Properties of New Parametric Generalized Entropy of Order Statistics

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Abstract: In this article, we introduce a new two parametric generalized uncertainty measure based on order statistics and also its residual version. Some general expressions of this proposed measure corresponding to particular probability distributions are derived. Finally, we study a lower bound for the proposed dynamic measure.

Keywords- *Shannon's Entropy; Order Statistics; Probability Integral Transformation; Residual Entropy; Generalized Information.*

I. INTRODUCTION

Shannon's [8] entropy is a significant concept introduced in information theory. It plays a decisive role in areas of engineering and physics as a measure of complexity and uncertainty in order to define and control many chaotic systems. The basic uncertainty measure is defined by Shannon for random variable (r.v.) W as,

$$H(W) = -\int_0^\infty f(w)\log f(w)du = -E[\log f(w)]$$

$$(1.1)$$

where $f(w)$ is the probability density function (pdf).

Assume that $W_1, W_2, ..., W_n$ be a random sample from a distribution function (cdf) $F(w)$ with pdf $f(w)$. By arranging $W_1, W_2, ..., W_n$ from the lowest to highest, the order statistics of the sample is defined as $W_{1:n} \le W_{2:n} \le ... \le W_{n:n}$. The density of kth order statistics [1] is given by,

$$f_{k:n}(w) = \frac{1}{B(k, n-k+1)}(F_w(w))^{k-1}(1-F_w(w))^{n-k} f_w(w)$$

$$(1.2)$$

where

$$B(u,v) = \frac{\Gamma u \Gamma v}{\Gamma(u+v)}$$

Order statistics finds its application in the wide range of real world problems like robust statistical estimation, detection of outliers [3], description of probability distribution of record values and order statistics [2], case-study of censored samples. Attributes of information pertaining to order statistics that are based on Kullback-Leibler [7] and Shannon entropy [8] and measure using probability integral transformation have been thoroughly investigated by Ebrahimi et.al [6]. Two parametric

generalized entropy, the Verma entropy [10] and [9] studied in context with order statistics. Various abstractions of Shannon's entropy [8] are present in the literature of information theory. There is a scope to further generalize the measure (1.1) in many ways and accordingly in this paper a new generalized Information measure is proposed.

In section II, we tend to specific generalized entropy of Kth order statistics in terms of the generalized entropy of Kth order statistics of exponential distribution and study a number of its properties. Section III, provide lower bound for entropy of order statistics. In section IV, we tend to derive an associate expression for residual generalized entropy of order statistics using residual generalized entropy for uniform distribution. In section V, some characterized results are analyzed for the generalized residual entropy of the proposed measure.

II. A NEW TWO PARAMETRIC GENERALIZED ENTROPY OF ORDER STATISTICS

Presume W be a continuous r.v. with pdf $f(w)$ and cdf $F(w)$, then the entropy of the type (θ, ϕ) of the r.v. W is given by,

$$H_{\theta,\phi}(W) = -\frac{\phi}{\theta - \phi}\log\int_0^\infty f^{\frac{\theta}{\phi}}(w)dw$$

$$\forall \phi \ge 1, \ \theta \ne \phi, \ 0 < \theta < \phi \qquad (2.1)$$

where

$$\lim_{\phi=1} H_{\theta,\phi}(W) = H_\theta(W) = \frac{1}{1-\theta}\log\int_0^\infty f^\theta(w)dw \qquad \text{is the}$$

Renyi's entropy

and $\lim\limits_{\phi=1,\theta\to1} H_{\theta,\phi}(W) = -\int_{0}^{\infty} f(w)\log f(w)d\,w$ is   the

Shannon entropy.

Using the substitution, $V = F(w)$ in (2.1) where V has a standard uniform distribution. Let $V_1,V_2,...,V_n$ be a random sample from a uniform distribution [0,1] with the order statistics $z_1 \le z_2 \le ... \le z_n$, then z_k, $k = 1,2,...,n$ has a beta distribution with pdf,

$$g(z_k) = \frac{1}{B(k,n-k+1)}(z_k)^{k-1}(1-z_k)^{n-k},$$
$$0 \le z_k \le 1 \qquad\qquad (2.2)$$

where

$B(u,v) = \int_{0}^{1} y^{u-1}(1-y)^{v-1}dy,\ \ u>0, v>0$ follows   a   beta

distribution with u and v as parameters.

