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Improving the Reliability of System by Standby Redundancy Method

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Abstract- Safety is main concern before designing the system. Most of the theories' are applied for improving the safety of the system, but the reliability theory plays a important role for designing the system as such, that we reach the higher levels of safety.In this paper, a Series- Parallel system is considered for reliability improvement. The said system is a combination of three items in which, two items are connected in parallel and third item is connected in series. It is assumed that the items are independent and follows alpha power Rama distribution. Two methods are applied to this system in order to enhance its reliability. In each method, different set of items are considered for improvement. A data analysis is performed, to compare different improvement methods. At last we find that among these methods, reduction method is proved to be best method for improving the system reliability.

Keywords-Reliability Analysis, Reduction Method, Hot Duplication Method.

I. INTRODUCTION

In recent years, the indusrial world is facing the problem of safety issues in regard to design of system. The safety of the system is the main concern nowadays and has gained much attention for so many years. Thus, there is an increasing need to work on system safety leading reliability theory.Statistical and reliability analysis methods are also applied in system performance evaluation in a wide range of industries including mechanical, civil and electrical engineering. For this purpose, different distribution functions, such as normal, lognormal, Weibull, exponential and linear-exponential distribution functions, have been considered in the literature to improve system reliability.

Moreover, a Monte Carlo method is introduced in the literature to estimate the failure probability of a system. A multi-state criticality analysis in multi-state systems has been done to improve reliability. Further, failure data has been analyzed to provide new reliability improvement methods. Linear-exponential distribution function is a commonly used probability distribution for modeling lifetime data.

In this paper, we apply alpha power Rama distribution function in a system to investigate system reliability. The mentioned system is also considered in the literature of a radar system of an aircraft. Three improvement methods are often considered for system reliability improvement.
These methods can be summarised as:

1. Reduction Method (RM): In this method, the failure parameter of
2. items are reduced by multiplying with a factor r where $0 < r < 1$.

A selected subset of system items will be hot duplicated in this method. The item(s) selected for improvement must be added to itself (themselves) in parallel.

II. RELATED WORK

The concept of reliability equivalence factor was proposed by Rade [1] and he has applied the reliability of equivalence to various systems. Rade [1,4] also applied the concept of to reliability of equivalence the parallel and series system whose components are independent and identical. Sarhan [5,8,9] considered the many systems for reliability whose components are independent and identical viz, series system, parallel system, network bridge system etc.. Abdelfattah Mustafa [10] discussed the modified Weibull distribution for improving the reliability of system. A. Mustafa and Adel El-Faheem [11] discussed the reliability equivalence factor for a system consist of mixture of n independent and non-identical lifetime with deferral time.

M. Hatim, A. A. Mustafa [12] The reliability equivalence factor of series system studies in when the failure rates of the system components are functions of time t., introduced two cases of non- constant failure rate (i) Weibull distribution (ii) linear increasing failure rate distribution. H. Mohammad, et.al [13] discussed the Reliability Performance of Improved General Series-Parallel Systems in the Generalized Exponential Lifetime Model.
A. Yousry, M.L. Al-Ohally [14] discussed the Reliability equivalence factors for exponentiated lifetime distribution.

III. METHODOLOGY

Reliability of complex electric system:
Reliability function of basic complex system is one of the most important concerns in designing a safety of the system. This concern is defined using a function that is called reliability function.

To define a reliability function of a system of n independent items are considered. If $v(t)$ was presumed as a subset of all items which have had no fault until time t, then $v(t)/n$ is a random variable showing system reliability when n tends to infinity; i.e. $\lim n \to \infty, \frac{v(t)}{n}$ .In other words, reliability is a probability with which an items works over a time period (t). The reliability function is also defined as

$$R(t) = P(T \geq t) = \int_{t}^{\infty} f(z)\,dz \qquad (1)$$

where T is a continuous random variable indicating time and $f(z)$ is probability density function.

Reliability of the systems are always taken as main issues in designing electrical systems. These systems are basically considered as series or parallel systems. Reliability function of series systems is as follows,

$$R_s(t) = \prod_{j=1}^{n} R_j(t) \qquad (2)$$

where $R_j(t)$ is reliability function of item j. A series system works when all items are working. Also, reliability function of a parallel system is defined as

$$R_p(t) = 1 - \prod_{j=1}^{n} Q_j(t) \qquad (3)$$

where $Q_j\ (t)$ is failure function of item j. Failure function is often supposed as a complementary function of the reliability function (i.e.$1 - R_j(t)$). A parallel system works when at least one item is working.

