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Research Article

Progression of Optimizational Principles in Portfolio Analysis through Isolated Entropic Models

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Abstract— Even if there is a vast array of parametric and non-parametric information models, predictability still emerges to expand further parametric models to strengthen plasticity in the structure under study. Additionally, there looks to be a well-built association sandwiched between information entropy and the theory of "Portfolio Analysis". Moreover, innumerable methodologies of risk assessment, comprising entropy technique, divergence procedure, unified approach, etc. are accessible within the existing corpus of portfolio analysis literature. The present paper is a step towards making progress on some well-known optimizational principles by using new discrete entropic models and then showing how they can be used in portfolio analysis. In addition, the entrenched principle has been elucidated through the support of a well-managed numerical example.

Keywords—Portfolio analysis, Modern portfolio theory, Entropy, Variance, Covariance matrix, Uncertainty

1. Introduction

Portfolio analysis is a quantifiable procedure for selecting the best portfolio to strike an equilibrium bordered by excellent return and the least risk in numerous ambiguous situations. For the selection of an optimal portfolio, the "return of a portfolio" and "risk convoluted in the portfolio" are the greatest momentous matters. Portfolio analysis is an exploration of constituents comprehended in an amalgamation of products through the determination of generating decisions that are expected to advance comprehensive returns. The word relates to a technique that permits an administrator to identify heightened approaches to assign resources with the objective of cumulating profits. This is an endorsed statement that portfolio choice is apprehensive to assign one's wealth surrounded by divergent securities to undertake the investment objective.

In portfolio selection analysis, we adopt the perspective of Markowitz, who pioneered the mean-variance approach [1], [2]. However, this mean-variance model typically generates significant portfolio revenue because the required data-generating process deviates from regularity, limiting its application to robust investment structures. It is well-accredited confirmation that although the variance is merely an average deviation measure of information, it is supposed to be a conjoint and profitable risk measure in such metaphysical models. To address this problem effectively, it is essential to understand diversification models. Encouraged by this impression, some investigators became familiar with Shannon's entropy for evaluating investment extent in the securities because of its operative instrumentation nature [3], [4].

2. Related Work

Numerous well-established entropy models, primarily derived from Shannon's model, are prevalent in information theory literature [3]. This entropic model with amazingly pleasant possessions is defined by the following expression:

$$S(P) = -\sum_{i=1}^{n} p_i \log p_i , \tag{1}$$

where $P = (p_1, p_2, \ldots, p_n)$.

Some supplementary modernizers who contributed a proportion to the enhancement of the portfolio analysis by introducing their distinctive entropic models include Stuart and Markowitz, Whitelaw, Ou, Soyer and Tanyeri, Bera and Park, Xu et al., Usta and Kantar, Lassance, Rau-Bredow, MacLean, etc. [5], [6], [7], [8], [9], [10], [11], [12], [13], [14].

In recent times, Mercurio et al. made available a modified technique for demonstrating the entropy model as a risk-convoluted portfolio problem [15]. The authors are accustomed to a pioneering assortment of problems entitled return-entropy optimization problems. This procedure rationalizes calculations using a combinatorial methodology which addresses five foremost concrete apprehensions with the mean-variance optimization. Lu et al. mentioned that predictability measures in physical pointers grounded on metrics of entropy have been comprehensively second-handed in the presentation purview of remedial assessment and investigational identification [16]. Li et al. well-thought-out the problem of expanded portfolio selection and revealed that this problem provides a significant apprehension in indeterminate economic conditions [17]. In their discoveries, the authors deliberated the problem bounded by the structure of uncertainty theory and, as a consequence, projected an uncertain extension mean-variance divergence model by picking the mean as an objective function along with variance and entropy by means of risk and diversity limits. The two divergences were then discovered to determine the smallest amount of risk and intense return.