Now the generalized entropy of $W_{k:n}$ is represented as,

$$H_{\theta,\phi}(W) = -\frac{\phi}{\theta-\phi}\log \int_{0}^{1} f^{\frac{\theta}{\phi}-1}\left(F^{-1}(V)\right)dV$$

$$(2.3)$$

Table No.1: The expressions of GE for some lifetime distributions

Distribution	$f(w)$	$H_{\theta,\phi}(W)$
Uniform	$\dfrac{1}{b-a}$ $a<w<b$	$\log(b-a)$
Exponential	$\lambda e^{-\lambda w}$ $w>0$ $\lambda>0$	$\dfrac{\phi}{\phi-\theta}\log\left[\dfrac{\phi\lambda^{\frac{\theta}{\phi}-1}}{\theta}\right]$
Gamma	$\dfrac{1}{\Gamma\omega}e^{-w}u^{\omega-1}$ $0<w<\infty$ $\omega>0$	$\dfrac{\phi}{\phi-\theta}\log\left[\dfrac{\gamma\left(\frac{\theta\lambda}{\phi}-\frac{\theta}{\phi}+1\right)}{\left(\frac{\theta}{\phi}\right)^{\frac{\theta\lambda}{\phi}-\frac{\theta}{\phi}+1}(\Gamma\omega)^{\frac{\theta}{\phi}}}\right]$
Pareto	$\dfrac{\omega\lambda^{\omega}}{w^{\omega+1}}$ $w\ge\lambda$ $\lambda>0$ $\omega>0$	$\dfrac{\phi}{\phi-\theta}\log\left[\left(\dfrac{\omega}{\lambda}\right)^{\frac{\theta}{\phi}}\dfrac{1}{\lambda\left(\frac{\theta}{\phi}(\omega+1)+1\right)}\right]$
Weibull	$\dfrac{1}{\lambda}e^{-\left(\frac{w-\omega}{\lambda}\right)}$ $w>\omega$ $\lambda>0$ $\omega>0$	$\dfrac{\phi}{\phi-\theta}\left[\log\left(\dfrac{\phi}{\theta\lambda^{\frac{\theta}{\phi}-1}}\right)\right]$
Lomax	$\dfrac{\omega}{(1+w)^{\omega+1}}$ $w\ge0$ $\omega>0$	$\dfrac{\phi}{\phi-\theta}\log\left[\dfrac{\omega^{\frac{\theta}{\phi}}}{\left(\frac{\theta}{\phi}(\omega+1)+1\right)}\right]$

Theorem 2.1: The new two parametric generalized entropy of $w_{k:n}$ can be defined as,

$$H_{\theta,\phi}(w_{k:n}) = H_{\theta,\phi}(z_k)$$
$$-\frac{\phi}{\theta-\phi}\log E_{g_k}\left[f^{\frac{\theta}{\phi}-1}\left(F^{-1}(X_k)\right)\right] \qquad (2.4)$$

where $H_{\theta,\phi}(z_k)$ denotes the entropy of beta distribution with parameters k and $(n-k+1)$, $E_{g_k}(w)$ denotes expectation of w over g_k and $X_k \sim g_k$ is the beta density with parameters $\left(\frac{\theta}{\phi}(k-1)+1\right)$ and $\left(\frac{\theta}{\phi}(n-k)+1\right)$.

