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An Exponential Strategy for Estimation of Population Mean in Systematic Sampling

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Abstract- Using exponential function, an estimator for mean has been developed and it's Properties are studied. The mean squared error of the developed estimator has been derived. To compare developed estimator with existing estimators. An empirical comparative study is also conducted.

Keywords :- population mean, Exponential estimator ,Mean squared error.

I. INTRODUCTION

In systematic sampling, Shukla (1971) worked on classical ratio .An almost unbiased ratio and product estimators were envisaged by Singh and Singh (1998). Recently, Singh et al. (2011) studied ratio and product type exponential estimators in systematic sampling. In this paper a ratio-cum-product –type exponential estimator for population mean is suggested and it's properties are studied. For the present work, Let us consider a population $U=\{U_1, U_2, U_3, \ldots U_N\}$ of size N serially numbered from 1 to N. To draw a systematic sample, first unit is selected at random between 1 to K, where $K=\frac{N}{n}$. This unit is denoted by i and then every K^{th} unit is selected. This systematic sample is obtained as i, i+k, i+2k, i+3k ----- i+(n-1)k units.

Swain (1964) suggested ratio estimator for $\overline{Y}$ in systematic sampling as

$$\hat{\overline{Y}}_{Rsys}=\bar{y}_{sys}\left(\frac{\overline{X}}{\bar{x}_{sys}}\right) \qquad (1.1)$$

where $\bar{x}_{sys}=\frac{1}{n}\sum X_{ij}$ is an unbiased estimator of population mean $\overline{Y}$.

$\overline{X}=\frac{1}{N}\sum_{j=1}^{n}X_{ij}$, the population mean of the auxiliary variate x. Here , $\overline{X}$ is assumed to be known. .

Shukla (1971) worked out product type estimator for population mean as

$$\hat{\overline{Y}}_{Psys}=\bar{y}_{sys}\left(\frac{\bar{z}_{sys}}{\overline{Z}}\right). \qquad (1.2)$$

Using exponential function a ratio- type and a product- type estimator were worked out by Bahl and Tuteja (1991) as

$$\hat{\bar{Y}}_{Re} = \bar{y} \times \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{1.3}$$

and

$$\hat{\bar{Y}}_{Pe} = \bar{y} \times \exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}}\right). \tag{1.4}$$

$\hat{\bar{Y}}_{Resys}$ and $\hat{\bar{Y}}_{Pesys}$ was studied in systematic sampling by Singh et al. (2011) as

$$\hat{\bar{Y}}_{Resy} = \bar{y}_{sys} \times \exp\left(\frac{\bar{X} - \bar{x}_{sys}}{\bar{X} + \bar{x}_{sys}}\right) \tag{1.5}$$

and

$$\hat{\bar{Y}}_{Pesy} = \bar{y}_{sys} \times \exp\left(\frac{\bar{z}_{sys} - \bar{z}}{\bar{z}_{sys} + \bar{z}}\right). \tag{1.6}$$

Variances of the ratio estimators $\hat{\bar{Y}}_R^{sys}, \hat{\bar{Y}}_P^{sys}$, $\hat{\bar{Y}}_{Resy}$ and $\hat{\bar{Y}}_{Pesy}$ are written as

$$V(\hat{\bar{Y}}_{Rsys}) = \frac{(N-1)}{nN}\bar{Y}^2\left[\rho_y^* C_y^2 + \rho_x^* C_x^2 (1 - 2K\sqrt{\rho^{**}})\right]. \tag{1.7}$$

and

$$V(\hat{\bar{Y}}_{Resy}) = \frac{(N-1)}{nN}\bar{Y}^2\left[\rho_y^* C_y^2 + \rho_x^* (C_x^2/4)(1 - 4K\sqrt{\rho^{**}})\right]. \tag{1.8}$$

$$V(\hat{\bar{Y}}_{Psys}) = \frac{(N-1)}{nN}\bar{Y}^2\left[\rho_y^* c_y^2 + \rho_z^* c_z^2 (1 + 2k\sqrt{\rho_2^{**}})\right]. \tag{1.9}$$