Zhang and Shi stressed that Shannon's entropy is a fabricating block of information theory and an obligatory feature of machine learning procedures [18]. The authors recognized asymptotic belongings that impose no settlements on the inventive dissemination, and these properties permit statistical analysis with wide-ranging Shannon's entropy. Saraiva provided a concise and informal overview of Shannon's entropy, including specific properties, and conveyed the applications of the model [19].

Bisht and Kumar revealed that the various sector-based investment portfolios are more significant and provide an integrated process of portfolio development as the economy expands [4]. The authors developed a model workable in four stages and the consequences accomplished by the predicted portfolio are detected as remarkable which endorses the efficacy of the projected model. From the application point of understanding, the authors delivered the investigational study over the prevailing models.

Recently, Vikramjit et al. wrought a newfangled entropic model for the isolated probability distributions acknowledged by the consequent appearance [20]

$$H_\gamma(P) = -\sum_{i=1}^{n} p_i \log\ p_i + \frac{1}{\gamma-1}\sum_{i=1}^{n}\log\,[1+(\gamma-1)p_i] - \frac{\log\gamma}{\gamma-1};\ \gamma>1. \quad (2)$$

By commissioning this model, the authors made accessible communications with the learning of dissimilarities of uncertainty in the steady state and non-steady state queuing processes. Additionally, the investigators delivered an association of solicitations of the maximum entropy principle by engaging their precise exposed model.

Some other researchers who contributed towards the variety of investigational projections include Elgawad et al., Parkash et al, Stoyanov et al., Shwartz and LeCun, etc. [21], [22], [23], [24].

3. Methodology

In the present communication, we make a practice of numerous entropic models for the development of optimizational principles, but in advance, we make available a temporary outline of the commencement of mean-efficient perspective outstanding to Markowitz [1]. To elucidate this straightforward commencement, we progress subsequently.

Let p_j entitle the probability of j^{th} security outcome and ρ_{ij} represent the security return on i^{th} security. Then the expected return on i^{th} security is quantified by the succeeding mathematical expression:

$$\overline{\rho}_i = \sum_{j=1}^{m} p_j \rho_{ij}. \quad (3)$$

Additionally, the variances and the covariances of security returns are established by the consequent expressions:

$$\sigma_i^2 = \sum_{j=1}^{m} p_j \left(\rho_{ij} - \overline{\rho_i}\right)^2, \quad (4)$$

$$r_{ik}\sigma_i\sigma_k = \sum_{j=1}^{m} p_j \left(\rho_{ij} - \overline{\rho_i}\right)\left(\rho_{kj} - \overline{\rho_k}\right). \quad (5)$$

Let a person aim to invest extents $z_1, z_2, \ldots, z_n$ of his complete resources in n securities. Then

$$\sum_{i=1}^{n} z_i = 1;\ z_i \geq 0. \quad (6)$$

Therefore the expected return on securities and variance return are quantified by

$$E = \sum_{i=1}^{n} z_i \overline{\rho_i} \quad (7)$$

and

$$V = \sum_{i=1}^{n} z_i^2 \sigma_i^2 + 2\sum_{k=1}^{n}\sum_{\substack{i=1\\ i<k}}^{n} z_i z_k r_{ik} \sigma_i \sigma_k. \quad (8)$$

Markowitz's portfolio theory is based on the principle that $z_1, z_2, \ldots, z_n$ are so selected to capitalize on the expected return E and to decrease the variance V, or, otherwise, to decrease V while keeping E at a stationary value. Now

$$V = \sum_{j=1}^{m} p_j \left(\rho_j - \overline{\rho}\right)^2, \quad (9)$$

where

$$\rho_j = \sum_{i=1}^{n} z_i \rho_{ij} \quad \text{and} \quad \overline{\rho} = \sum_{i=1}^{n} z_i \overline{\rho_i}. \tag{10}$$

Markowitz's principle can be explained geometrically, where for each vector $(z_1, z_2, \ldots, z_n)$, one can ascertain the values E and V and then exemplify a point in the $E-V$ plane. All points have normally been circumscribed by a sealed convex curve as exhibited in Figure 1.The lower portion on this curve, representing "mean-variance efficient frontier" intellects that collections $(z_1, z_2, \,.\, z_n.)$ compatible to points on this curve are superior to all supplementary collections.