Proof: We have

$$H_{\theta,\phi}(z_k) = \frac{\theta}{\theta-\phi}\log B(k,n-k+1)$$
$$-\frac{\phi}{\theta-\phi}\log B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right) \qquad (2.5)$$

Putting $z_k = F(w_{k:n})$, $k = 1,2,...,n$ which implies that $w_{k:n} = F^{-1}(z_k)$

Therefore, the generalized entropy of $w_{k:n}$ is given by,

$$H_{\theta,\phi}(w_{k:n}) = -\frac{\phi}{\theta-\phi}\log\int_0^1 f^{-1}\left(F^{-1}(z_k)\right)$$

$$\left(\frac{1}{B(k,n-k+1)}(z_k)^{k-1}(1-z_k)^{n-k}f\left(F^{-1}(z_k)\right)\right)^{\frac{\theta}{\phi}} dz_k$$

$$= \frac{\theta}{\theta-\phi}\log B(k,n-k+1)$$

$$-\frac{\phi}{\theta-\phi}\log B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)$$

$$-\frac{\phi}{\theta-\phi}\log\int_0^1 f^{\frac{\theta}{\phi}-1}\left(F^{-1}(z_k)\right)(z_k)^{\frac{\theta}{\phi}(k-1)}(1-z_k)^{\frac{\theta}{\phi}(n-k)}$$

$$\frac{1}{B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)} dz_k \qquad (2.6)$$

Using (2.5) in (2.6) the required result (2.4) follows,

Example 2.1: Suppose W is a r.v. that has an exponential distribution (ED) with pdf

$$f(w)=\lambda e^{-\lambda w}, \ \lambda>0, w\geq 0$$

Here $F(w)=1-e^{-\lambda w}$

Putting $X_k = F(w) \Rightarrow w = F^{-1}(X_k)$

$$E_{g_k}\left[f^{\frac{\theta}{\phi}-1}\left(F^{-1}(X_k)\right)\right]$$

$$= E_{g_k}\left[f^{\frac{\theta}{\phi}-1}\left(-\frac{1}{\lambda}\log(1-X_k)\right)\right]$$

$$= \lambda^{\frac{\theta}{\phi}-1}\left(\frac{B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k+1)\right)}{B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)}\right)$$

$$\qquad (2.7)$$

Putting $k=1$ in (2.5), we get

$$H_{\theta,\phi}(z_1) = -\frac{\theta}{\theta-\phi}\log(n)$$

$$+\frac{\phi}{\theta-\phi}\log\left(\frac{\theta}{\phi}(n-1)+1\right) \qquad (2.8)$$

Putting $k=1$ in (2.7), we get

$$E_{g_1}\left[f^{\frac{\theta}{\phi}-1}\left(F^{-1}(X_1)\right)\right] = \lambda^{\frac{\theta}{\phi}-1}\left(\frac{\frac{\theta}{\phi}(n-1)+1}{n\left(\frac{\theta}{\phi}\right)}\right)$$

$$\qquad (2.9)$$

Using (2.8) and (2.9) in (2.4), we have

$$H_{\theta,\phi}(W_{1:n}) = -\log(n)-\log(\lambda)+\frac{\phi}{\theta-\phi}\log\left(\frac{\theta}{\phi}\right)$$

$$H_{\theta,\phi}(W) = -\frac{\phi}{\theta-\phi}\log\left(\frac{\phi}{\theta}\right)-\log(\lambda)$$

$$H_{\theta,\phi}(W_{1:n})-H_{\theta,\phi}(W) = -\log(n)$$

$$H_{\theta,\phi}(W_{n:n}) = -\log(n)-\log(\lambda)-\frac{\phi}{\theta-\phi}\log\Gamma\left(\frac{\theta}{\phi}\right)$$

$$+\frac{\phi}{\theta-\phi}\log\left(\frac{\theta}{\phi}\right)$$

$$H_{\theta,\phi}(W_{n:n})-H_{\theta,\phi}(W) = -\log(n)$$

$$-\frac{\phi}{\theta-\phi}\log\Gamma\left(\frac{\theta}{\phi}\right)$$

III. LOWER BOUND FOR THE NEW TWO PARAMETRIC GENERALIZED ENTROPY OF ORDER STATISTICS

Theorem 3.1: A r.v. W with $H_{\theta,\phi}(W)<\infty$ an entropy of the kth order statistics $W_{k:n}$, $k=1,2,\dots,n$ is bounded below as,

$$H_{\theta,\phi}(W_{k:n}) \geq H_{\theta,\phi}(z_k)-\log(M)$$

where $M=f(m)<\infty$, f is the pdf of U and m is the mode of the distribution.