In this paper, an electrical system is considered for reliability improvement. Reliability function of this system is formulated as below.

$$R(t) = \left(1 + \frac{\theta_1^3 t^3 + 3\theta_1^2 t^2 + 6\theta_1 t}{\theta_1^3 + 6}\right) e^{-\theta_1 t} \left\{ \begin{array}{l} \left(1 + \frac{\theta_2^3 t^3 + 3\theta_2^2 t^2 + 6\theta_2 t}{\theta_2^3 + 6}\right) e^{-\theta_2 t} + \\ \left(1 + \frac{\theta_3^3 t^3 + 3\theta_3^2 t^2 + 6\theta_3 t}{\theta_3^3 + 6}\right) e^{-\theta_3 t} + \\ \left(1 + \frac{\theta_2^3 t^3 + 3\theta_2^2 t^2 + 6\theta_2 t}{\theta_2^3 + 6}\right) \\ \left(1 + \frac{\theta_3^3 t^3 + 3\theta_3^2 t^2 + 6\theta_3 t}{\theta_3^3 + 6}\right) e^{-(\theta_2 + \theta_3)t} \end{array} \right\} \qquad (4)$$

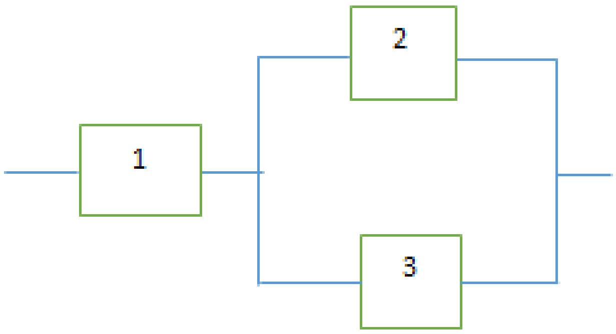

Figure. 1 Basic Complex System

In this paper, we consider the reliability function of alpha power rama distribution into reliability of this system. Then, the mentioned improvement methods are reduction method and one of the redundacy method viz hot duplication method are considered to enhance the reliability of this system.In each improvement methods, the following set of items is taken for improvement.

$$A_1 = \{1\}$$
$$A_2 = \{2\}$$
$$A_3 = \{1, 2\}$$
$$A_4 = \{2, 3\}$$

Reliability improvements:
It's supposed in this paper that random variables of the basic complex system, is shown in the figure 1.

$$f(t) = \left(\frac{\log \alpha}{\alpha - 1}\right)\left(\frac{\theta^4}{\theta^3 + 6}\right)(1 + t^3) e^{-\theta t} \alpha^{1 - \left(1 + \frac{\theta^3 t^3 + 3\theta^2 t^2 + 6\theta t}{\theta^3 + 6}\right) e^{-\theta t}} \qquad (5)$$

$$R(t) = \left(\frac{\alpha}{\alpha - 1}\right) \alpha^{-\left(1 + \frac{\theta^3 t^3 + 3\theta^2 t^2 + 6\theta t}{\theta^3 + 6}\right) e^{-\theta t}} \qquad (6)$$

Reliability function of above complex system is generally written as

$$R(t) = R_1(t)\ R_4(t)$$

$R_1(t)$ is reliability function of item 1 and $R_4(t)$ reliability function of item 2 and item 3. Therefore reliability function of system.

$$R_s(t) = R_1(t)\big(R_2(t) + R_3(t) - R_2(t) R_3(t)\big) \qquad (7)$$

$$R_s(t)=\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)\left\{\begin{array}{l}\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\\+\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\\-\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(\frac{\alpha_2}{\alpha_2-1}\right)\\\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\\\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\end{array}\right\} \quad (8)$$

1. Reduction Method:

The reduction method is applied to improve system reliability. In this method, the failure parameters is multiplied with a reduction factor r, $0 \le r \le 1$.