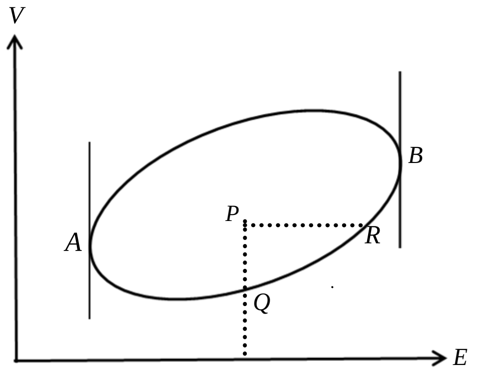

Figure 1. Mean-Variance efficient frontier

If point P does not lie on boundary AB and Q, R lie on AB, then a portfolio analogous to Q is superior to a portfolio analogous to P. Subsequently, it has undergone a condensed amendment to match the expected return. Likewise, R is more amended than P because it has a superior expected return. Nevertheless, the purpose of the outstanding capable portfolios is uncontrollable. Furthermore, Markowitz pointed out that one could expect uncertainty in concluding outstanding, which can be determined introverted by persuading the arrogance of an investor's risk.

This elucidation owed to Markowitz's principle specifies that uncertainty or entropy models can be engaged marvelously in portfolio analysis. Observing this understanding, we progress toward the expansion of optimization principles using our own entropy models. We introduce two newfangled entropic models for the discrete probability distributions, and we deliver transactions of these models for the enlargement of optimizational principles.

Newfangled Entropic Models for Discrete Probability Distributions

1. We introduce a newfangled parametric entropic model specified by the succeeding manifestation:

$$H_{\alpha,\beta}(P) = -\sum_{i=1}^{n} p_i \log p_i + \frac{1}{\beta-\alpha}\log\sum_{i=1}^{n} p_i^{\alpha} - \frac{1}{\beta-\alpha}\log\sum_{i=1}^{n} p_i^{\beta};$$

$$\alpha < 1, \beta > 1 \;\text{ or }\; \alpha > 1, \beta < 1. \tag{11}$$

To derive this measure, we use Renyi's entropy specified by [25]

$$H_{\alpha}(P) = \frac{1}{1-\alpha}\log\sum_{i=1}^{n} p_i^{\alpha}\;;\; \alpha \neq 1, \alpha > 0. \tag{12}$$

Similarly, we have

$$H_{\beta}(P) = \frac{1}{1-\beta}\log\sum_{i=1}^{n} p_i^{\beta}\;;\; \beta \neq 1, \beta > 0. \tag{13}$$

Now, let us contemplate an innovative function prearranged by the succeeding manifestation:

$$H_{\alpha,\beta}(P) = -\sum_{i=1}^{n} p_i \log p_i + \frac{\lambda}{1-\alpha}\log\sum_{i=1}^{n} p_i^{\alpha} + \frac{\mu}{1-\beta}\log\sum_{i=1}^{n} p_i^{\beta}, \tag{14}$$

where $\lambda, \mu > 0$.

Obviously, $H_{\alpha,\beta}(P)$ attains its maximum value when $p_i = \frac{1}{n}\; \forall i = 1,2,\ldots,n$, that is, at the uniform distribution. Therefore we can conclude that $H_{\alpha,\beta}(P)$ is maximum at the uniform distribution.