Proof: The mode of the beta distribution g_k is

$$m_k = \frac{k-1}{n-1}$$

Thus, $g_k(X) \leq B_k = g_k(m_k)$

$$= \frac{1}{B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)}(m_k)^{\frac{\theta}{\phi}(k-1)}(1-m_k)^{\frac{\theta}{\phi}(n-k)}$$

For $\phi>1$, $0<\theta<\phi$ from (2.4), we have

$$H_{\theta,\phi}(W_{k:n}) \geq H_{\theta,\phi}(z_k)$$

$$-\frac{\phi}{\theta-\phi}\log\int_0^1 g_k(z_k)(M)^{\frac{\theta}{\phi}-1}dw_k$$

$$H_{\theta,\phi}(W_{k:n}) \geq H_{\theta,\phi}(z_k)$$

$$-\frac{\phi}{\theta-\phi}\int_0^1 \frac{1}{B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)}(z_k)^{\frac{\theta}{\phi}(k-1)}$$

$$\left((1-z_k)^{\frac{\theta}{\phi}(n-k)}f^{\frac{\theta}{\phi}-1}(m_k)\right)dz_k$$

$$H_{\theta,\phi}(W_{k:n}) \geq H_{\theta,\phi}(z_k)-\log(M)$$

Example 3.1: Over the interval [a, b] in the lifetime(uniform) distribution, we have

$$H_{\theta,\phi}(W) = \log(b-a)$$

$$H_{\theta,\phi}(z_1) = H_{\theta,\phi}(z_n) = -\frac{\theta}{\theta-\phi}\log(n)$$

$$+ \frac{\phi}{\theta-\phi}\log\left(\frac{\theta}{\phi}(n-1)+1\right)$$

$$H_{\theta,\phi}(W_{k:n}) - H_{\theta,\phi}(z_k) \geq$$

$$-\frac{\phi}{\theta-\phi}\int_0^1 \frac{1}{B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)}(z_k)^{\frac{\theta}{\phi}(k-1)}\left(1-z_k\right)^{\frac{\theta}{\phi}(n-k)} f^{\frac{\theta}{\phi}-1}(m_k)$$

$$H_{\theta,\phi}(W_{k:n}) \geq H_{\theta,\phi}(z_k) + \log(b-a)$$

Substituting $k=1$ in above equation, we obtain

$$H_{\theta,\phi}(W_{1:n}) \geq H_{\theta,\phi}(z_1) + \log(b-a)$$

$$H_{\theta,\phi}(W_{n:n}) \geq H_{\theta,\phi}(z_n) + \log(b-a)$$

We conclude that the bounds for $H_{\theta,\phi}(W_{1:n})$ and $H_{\theta,\phi}(W_{n:n})$ are similar.

IV. RESIDUAL ENTROPY FOR THE NEW TWO PARAMETRIC GENERALIZED ENTROPY OF ORDER STATISTICS

Ebrahim [5] for a random life time of W of a system, at time t the residual entropy is defined as,

$$H(w;t) = -\int_t^\infty f_t(w)\log f_t(w)\,du \qquad (4.1)$$

Equation (4.1) evaluates the uncertainty in the remaining lifetime of the component which is believed to have survived to a lifetime of t.

where $f_t(w)$ is the pdf of the r.v. $W_t = (W - t / W > t)$ and is given by,

$$f_t(w) = \left\{\frac{f(w)}{\overline{F}(t)}, \; if\; w > t\right.$$

Using above result (4.1) can be rewritten as,

$$H(w;t) = -\int_t^\infty \frac{f(w)}{\overline{F}(t)}\log\frac{f(w)}{\overline{F}(t)}\,du, \; t > 0$$

$$\qquad (4.2)$$

where $\overline{F}(t) = 1 - F(t)$ is the survival function (s.f.) of W.
In the same way the generalized entropy of the residual life time W_t is given by,

$$H_{\theta,\phi}(w;t) = -\frac{\phi}{\theta-\phi}\log\int_t^\infty \left(\frac{f(w)}{\overline{F}(t)}\right)^{\frac{\theta}{\phi}}dw.$$

$$0 < \theta < \phi, \; \phi \geq 1 \qquad (4.3)$$

when $t = 0$, it reduces to (2.1).
We note that the pdf and s.f. of $W_{k:n}$ (refer to [4]), denoted by $f_{k:n}(w)$ and $\overline{F}_{k:n}(w)$, $k = 1,2,\dots,n$ respectively are

$$f_{k:n}(w)$$

$$= \frac{1}{B(k,n-k+1)}(F(w))^{k-1}(1-F(w))^{n-k}f(w) \qquad (4.4)$$

where

$$B(u,v) = \int_0^1 y^{u-1}(1-y)^{v-1}dy, \; u > 0, v > 0 \quad , \quad \text{follows beta}$$

distribution.