If $A_1 = \{1\}$ is selected for reliability improvement, then reliability function of improved system is

$$R_{A_1}^r(t)=R_1^r(t)\big(R_2(t)+R_3(t)-R_2(t)R_3(t)\big)$$

where $R_1^r(t)$ is reliability function of reduced item. Also $R_2(t)$, $R_3(t)$ are main reliability function of item 2 and item 3 respectively.

$$R_s(t)=\left(\frac{r\alpha_1}{r\alpha_1-1}\right)\left(1-r\alpha_1^{\left(1+\frac{r^3\theta_1^3t^3+3r^2\theta_1^2t^2+6r\theta_1t}{r^3\theta_1^3+6}\right)e^{-r\theta_1t}}\right)\left\{\begin{array}{l}\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)+\left(\frac{\alpha_3}{\alpha_3-1}\right)\\\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)-\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(\frac{\alpha_2}{\alpha_2-1}\right)\\\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\\\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\end{array}\right\} \quad (9)$$

Also, $A_2 = \{2\}$, is supposed selected for reliability improvement. Then the reliability function of improved system would be as:

$$R_s(t)=\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)\left\{\begin{array}{l}\left(\frac{r\alpha_2}{r\alpha_2-1}\right)\left(1-r\alpha_2^{\left(1+\frac{r^3\theta_2^3t^3+3r^2\theta_2^2t^2+6r\theta_2t}{r^3\theta_2^3+6}\right)e^{-r\theta_2t}}\right)\\+\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)-\left(\frac{\alpha_3}{\alpha_3-1}\right)\\\left(\frac{r\alpha_2}{r\alpha_2-1}\right)\left(1-r\alpha_2^{\left(1+\frac{r^3\theta_2^3t^3+3r^2\theta_2^2t^2+6r\theta_2t}{r^3\theta_2^3+6}\right)e^{-r\theta_2t}}\right)\\\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\end{array}\right\} \quad (10)$$

Also, $A_3 = \{1,2\}$, is supposed selected for reliability improvement. Then the reliability function of improved system would be as:

$$R_s(t)=\left(\frac{r\alpha_1}{r\alpha_1-1}\right)\left(1-r\alpha_1^{\left(1+\frac{r^3\theta_1^3t^3+3r^2\theta_1^2t^2+6r\theta_1t}{r^3\theta_1^3+6}\right)e^{-r\theta_1t}}\right)\left\{\begin{array}{l}\left(\frac{r\alpha_2}{r\alpha_2-1}\right)\left(1-r\alpha_2^{\left(1+\frac{r^3\theta_2^3t^3+3r^2\theta_2^2t^2+6r\theta_2t}{r^3\theta_2^3+6}\right)e^{-r\theta_2t}}\right)\\+\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\\-\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(\frac{r\alpha_2}{r\alpha_2-1}\right)\\\left(1-r\alpha_2^{\left(1+\frac{r^3\theta_2^3t^3+3r^2\theta_2^2t^2+6r\theta_2t}{r^3\theta_2^3+6}\right)e^{-r\theta_2t}}\right)\\\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\end{array}\right\} \quad (11)$$

The set $A_4 = \{2,3\}$, is supposed selected for reliability improvement. Then the reliability function of improved system would be as:

$$R_s(t)=\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)$$

$$\left\{\begin{array}{l}\left(\frac{r\alpha_2}{r\alpha_2-1}\right)\left(1-r\alpha_2^{\left(1+\frac{r^3\theta_2^3t^3+3r^2\theta_2^2t^2+6r\theta_2t}{r^3\theta_2^3+6}\right)e^{-r\theta_2t}}\right)\\ +\left(\frac{r\alpha_3}{r\alpha_3-1}\right)\left(1-r\alpha_3^{\left(1+\frac{r^3\theta_3^3t^3+3\theta_3^2r^2t^2+6\theta_3rt}{r^3\theta_3^3+6}\right)e^{-\theta_3rt}}\right)\\ -\left(\frac{r\alpha_3}{r\alpha_3-1}\right)\left(\frac{r\alpha_2}{r\alpha_2-1}\right)\\ \left(1-r\alpha_2^{\left(1+\frac{r^3\theta_2^3t^3+3r^2\theta_2^2t^2+6r\theta_2t}{r^3\theta_2^3+6}\right)e^{-r\theta_2t}}\right)\\ \left(1-r\alpha_3^{\left(1+\frac{r^3\theta_3^3t^3+3\theta_3^2r^2t^2+6\theta_3rt}{r^3\theta_3^3+6}\right)e^{-\theta_3rt}}\right)\end{array}\right\} \quad (12)$$

2. **Hot Duplication Method:**

A Hot Duplication method is applied to the items of system to improve the system reliability.In this method, the items are hot duplicated, i.e. an identical items are added to the items in parallel in the said system. Thus the following sets are considered for reliability improvement, then the reliability functions can be obtained as,