Since $\lambda, \mu > 0, 1-\alpha$ and $1-\beta$ must have opposite signs if $\alpha < 1, \beta > 1$ or if $\alpha > 1, \beta < 1$. Hence we can suppose that

$$\frac{\mu}{1-\beta} = -\frac{k\lambda}{1-\alpha}, k > 0. \tag{15}$$

Thus equation (14) reduces to

$$\begin{aligned} H_{\alpha,\beta}(P) &= -\sum_{i=1}^{n} p_i \log p_i + \frac{\lambda}{1-\alpha}\left[\log\sum_{i=1}^{n} p_i^{\alpha} - k\log\sum_{i=1}^{n} p_i^{\beta}\right] \\ &= -\sum_{i=1}^{n} p_i \log p_i + \frac{\lambda}{1-\alpha}\log\sum_{i=1}^{n} p_i^{\alpha} - \frac{\lambda}{1-\alpha}\log\left\{\sum_{i=1}^{n} p_i^{\beta}\right\}^{k}. \end{aligned} \tag{16}$$

Now in whichever case, whether $\alpha < 1, \beta > 1$ or $\alpha > 1, \beta < 1$, $(1-\alpha)$ has the same sign as $(\beta-\alpha)$. Consequently, we have

$$H_{\alpha,\beta}(P) = -\sum_{i=1}^{n} p_i \log p_i + \frac{1}{\beta-\alpha}\log\sum_{i=1}^{n} p_i^{\alpha} - \frac{1}{\beta-\alpha}\log\left\{\sum_{i=1}^{n} p_i^{\beta}\right\}^k. \quad (17)$$

In particular, for $k=1$, we contract (11).

Accordingly, the entropy represented by equation (11) assumes its maximum value at the uniform distribution, and thus it is an effective and additive measure of entropy.

2. Next, we introduce a new parametric entropy of order α specified by the ensuing exact appearance:

$$H_{\alpha}(P) = -\sum_{i=1}^{n} p_i \log p_i + \frac{1}{\alpha}\sum_{i=1}^{n}\left(1-p_i^{\alpha p_i}\right);\ \alpha \neq 0, \alpha > 0. \quad (18)$$

Under contract, we proceed through $(0)^{\alpha 0} = 1$.

Apparently, we ensure $\lim_{\alpha\to 0} H_{\alpha}(P) = -2\sum_{i=1}^{n} p_i \log p_i$,

which is Shannon's entropy except a multiplicative constant [3]. Thus, the entropy represented by equation (18) is a generalized measure of entropy.

To ascertain that (18) is an effective measure of entropy, we study its vital properties subsequently.

(i) Non-negativity: By using the identity $e^{-x} \leq 1$ for $x \geq 0$, we get

$$\frac{1}{\alpha}\sum_{i=1}^{n}(1-p_{i_i}^{\alpha p_i}) \geq 0 \ \text{ for } \alpha > 0.$$

Also, $-\sum_{i=1}^{n} p_i \log p_i \geq 0.$

Hence $H_{\alpha}(P) \geq 0$.

(ii) Permutational symmetry: Obviously, $H_{\alpha}(P)$ is permutationally symmetric.

(iii) Concavity: We have

$$\frac{dH_{\alpha}(P)}{dp_i} = -1 - \log p_i - p_i^{\alpha p_i}(1+\log p_i).$$

Therefore

$$\frac{d^2 H_{\alpha}(P)}{dp_i^2} = -\frac{1}{p_i} - p_{i_i}^{\alpha p_i}\left\{\alpha(1+\log p_i)^2 + \frac{1}{p_i}\right\} < 0.$$

Accordingly, $H_{\alpha}(P)$ possesses concavity. Additionally, the concavity of $H_{\alpha}(P)$ is obvious from the graph of $H_{\alpha}(P)$ against p_i as presented in Figure 2 for $n = 2$ and $\alpha = 2$.