$$\overline{F}_{k:n}(w) = \frac{\overline{B}_{F(w)}(k,n-k+1)}{B(k,n-k+1)} \qquad (4.5)$$

where

$$B_w(u,v) = \int_w^1 w^{u-1}(1-w)^{v-1}dw, \; 0 < w < 1$$

follows, incomplete beta distribution.

Theorem 4.1: The new two parametric generalized residual entropy of the kth order statistics is expressed as

$$H_{\theta,\phi}(W_{k:n},t) = H_{\theta,\phi}(z_k, F(t))$$

$$- \frac{\phi}{\theta-\phi}\log E_{g_k}\left[f^{\frac{\theta}{\phi}-1}\left(F^{-1}(X_k)\right)\right] \qquad (4.6)$$

where $H_{\theta,\phi}(z_k, F(t))$ represents the generalized residual entropy of the beta variate with parameters $(k \,\&\, n-k+1)$, $E_{g_k}(z)$ represents the expectation of z over the random sample of size n from uniform distribution on $[0,1]$. Then

$$H_{\theta,\phi}(z_k, F(t)) = \frac{\theta}{\theta-\phi}\log \overline{B}_{F(t)}(k,n-k+1)$$

$$- \frac{\phi}{\theta-\phi}\log \overline{B}_{F(t)}\left(\frac{\theta}{\phi}(k-1)+1, \frac{\theta}{\phi}(n-k)+1\right) \qquad (4.7)$$

$g_k \,\& Z_k \sim g_k$ is the incomplete beta density with parameters $\left(\frac{\theta}{\phi}(k-1)+1, \frac{\theta}{\phi}(n-k)+1\right)$

Proof: Let z_k be the kth order statistics based on represents the generalized entropy of beta variate with parameters $(k \,\&\, n-k+1)$.

$$H_{\theta,\phi}(W_{k:n},t) = -\frac{\phi}{\theta-\phi}\log\left[\frac{\overline{B}_{F(t)}(k,n-k+1)}{B(k,n-k+1)}\right]^{\frac{\theta}{\phi}}$$

$$+ \frac{\phi}{\theta-\phi}\log(B(k,n-k+1))^{\frac{\theta}{\phi}}$$

$$- \frac{\phi}{\theta-\phi}\log\int_0^\infty \left((F(w))^{k-1}(1-F(w))^{n-k}f(w)\right)^{\frac{\theta}{\phi}}dw$$

Putting $X_k = F(W_{k:n})$ we have $F^{-1}(X_k) = W_{k:n}$

$$H_{\theta,\phi}(W_{k:n},t) = \frac{\theta}{\theta-\phi}\log \overline{B}_{F(t)}(k,n-k+1)$$

$$-\frac{\phi}{\theta-\phi}\log \overline{B}_{F(t)}\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)$$

$$-\frac{\phi}{\theta-\phi}\int_{F(t)}^{1}\frac{1}{B\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)}(X_k)^{\frac{\theta}{\phi}(k-1)}(1-X_k)^{\frac{\theta}{\phi}(n-k)}f^{\frac{\theta}{\phi}-1}\left(F^{-1}(X_k)\right)dX_k$$

$$(4.8)$$

using (4.7) in (4.8), we get the desired result (4.6).