$$V(\hat{\bar{Y}}_{Pesys}) = \frac{(N-1)}{nN}\bar{Y}^2\left[\rho_y^* c_y^2 + \rho_z^* (c_z^2/4)(1 - 4k\sqrt{\rho_2^{**}})\right]. \tag{1.10}$$

where

$$k = \rho_{yx}\frac{C_y}{C_x}, \quad \rho_y^* = [1 + \rho_y(n-1)], \quad \rho_x^* = [1 + \rho_x(n-1)], \quad \rho_z^* = [1 + \rho_z(n-1)],$$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{K}\sum_{j=1}^{n}(x_{ij} - \bar{X}) \times (y_{ij} - \bar{Y}), \quad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \bar{X})^2,$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{Y})^2, \quad S_z^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(z_{ij} - \bar{Z})^2,$$

$$k^* = \rho_{yz}\frac{C_y}{C_z} \quad \text{and} \quad \rho_2^{**} = \frac{\rho_y^*}{\rho_z^*}.$$

and (ρ_y, ρ_x, ρ_z) are the intra- class correlation coefficient between the units of a cluster corresponding the (y, x, z) variates.

II. SUGGESTED ESTIMATOR

Singh (1967) envisaged ratio-cum –product estimator for population mean in simple random sampling. Tailor et al.(2011) studied properties of singh (1967) estimators in stratified random sampling. Singh et al.(2009) studied ratio –cum- product type exponential estimators. In simple random sampling assuming that the auxiliary variate x is positively correlated with the study variate y and the auxiliary variate z is negatively correlated with the y, motivated by above cited work, the suggested raio-cum-product type exponential estimator for the population mean in systematic sampling is expressed as

$$\hat{\bar{Y}}_{RPe}^{sys} = \bar{y}_{sys} \exp\left(\frac{\bar{X}-\bar{x}_{sys}}{\bar{X}+\bar{x}_{sys}}\right)\exp\left(\frac{\bar{z}_{sys}-\bar{Z}}{\bar{Z}_{sys}+\bar{Z}}\right). \quad (2.1)$$

To get essential property of $\hat{\bar{Y}}_{RPe}^{sys}$ .i.e. bias and mean squared error ,it is assumed that

$\bar{y}_{sys} = \bar{Y}(1+e_0)$, $\bar{x}_{sys} = \bar{X}(1+e_1)$, $\bar{z}_{sys} = \bar{Z}(1+e_2)$ such that

$$E(e_0) = E(e_1) = E(e_2) = 0$$

$$E(e_0^2) = \theta C_y^2 \rho_y^* , \quad E(e_1^2) = \theta C_x^2 \rho_x^* , \quad E(e_2^2) = \theta C_z^2 \rho_z^* ,$$

$$E(e_0 e_1) = \theta k C_x^2 \sqrt{\rho_y^*}\rho_x^* , \; E(e_0 e_2) = \theta k^* C_z^2 \sqrt{\rho_y^*}\rho_z^* , \; E(e_1 e_2) = \theta k^{**} C_z^2 \sqrt{\rho_x^*}\rho_z^*$$

$$C_z^2 = \frac{S_z^2}{\bar{Z}^2}, \quad C_x^2 = \frac{S_x^2}{\bar{X}^2}, \quad C_y^2 = \frac{S_y^2}{\bar{Y}^2}, \; \theta = \frac{N-1}{nN} , \; \rho_{yz} = \frac{S_{yz}}{S_y S_z}, \; \rho_{xz} = \frac{S_{xz}}{S_x S_z},$$

$$\rho_{yx} = \frac{S_{yx}}{S_y S_x}, \quad \rho_x^* = (1+(n-1)\rho_x), \quad \rho_y^* = (1+(n-1)\rho_y), \; \rho_z^* = (1+(n-1)\rho_z)$$

$$k^{**} = \rho_{xz}\left(\frac{c_x}{c_z}\right), \quad k^* = \rho_{yz}\left(\frac{c_y}{c_z}\right), \; k = \rho_{yx}\left(\frac{c_y}{c_x}\right),$$