Table 1. $H_{\alpha}(P)$ against p_i for $n=2$ and $\alpha=2$

p_i	$H_{\alpha}(P)$
0	0.00000
0.1	0.27145
0.2	0.38856
0.3	0.45434
0.4	0.48957
0.5	0.50000
0.6	0.48956
0.7	0.45437
0.8	0.38829
0.9	0.27168
1	0.00000

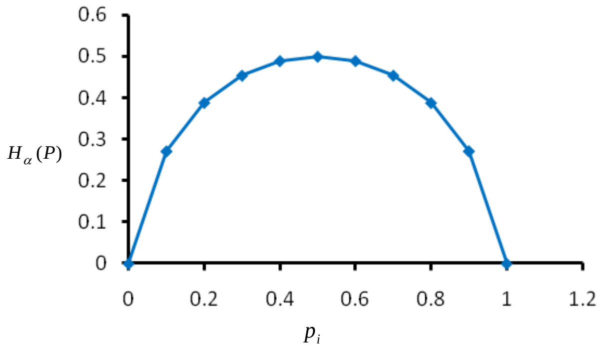

Figure 2. Concavity of $H_{\alpha}(P)$ with respect to p_i

Henceforth, $H_{\alpha}(P)$ is an accurate measure of entropy.

(iv) Continuity: It can be seen that $H_{\alpha}(P)$ is a continuous function of p_i for all p_i's.

(v) Maximization: Consider the conforming Lagrange's function

$$L = -\sum_{i=1}^{n} p_i \log p_i + \frac{1}{\alpha}\sum_{i=1}^{n}(1-p_i^{\alpha p_i}) - \lambda\left(\sum_{i=1}^{n} p_i - 1\right). \quad (19)$$

Differentiating equation (19) w.r.to p_i and equating the derivative to zero, we get

$$\log p_i + p_i^{\alpha p_i}(1+\log p_i) = -(1+\lambda),$$

which is conceivable only if $p_1 = p_2 = \ldots = p_n$.

Further, since $\sum_{i=1}^{n} p_i = 1$, , we have

$$p_i = \frac{1}{n},\ \forall\, i = 1,2,\ldots,n.$$

Consequently, we perceive that $\left[H_\alpha(P)\right]_{\max}$ ascends for the uniform distribution, and this consequence is further anticipated.

(vi) Expansibility: We have

$$H_\alpha(p_1, p_2, \ldots, p_n, 0) = -\sum_{i=1}^{n} p_i \log p_i + \frac{1}{\alpha}\sum_{i=1}^{n}(1 - p_i^{\alpha p_i}) + \frac{1}{\alpha}\left[1 - (0)^{\alpha 0}\right]$$

$$= -\sum_{i=1}^{n} p_i \log p_i + \frac{1}{\alpha}\sum_{i=1}^{n}(1 - p_i^{\alpha p_i})$$

$$= H_\alpha(p_1, p_2, \ldots, p_n)$$

That is, the entropy is unaffected by adding an impossible event.

(vii) The maximum value $\psi(n)$ of the entropy is prearranged by

$$\psi(n) = \log n + \frac{n}{\alpha}\left[1 - \left(\frac{1}{n}\right)^{\frac{\alpha}{n}}\right]. \tag{20}$$

Let $f(x) = \frac{1}{\alpha x}\left(1 - e^{\alpha x \log x}\right)$, where $x = \frac{1}{n}$, $x \in (0, 1]$.

Accordingly, we ensure

$$f'(x) = -\frac{1}{\alpha x}\left[\alpha x^{\alpha x}(1 + \log x) + \frac{1}{x}(1 - x^{\alpha x})\right] < 0,$$

which provides an illustration that $f(x)$ decreases as x increases. Consequently, the term $\frac{n}{\alpha}\left[1 - \left(\frac{1}{n}\right)^{\frac{\alpha}{n}}\right]$ in equation (20) increases as *n* increases. Therefore $\psi(n)$ is an increasing function of *n*, which is once more a looked-for consequence as the maximum value of entropy should continuously increase.

4. Results and Discussion

Now we proceed to develop optimization principles through our proposed entropy measures.