For set 1 $A_1=\{1\}$

$$R_{A_1}^H(t)=R_1(t)\left(2-R_1(t)\right)\left(R_2(t)+R_3(t)-R_2(t)R_3(t)\right)$$

$$R_{A_1}^H(t)=$$

$$\left[\begin{array}{l}\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)\\ \left(2-\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)\right)\end{array}\right]$$

$$\left\{\begin{array}{l}\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\\ +\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\\ -\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(\frac{\alpha_2}{\alpha_2-1}\right)\\ \left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\\ \left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\end{array}\right\} \quad (13)$$

For set $A_2=\{2\}$,

$$R_{A_1}^H(t)=R_1(t)\begin{pmatrix}R_2(t)\left(2-R_2(t)\right)+R_3(t)\\ \left(1-R_2(t)\right)\left(2-R_2(t)\right)\end{pmatrix}$$

$$R_{A_2}^H(t)=\left(\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)\right)$$

$$\left\{\begin{array}{l}\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\\ \left(2-\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\right)\\ +\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\\ \left(\left(1-\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\right)\right.\\ \left.\left(2-\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\right)\right)\end{array}\right\}$$

(14)

For set $A_3=\{1,2\}$,

$$R_{A_1}^H(t)=\left(R_1(t)\left(2-R_1(t)\right)\right)\begin{pmatrix}R_3(t)+R_2(t)\left(2-R_2(t)\right)\\ \left(1-R_3(t)\right)\end{pmatrix}$$

$$R_{A_3}^H(t)=$$

$$\left[\begin{array}{l}\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)\\ \left(2-\left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1t}}\right)\right)\end{array}\right]$$

$$\left\{\begin{array}{l}\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\\ +\left(\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\right)\\ \left(2-\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2t}}\right)\right)\\ \left(1-\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3t}}\right)\right)\end{array}\right\}$$

(15)

For set $A_4 = \{2,3\}$,

$$R_{A_4}^{H}(t) = R_1(t)\begin{pmatrix} R_3(t)\left(2-R_3(t)\right)+R_2(t) \\ \left(1-R_3(t)\right)\left(2-R_3(t)\right) \end{pmatrix}$$

$$R_{A_4}^{H}(t) = \left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1 t}}\right)$$

$$\left\{\begin{array}{l} \left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3 t}}\right) \\ \left(2-\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3 t}}\right)\right) \\ +\left(\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2 t}}\right)\right) \\ \left(1-\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3 t}}\right)\right) \\ \left(2-\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3 t}}\right)\right) \end{array}\right\} \quad (16)$$

III. RESULTS AND DISCUSSION

In this section, we will discuss methods of reliability improvement of a complex system and make a comparison among different improved methods.

Reduction Method for different items:

The basic complex system is improved in the section using reduction method. Four different sets of items are selected for improvement.

$R_s(t) = R_1(t)\,(R_2(t)+R_3(t)-R_2(t)\,R_3(t))$ Suppose that the following failure parameters are given:

$$\alpha_1 = 0.8, \alpha_2 = 0.65, \alpha_3 = 0.85, \theta_1 = 1.1, \theta_2 = 0.9,$$
$$\theta_3 = 1.25, t = 2 hours$$

Let's supposed that $r = 0.7$, the reliability function of the complex electric system would be as :

$$R_s(t) = \left(\frac{\alpha_1}{\alpha_1-1}\right)\left(1-\alpha_1^{\left(1+\frac{\theta_1^3t^3+3\theta_1^2t^2+6\theta_1t}{\theta_1^3+6}\right)e^{-\theta_1 t}}\right)$$

$$\left\{\begin{array}{l} \left(\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3 t}}\right)\right) \\ +\left(\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2 t}}\right)\right) \\ -\left(\left(\frac{\alpha_3}{\alpha_3-1}\right)\left(1-\alpha_3^{\left(1+\frac{\theta_3^3t^3+3\theta_3^2t^2+6\theta_3t}{\theta_3^3+6}\right)e^{-\theta_3 t}}\right)\right) \\ \left(\left(\frac{\alpha_2}{\alpha_2-1}\right)\left(1-\alpha_2^{\left(1+\frac{\theta_2^3t^3+3\theta_2^2t^2+6\theta_2t}{\theta_2^3+6}\right)e^{-\theta_2 t}}\right)\right) \end{array}\right\} \quad (17)$$

The reliability function of the complex system by inserting reliability of an items 1, 2 and 3 by using equations (i), (j) and (k) to equations (17), The reliability of original system is obtained as,

$$R_1(t) = 0.66655 \quad (i)$$
$$R_2(t) = 0.8016 \quad (j)$$
$$R_3(t) = 0.5577 \quad (k)$$

thus the reliability of original system is

$$R_s(t) = 0.60803$$

Therefore, if different sets are selected for improvement, the following reliability functions are obtained for the improved systems respectively.

$$R_1^r(t) = 0.84519$$
$$R_2^r(t) = 0.8801$$
$$R_3^r(t) = 0.77945$$

The Reliability functions of an improved system after reducing the items of complex system.

$$R_s^r(t) = 0.77098$$
$$R_s^r(t) = 0.63402$$
$$R_s^r(t) = 0.80369$$

Graphs for reliability functions of the original and improved systems are shown in the figure 2.