Example 4.1: Let W be an exponentially distributed r.v. with pdf $f(w)=\lambda e^{-\lambda w}$, $w\geq 0, \lambda >0$, then
$F(w)=1-e^{-\lambda w}$ putting $X_k=F(w)$

$$E_{g_k}\left(f^{\frac{\theta}{\phi}-1}\left(F^{-1}(X_k)\right)\right)$$

$$=\lambda^{\frac{\theta}{\phi}-1}\frac{\overline{B}_{F(t)}\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k+1)\right)}{\overline{B}_{F(t)}\left(\frac{\theta}{\phi}(k-1)+1,\frac{\theta}{\phi}(n-k)+1\right)} \qquad (4.9)$$

Putting $k=1$ (4.7) and (4.9), we get

$$H_{\theta,\phi}(z_k,F(t))=\frac{\theta}{\theta-\phi}\log\left(\overline{B}_{F(t)}(1,n)\right)$$

$$-\frac{\phi}{\theta-\phi}\log\left(\overline{B}_{F(t)}\left(1,\frac{\theta}{\phi}(n-1)+1\right)\right)$$

$$E_{g_1}\left(f^{\frac{\theta}{\phi}-1}\left(F^{-1}(X_1)\right)\right)=\lambda^{\frac{\theta}{\phi}-1}\frac{\overline{B}_{F(t)}\left(1,\frac{\theta}{\phi}n\right)}{\overline{B}_{F(t)}\left(1,\frac{\theta}{\phi}(n-1)+1\right)}$$

Using above results in (4.6), we get

$$H_{\theta,\phi}(W_{1:n},t)=-\log(n\lambda)+\frac{\phi}{\theta-\phi}\log\left(\frac{\theta}{\phi}\right)$$

Also, $H_{\theta,\phi}(W,t)=-\frac{\phi}{\theta-\phi}\log\left(\frac{\phi}{\theta}\right)-\log(\lambda)$

Hence, $H_{\theta,\phi}(W_{1:n},t)-H_{\theta,\phi}(W,t)=-\log n$

Therefore, the divergence in the above two discussed entropies is independent of time in the exponential case.

V. PROPERTIES OF NEW TWO PARAMETRIC GENERALIZED RESIDUAL ENTROPY

Theorem 5.1: Let $\overline{F}$ (survival function) be IGE (DGE) and $\phi>\theta$, then

$$\lambda_{F_{k:n}}(t)\leq(\geq)\left[\frac{\theta}{\phi}\exp\frac{(\phi-\theta)}{\phi}H_{\theta,\phi}\left(H_{\theta,\phi}(W_{k:n},t)\right)\right]^{\frac{\phi}{\theta-\phi}}$$

$$t>0$$

Proof: From (4.3), we have.

$$\frac{(\phi-\theta)}{\phi}H'_{\theta,\phi}(W_{k:n},t)$$

$$=\frac{\theta}{\phi}\lambda_{F_{k:n}}(t)-\frac{\lambda^{\frac{\theta}{\phi}}F_{k:n}(t)}{\exp\left[\frac{(\phi-\theta)}{\phi}H'_{\theta,\phi}(W_{k:n},t)\right]}$$

Since $\overline{F}$ is IGE (DGE) and $\phi>\theta$, therefore, we have

$$\lambda_{F_{k:n}}(t)\left[\frac{\theta}{\phi}\exp\left(\left[\frac{(\phi-\theta)}{\phi}H'_{\theta,\phi}(W_{k:n},t)\right]\right)\right.$$

$$\left.-\lambda^{\frac{\theta}{\phi}-1}F_{k:n}(t)\right]\geq(\leq)0$$

which leads to
$$\lambda_{F_{k:n}}(t)$$

$$\geq(\leq)\left[\frac{\theta}{\phi}\exp\frac{(\phi-\theta)}{\phi}H_{\theta,\phi}\left(H_{\theta,\phi}(W_{k:n},t)\right)\right]^{\frac{\phi}{\theta-\phi}} (5.1)$$

Theorem 5.2: Let W be the lifetime of a series system with pdf $f_{k:n}(w)$ and s.f. $\overline{F}_{k:n}(t)$, $t>0$, then for $\phi>\theta$, the following un equivalence holds
$$H_{\theta,\phi}(W_{k:n},t)\geq H(W_{k:n},t)$$