Markowitz endorsed his principle by reflecting the choice from investment proportions $z_1, z_2, \ldots, z_n$ in such a methodology that minimizes the variance, that is, to make $\rho_1, \rho_2, \ldots, \rho_m$ as undistinguishable as probable [1]. Consequently, any withdrawal of $\rho_1, \rho_2, \ldots, \rho_m$ from egalitarianism was reflected as a risk. The comparable perseverance can be enhanced if we select $z_1, z_2, \ldots, z_n$ so as to maximize the entropy function. Since the probabilities have not been convoluted in the above discussion, but there is an influenced requirement for their insertion, we can instead contemplate the maximization of the consequent entropy model

$$H(P) = -\sum_{j=1}^{m} \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k} \log \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k}. \tag{21}$$

Equation (21) characterizes Shannon's entropy function and ascertains incredible applications in a diversity of disciplines [3].

For our proposed entropy measure represented by equation (17), the model (21) takes the succeeding form:

$$H_{\alpha,\beta}(P) = -\sum_{j=1}^{m} \frac{p_j \rho_j}{\bar{\rho}} \log \frac{p_j \rho_j}{\bar{\rho}} + \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \left\{ \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k} \right\}^{\alpha}$$

$$- \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \left\{ \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k} \right\}^{\beta},$$

where

$$\sum_{j=1}^{m} p_j \rho_j = \sum_{j=1}^{m} p_j \sum_{i=1}^{n} z_i \rho_{ij} = \sum_{i=1}^{n} z_i \left\{ \sum_{j=1}^{m} p_j \rho_{ij} \right\} = \sum_{i=1}^{n} z_i \bar{\rho}_i = \bar{\rho}$$

i.e., we have

$$H_{\alpha,\beta}(P) = -\sum_{j=1}^{m} \frac{p_j \rho_j}{\bar{\rho}} \log p_j \rho_j + \sum_{j=1}^{m} \frac{p_j \rho_j}{\bar{\rho}} \log \bar{\rho}$$

$$+ \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \left\{ \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k} \right\}^{\alpha} - \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \left\{ \frac{p_j \rho_j}{\sum_{k=1}^{m} p_k \rho_k} \right\}^{\beta}$$

$$= \log \bar{\rho} - \frac{1}{\bar{\rho}} \sum_{j=1}^{m} p_j \rho_j \log p_j \rho_j$$

$$+ \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \{p_j \rho_j\}^{\alpha} - \frac{1}{\beta - \alpha} \log m \bar{\rho}^{\alpha}$$

$$- \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \{p_j \rho_j\}^{\beta} + \frac{1}{\beta - \alpha} \log m \bar{\rho}^{\beta}$$

$$= \text{Constant} - \frac{1}{\bar{\rho}} \sum_{j=1}^{m} p_j \rho_j \log p_j \rho_j$$

$$+ \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \{p_j \rho_j\}^{\alpha} - \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \{p_j \rho_j\}^{\beta}.$$

Accordingly, we formulate the subsequent entropy-grounded principle as follows:

Optimizational Principle-I

Choose $z_1, z_2, ..., z_n$ so as to maximize

$$Z = -\frac{1}{\bar{\rho}} \sum_{j=1}^{m} p_j(z_1\rho_{1j} + z_2\rho_{2j} + ... + z_n\rho_{nj}) \log \; p_j(z_1\rho_{1j} + z_2\rho_{2j} + ... + z_n\rho_{nj})$$

$$+ \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \left\{ p_j(z_1\rho_{1j} + z_2\rho_{2j} + ... + z_n\rho_{nj}) \right\}^{\alpha}$$

$$- \frac{1}{\beta - \alpha} \log \sum_{j=1}^{m} \left\{ p_j(z_1\rho_{1j} + z_2\rho_{2j} + ... + z_n\rho_{nj}) \right\}^{\beta} \quad (22)$$

subject to the succeeding restrictions:

$$\sum_{j=1}^{m} p_j(z_1\rho_{1j} + z_2\rho_{2j} + ... + z_n\rho_{nj}) = \text{Constant} \quad (23)$$

and

$$\sum_{i=1}^{n} z_i = 1, \quad (24)$$

$$z_i \geq 0, \; \forall i = 1, 2, ..., n. \quad (25)$$

The aforementioned hypothesis is illustrated through the following numerical example.