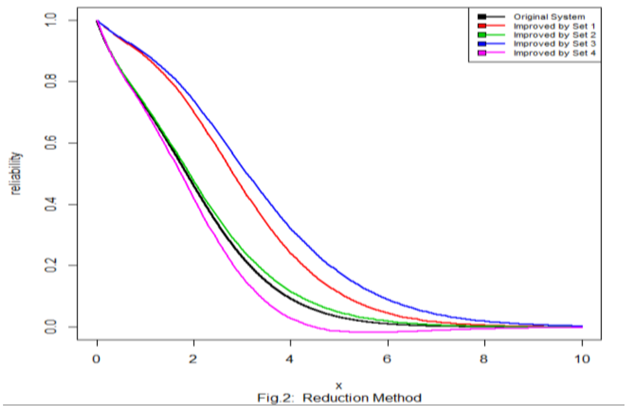

Figure 2. Reduction Method

It has been clearly seen in the figure 2 that the reduction method has been consistent trend to improve system reliability. Also improved systems have higher reliability than the original system. In this case, $A_3 = \{1,2\}$ reaches the best reliability level and A_1, A_4 are in the next positions respectively.

Reduction Method with Different Reduction Factors:
Various reduction factors are applied in this subsection for reliability improvement. The following failure parameters are considered.
Thus, reliability function of the complex electrical system including three independent and identical components is as follows

$$R_s(t) = \left(\frac{\alpha_1}{\alpha_1 - 1}\right)\left(1 - \alpha_1^{\left(1 + \frac{\theta_1^3 t^3 + 3\theta_1^2 t^2 + 6\theta_1 t}{\theta_1^3 + 6}\right)e^{-\theta_1 t}}\right)$$

$$\left\{ \begin{array}{l} \left(\frac{\alpha_2}{\alpha_2 - 1}\right)\left(1 - \alpha_2^{\left(1 + \frac{\theta_2^3 t^3 + 3\theta_2^2 t^2 + 6\theta_2 t}{\theta_2^3 + 6}\right)e^{-\theta_2 t}}\right) \\ + \left(\frac{\alpha_3}{\alpha_3 - 1}\right)\left(1 - \alpha_3^{\left(1 + \frac{\theta_3^3 t^3 + 3\theta_3^2 t^2 + 6\theta_3 t}{\theta_3^3 + 6}\right)e^{-\theta_3 t}}\right) \\ - \left(\frac{\alpha_3}{\alpha_3 - 1}\right)\left(\frac{\alpha_2}{\alpha_2 - 1}\right) \\ \left(1 - \alpha_2^{\left(1 + \frac{\theta_2^3 t^3 + 3\theta_2^2 t^2 + 6\theta_2 t}{\theta_2^3 + 6}\right)e^{-\theta_2 t}}\right) \\ \left(1 - \alpha_3^{\left(1 + \frac{\theta_3^3 t^3 + 3\theta_3^2 t^2 + 6\theta_3 t}{\theta_3^3 + 6}\right)e^{-\theta_3 t}}\right) \end{array} \right\}$$

Then, a system using a set $A_3 = \{1,2\}$, is considered for improving using reduction method. Various reduction factors are applied to compare effects of various reduced factors of the system.

$r_1 = 0.5$, $r_2 = 0.8$, $r_3 = 0.2$, $r_4 = 1$
Therefore, the reliability functions of improved systems using these three reducing factors will respectively be as follows

