SRS							
3	5.373	0.1	3.462	(1,0.5)	1.037	0.1	0.896
		-0.1	12.407			-0.1	1.215
				(1.5,1)	0.422	0.1	0.379
						-0.1	0.473
4	1.887	0.1	1.531	(1,0.5)	0.961	0.1	0.851
		-0.1	2.476			-0.1	1.091
				(1.5,1)	0.406	0.1	0.371
						-0.1	0.446
5	1.153	0.1	1.007	(1,0.5)	0.629	0.1	0.573
		-0.1	2.476			-0.1	1.091
				(1.5,1)	0.358	0.1	0.332
						-0.1	0.388
Based on RSS							
n	$M\hat{S}E_{RJS}$	c	$M\hat{S}E_{RJL}$	(a,b)	$M\hat{S}E_{RCS}$	c	$M\hat{S}E_{RCL}$
3	0.826	0.1	0.741	(1,0.5)	0.614	0.1	0.564
		-0.1	0.928			-0.1	0.671
				(1.5,1)	0.360	0.1	0.336
						-0.1	0.387
4	0.396	0.1	0.374	(1,0.5)	0.397	0.1	0.375
		-0.1	0.420			-0.1	0422
				(1.5,1)	0.253	0.1	0.241
						-0.1	0.265
5	0.215	0.1	0.208	(1,0.5)	0.209	0.1	0.201
		-0.1	0.223			-0.1	0.217
				(1.5,1)	0.158	0.1	0.154
						-0.1	0.163

Moreover, The relative efficiency under Rayleigh distribution of the Bayesian estimates based on ranked set sampling, defined as follows;

$$\text{eff}_{(RJS)} = \frac{MSE^*_{RJS}}{M\hat{S}E_{RJS}}; \qquad \text{eff}_{(RJL)} = \frac{MSE^*_{RJL}}{M\hat{S}E_{RJL}};$$

$$\text{eff}_{(RCS)} = \frac{MSE^*_{RCS}}{M\hat{S}E_{RCS}}; \qquad \text{eff}_{(RCL)} = \frac{MSE^*_{RCL}}{M\hat{S}E_{RCL}};$$

Table 13. Relative efficiency under Rayleigh distribution when $\alpha = 0.5$

n	$\text{eff}_{(RJS)}$	c	$\text{eff}_{(RJL)}$	(a,b)	$\text{eff}_{(RCS)}$	c	$\text{eff}_{(RCL)}$
3	4.554	0.1	4.153	(1,0.5)	3.637	0.1	3.508
		-0.1	5.142			-0.1	3.783
				(1.5,1)	2.384	0.1	2.327
						-0.1	2.447
4	4.347	0.1	4.172	(1,0.5)	3.693	0.1	3.594
		-0.1	4.554			-0.1	3.802
				(1.5,1)	3.429	0.1	3.349
						-0.1	3.515
5	5.359	0.1	5.160	(1,0.5)	4.599	0.1	4.493
		-0.1	5.557			-0.1	4.690
				(1.5,1)	4.658	0.1	4.567
						-0.1	4.770

Table 14. Relative efficiency under Rayleigh distribution when $\alpha = 1$

n	$\text{eff}_{(RJS)}$	c	$\text{eff}_{(RJL)}$	(a,b)	$\text{eff}_{(RCS)}$	c	$\text{eff}_{(RCL)}$
3	8.267	0.1	4.517	(1,0.5)	2.033	0.1	1.938
		-0.1	3.870			-0.1	2.142
				(1.5,1)	1.562	0.1	1.511
						-0.1	1.617
4	4.936	0.1	4.445	(1,0.5)	3.281	0.1	3.113
		-0.1	5.606			-0.1	3.469
				(1.5,1)	2.625	0.1	2.529
						-0.1	2.728
5	5.037	0.1	4.749	(1,0.5)	3.055	0.1	2.939
		-0.1	5.367			-0.1	3.181
				(1.5,1)	2.880	0.1	2.783
						-0.1	2.980

Table 15. Relative efficiency under Rayleigh distribution when $\alpha = 1.5$

n	$\text{eff}_{(RJS)}$	c	$\text{eff}_{(RJL)}$	(a,b)	$\text{eff}_{(RCS)}$	c	$\text{eff}_{(RCL)}$
3	6.504	0.1	4.670	(1,0.5)	1.689	0.1	1.587
		-0.1	13.359			-0.1	1.810
				(1.5,1)	1.169	0.1	1.125
						-0.1	1.220
4	4.760	0.1	4.089	(1,0.5)	2.415	0.1	2.267
		-0.1	5.887			-0.1	2.587
				(1.5,1)	1.605	0.1	1.534
						-0.1	1.683
5	5.351	0.1	4.843	(1,0.5)	3.006	0.1	2.842
		-0.1	11.077			-0.1	5.020
				(1.5,1)	2.256	0.1	2.157
						-0.1	2.367

Data Availability

There is no data to provide other than that given in the article.

Conflict of Interest

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