Numerical Example: Let us reflect upon the case of two securities, each with ten plausible significances of corresponding probabilities and returns as shown in Table 2.

Table 2. Probabilities and returns of securities

Probability	Return-I	Return-II
0.15	0.20	0.15
0.10	0.10	0.20
0.05	0.05	0.15
0.10	0.15	0.10
0.10	0.15	0.15
0.15	0.15	0.15
0.05	0.15	0.05
0.10	0.20	0.15
0.15	0.20	0.10
0.05	0.15	0.15

We need to ascertain the optimum values of z_1 and z_2 when the mean return is 0.14875. Thus, our mathematical optimization problem for $\alpha = 2, \beta = 3$ can be expressed successively:

Maximize

$$Z = -\frac{1}{\bar{\rho}} \sum_{j=1}^{10} p_j(z_1\rho_{1j} + z_2\rho_{2j}) \log \; p_j(z_1\rho_{1j} + z_2\rho_{2j})$$
$$+ \log \sum_{j=1}^{10} \left\{ p_j(z_1\rho_{1j} + z_2\rho_{2j}) \right\}^2 - \log \sum_{j=1}^{10} \left\{ p_j(z_1\rho_{1j} + z_2\rho_{2j}) \right\}^3 \quad (26)$$

subject to the set of restrictions:

$$\sum_{j=1}^{10} p_j(z_1\rho_{1j} + z_2\rho_{2j}) = 0.14875 \quad (27)$$

and

$$\sum_{i=1}^{2} z_i = 1, \quad (28)$$

$$z_i \geq 0, \; \forall \, i = 1, 2, \quad (29)$$

where

$p_1 = 0.15, \; p_2 = 0.10, \; p_3 = 0.05, \; p_4 = 0.10, \; p_5 = 0.10,$
$p_6 = 0.15, \; p_7 = 0.05, \; p_8 = 0.10, \; p_9 = 0.15, \; p_{10} = 0.05$

and

$\rho_{11} = 0.20, \; \rho_{12} = 0.10, \; \rho_{13} = 0.05, \; \rho_{14} = 0.15, \; \rho_{15} = 0.15,$
$\rho_{16} = 0.15, \; \rho_{17} = 0.15, \; \rho_{18} = 0.20, \; \rho_{19} = 0.20, \rho_{110} = 0.15,$
$\rho_{21} = 0.15, \; \rho_{22} = 0.20, \; \rho_{23} = 0.15, \; \rho_{24} = 0.10, \rho_{25} = 0.15,$
$\rho_{26} = 0.15, \; \rho_{27} = 0.05, \; \rho_{28} = 0.15, \; \rho_{29} = 0.10, \rho_{210} = 0.15.$

where $\bar{\rho} = 0.16 z_1 + 0.1375 z_2$.

Consider the Lagrange's function

$$L = -\frac{1}{\bar{\rho}} \sum_{j=1}^{10} p_j(z_1\rho_{1j} + z_2\rho_{2j}) \log(z_1\rho_{1j} + z_2\rho_{2j})$$
$$+ \log \sum_{j=1}^{10} \left\{ p_j(z_1\rho_{1j} + z_2\rho_{2j}) \right\}^2 - \log \sum_{j=1}^{10} \left\{ p_j(z_1\rho_{1j} + z_2\rho_{2j}) \right\}^3$$
$$- \lambda \left\{ \sum_{j=1}^{10} p_j(z_1\rho_{1j} + z_2\rho_{2j}) - 0.14875 \right\} - \mu \{ z_1 + z_2 - 1 \}.$$