$$R_{A_3}^{r_1}(t) = 0.63449$$

$$R_{A_3}^{r_2}(t) = 0.31479$$

$$R_{A_3}^{r_3}(t) = 0.98981$$

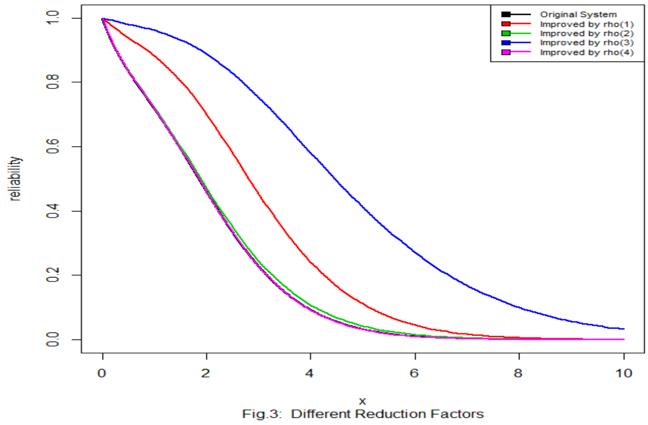

Figure 3a. Different Reduction Factors

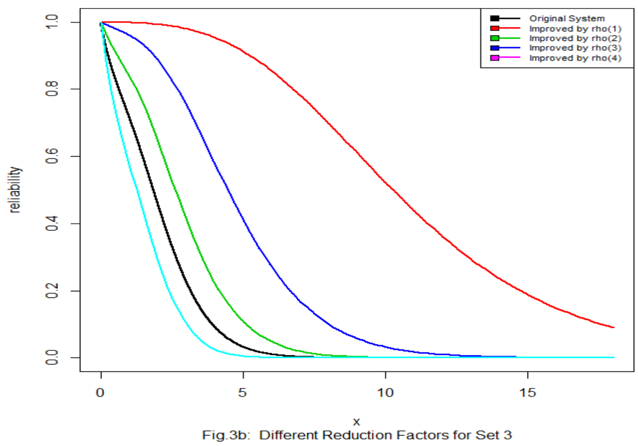

Figure 3b. Different Reduction Factors for set A_3

When reduction factor tends to zero, reliability improvement is increased. It is displayed in figure that the best reliability is obtained when $r_3 = 0.2$. Second and third reduction factors reach second and third reliability levels respectively.

For Hot Duplication Method:
Now for Set $A_1 = \{1\}$
Inserting the values in equation (1) we will find out the reliability of improved systems.

$$R_{A_1}^{H}(t) = 0.8573$$

Similarly for Set $A_3 = \{1,2\}$

$$R_{A_3}^{H}(t) = 0.7057$$

Similarly for Set $A_4 = \{2,3\}$

$$R_{A_4}^{H}(t) = 0.2364$$

Different Improvement Methods:

In this subsection, different improvement methods are applied to improve reliability of the mentioned system using just the first component. The following data are supposed in this experiment

$$\alpha_1 = 0.1,\ \alpha_2 = 0.3,\ \alpha_3 = 0.6,\ \theta_1 = 0.03,\ \theta_2 = 0.04,$$
$$\theta_3 = 0.001,\ t = 0.5$$

After substituting the value of the parameters in equation (8), we can obtain the reliability of complex system as follows

$$R_s(t) = 0.999996$$

If the system is improved using the reduction method where $r = 0.6$, for the second and first item of system, then its reliability functions is obtained as follows:

$$R^r_{A_3}(t) = 1.0024$$

Moreover, when the first and second item of system is duplicated using hot duplication method, the following reliability functions are obtained, respectively:

$$R^H_{A_1}(t) = 1$$

Reliability function of the main system and also improved system using different methods are displayed in the figure 4.

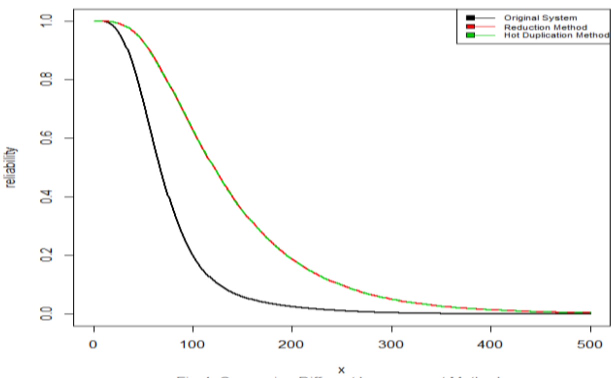

Fig.4: Comparing Different Improvement Methods

Figure 4. Comparing Different Reduction Method

The main system has always a minimum reliability level. Thus it can be resulted that different improvements methods provide various levels of reliability and also order of reliability levels sometimes changed.

IV. CONCLUSION

The authors conclude that the reliability of the improved system is much higher than the reliability of original system. In reduction method the reliability of the system is higher with smaller reduction factors.The reduction methods provide a better reliability than the hot duplication method.

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