Proof: We know that from log sum inequality,

$$\int_t^\infty f_{k:n}(w)\log\frac{f_{k:n}(w)}{\left(\frac{f_{k:n}(w)}{\overline{F}_{k:n}}\right)^{\frac{\theta}{\phi}}}dw$$

$$\geq\left(\int_t^\infty f_{k:n}(w)dw\right)\log\left(\frac{\int_t^\infty f_{k:n}(w)dw}{\int_t^\infty\left(\frac{f_{k:n}(w)}{\overline{F}_{k:n}}\right)^{\frac{\theta}{\phi}}dw}\right)$$

$$=\overline{F}_{k:n}(t)\left(\log\overline{F}_{k:n}(t)-\frac{(\phi-\theta)}{\phi}H_{\theta,\phi}(W_{k:n},t)\right) (5.2)$$

The LHS of (5.2) contributes to.

$$\int_t^\infty f_{k:n}(w)\log f_{k:n}(w)dw+\frac{\theta}{\phi}\overline{F}_{k:n}(t)H(W_{k:n},t).$$

$$(5.3)$$

Put (5.3) in (5.2), we obtain the desired result.

Table No. 2: Generalized residual entropy of some lifetime distributions

Distribution	$f(w)$	$H_{\theta,\phi}(W;t)$
Uniform	$\dfrac{1}{b-a}$ $a < w < b$	$\log(b-t)$
Exponential	$\gamma e^{-\gamma w}$ $w > 0$ $\gamma > 0$	$\dfrac{\phi}{\theta - \phi} \log \left[\dfrac{\phi \gamma^{\frac{\theta}{\phi}-1}}{\theta} \right]$
Gamma	$\dfrac{1}{\Gamma \mu} e^{-w} w^{\mu-1}$ $0 < w < \infty$	$\dfrac{\phi}{\phi-\theta} \log \left[\dfrac{\gamma\left(\frac{\theta\mu}{\phi}-\frac{\theta}{\phi}+1, \frac{\theta t}{\phi}\right)}{\left(\frac{\theta}{\phi}\right)^{\frac{\theta\mu}{\phi}-\frac{\theta}{\phi}+1} (\Gamma\mu - \gamma(\mu,t))^{\frac{\theta}{\phi}}} \right]$
Weibull	$\dfrac{1}{\lambda} e^{-\left(\frac{w-\mu}{\lambda}\right)}$ $w > \mu$ $\lambda > 0$ $\mu > 0$	$\dfrac{\phi}{\theta - \phi} \left[\log\left(\dfrac{\phi}{\theta}\right) + \dfrac{\theta}{\phi} \right]$

Fig. 1(a): $H_{(\theta,\phi)}(W_{1:n})$ of Exponential Distribution
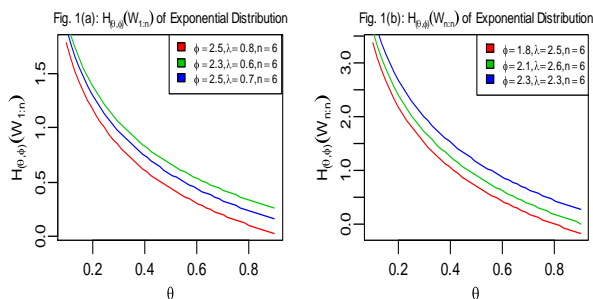Fig. 1(b): $H_{(\theta,\phi)}(W_{n:n})$ of Exponential Distribution
Fig.1: $H_{(\theta,\phi)}(W_{1:n})$, and $H_{(\theta,\phi)}(W_{n:n})$ plots of Exponential Distribution for $0.1 \leq \theta \leq 0.9$.

It is clear from Fig (1) that both the $H_{\theta,\phi}(W_{1:n})$ and $H_{\theta,\phi}(W_{n:n})$ generalized entropies with respect to Exponential distribution are monotonic decreasing.

VI. CONCLUSION

A new generalized two parametric entropy of order statistics is proposed in this paper. Various characterizations based on this measure are also studied. The generalized entropy finds its application in many practical situations like areas of engineering, physics etc. Some results of this new generalized entropy are also derived and the behavior of generalized expressions are mentioned graphically. The proposed two parametric measure can be of utmost research interest in future.

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