Differentiating L w.r.to z_1 and z_2, and equating the derivatives to zero, we get

$$-\frac{1}{\bar{\rho}} \sum_{j=1}^{10} p_j \{ 1 + \log\,(z_1\rho_{1j} + z_2\rho_{2j})\rho_{1j} \} + \frac{2\sum_{j=1}^{10} p_j(z_1\rho_{1j} + z_2\rho_{2j})}{\sum_{j=1}^{10} \{ p_j(z_1\rho_{1j} + z_2\rho_{2j}) \}^2} p_j\rho_{1j}$$
$$- \frac{3\sum_{j=1}^{10} \{ p_j(z_1\rho_{1j} + z_2\rho_{2j}) \}^2}{\sum_{j=1}^{10} \{ p_j(z_1\rho_{1j} + z_2\rho_{2j}) \}^3} p_j\rho_{1j} - \lambda \sum_{j=1}^{10} p_j\rho_{1j} - \mu = 0 \quad (30)$$

and

$$-\frac{1}{\bar{\rho}}\sum_{j=1}^{10} p_j\{1+\log\,(z_1\rho_{1j}+z_2\rho_{2j})\rho_{2j}\}+\frac{2\sum_{j=1}^{10} p_j(z_1\rho_{1j}+z_2\rho_{2j})}{\sum_{j=1}^{10}\{p_j(z_1\rho_{1j}+z_2\rho_{2j})\}^2}\,p_j\rho_{2j}$$
$$-\frac{3\sum_{j=1}^{10}\{p_j(z_1\rho_{1j}+z_2\rho_{2j})\}^2}{\sum_{j=1}^{10}\{p_j(z_1\rho_{1j}+z_2\rho_{2j})\}^3}\,p_j\rho_{2j}-\lambda\sum_{j=1}^{10} p_j\rho_{2j}-\mu=0. \tag{31}$$

Now, using the relation $\bar{\rho}=\sum_{i=1}^{2} z_i\bar{\rho}_i \;=0.16z_1+0.1375\,z_2$, and solving equations (30) and (31) for random values of λ and μ, we get $z_1=0.5117$ and $z_2=0.4883$, providing the optimum values of z_1 and z_2, with mean return 0.14875.

Next, we introduce an additional optimization principle, applying the quantitative parametric entropic model of order α, as developed in equation (18).

In the current context, model (18) takes the following form:

$$H_\alpha(P)=\text{Constant}-\frac{1}{\bar{\rho}}\sum_{j=1}^{m} p_j\rho_j\,\log\,p_j\rho_j-\frac{1}{\alpha}\sum_{j=1}^{m}\left\{\frac{p_j\rho_j}{\bar{\rho}}\right\}^{\alpha\frac{p_j\rho_j}{\bar{\rho}}}.$$

Therefore, we propose the following entropy-based principle:

Optimizational Principle-II

Choose $z_1, z_2,..., z_n$ so as to maximize

$$z=-\frac{1}{\bar{\rho}}\sum_{j=1}^{m} p_j\rho_j\,\log\,p_j\rho_j-\frac{1}{\alpha}\sum_{j=1}^{m}\left\{\frac{p_j\rho_j}{\bar{\rho}}\right\}^{\alpha\frac{p_j\rho_j}{\bar{\rho}}} \tag{32}$$

subject to the succeeding restrictions:

$$\sum_{j=1}^{m} p_j(z_1\rho_{1j}+z_2\rho_{2j}+...+z_n\rho_{nj})=\text{Constant} \tag{33}$$

and

$$\sum_{i=1}^{n} z_i=1, \tag{34}$$

$$z_i\ge 0,\;\forall i=1,2,...,n. \tag{35}$$

5. Conclusion

Our findings demonstrate the potential of parametric and non-parametric information-theoretic entropy models to advance optimization principles across various domains of Operations Research. Furthermore, while these parametric models, with their unique advantages, persuade elasticity in the arrangement under study, entropy models can be adapted to address the challenges of portfolio analysis. The newly anticipated discrete entropy models introduce a groundbreaking approach to decision-making in uncertain portfolio selection problems, offering a significant advancement in the field. The future scope of the study is to investigate how these entropic models can be used to adapt portfolio strategies to changing market conditions.

Data Availability
None

Conflict of Interest
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