Finally, upto the first degree of approximation, the bias and mean squared error of the suggested estimator are $\hat{\bar{Y}}_{RPe}^{sys}$ obtained as

$$B(\hat{\bar{Y}}_{RPe}^{sys}) = \frac{(N-1)}{nN}\bar{Y}^2\left[\frac{3}{8}\rho_x^* c_x^2 - \frac{1}{8}\rho_z^* c_z^2 + \frac{1}{2}k^* c_z^2\sqrt{\rho_y^*}\rho_z^* - \frac{1}{2}k c_x^2\sqrt{\rho_y^*}\rho_x^* - \frac{1}{4}k^{**} c_z^2\sqrt{\rho_x^*\rho_z^*}\right] \quad (2.2)$$

and

$$V(\hat{\bar{Y}}_{RPe}^{sys}) = \frac{(N-1)}{nN}\bar{Y}^2\left[\rho_y^* c_y^2 + \rho_x^*(c_x^2/4) + \rho_z^*(c_z^2/4) + k^* c_z^2\sqrt{\rho_y^*}\rho_z^* - k c_x^2\sqrt{\rho_y^*}\rho_x^* - \frac{1}{2}k^{**} c_z^2\sqrt{\rho_x^*\rho_z^*}\right]. \quad (2.3)$$

III. EFFICIENCY COMPARISON

Variance of classical-unbiased estimator for population mean is expressed as

$$V(\bar{y}_{sys}) = \frac{(N-1)}{nN}\bar{Y}^2 \rho_y^* C_y^2 . \quad (3.1)$$

Comparision of (3.1), (1.7), (1.9), (1.11) and (2.3) show that the developed estimator $\hat{\bar{Y}}_{RPe}^{sys}$ would be more efficient than

(i) $\bar{y}_{sys}$ if

$$Var(\hat{\bar{Y}}_{RPe}^{sys}) - Var(\bar{y}_{sys}) \prec 0,$$

$$\Rightarrow \left[\frac{(N-1)}{nN}\bar{Y}^2\left[\rho_y^* c_y^2 + \rho_x^*(c_x^2/4) + \rho_z^*(c_z^2/4) + k^* c_z^2\sqrt{\rho_y^*}\rho_z^* - k c_x^2\sqrt{\rho_y^*}\rho_x^* - \frac{1}{2}k^{**} c_z^2\sqrt{\rho_x^*\rho_z^*}\right] - \frac{(N-1)}{nN}\bar{Y}^2 \rho_y^* C_y^2 .\right.$$

$$\Rightarrow \frac{(N-1)}{nN}\ \overline{Y}^2\left[\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]<0$$

(ii) $\hat{\overline{Y}}_{Rsys}$ if

$$Var(\hat{\overline{Y}}_{RPe}^{sys})-Var(\hat{\overline{Y}}_{Rsys})\prec 0,$$

$$\Rightarrow \left[\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*c_y^2+\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]-\right.$$
$$\left.\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*C_y^2+\rho_x^*C_x^2(1-2K\sqrt{\rho^{**}})\right]\right]\prec 0$$

$$\Rightarrow\left[\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_x^*(c_x^2/4)-\rho_x^*C_x^2(1-2K\sqrt{\rho^{**}})+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]\right]\prec 0 \quad (4.3.1)$$

(iii) $\hat{\overline{Y}}_{Resys}$ if

$$Var(\hat{\overline{Y}}_{RPe}^{sys})-Var(\hat{\overline{Y}}_{Resys})\prec 0,$$

$$\Rightarrow \left[\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*c_y^2+\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]-\right.$$
$$\left.\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*C_y^2+\rho_x^*(C_x^2/4)(1-4K\sqrt{\rho^{**}})\right]\right]\prec 0$$

$$\Rightarrow\left[\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_x^*c_x^2k\sqrt{\rho^{**}}+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]\right]\prec 0 \quad (4.3.2)$$

(iv) $\hat{\overline{Y}}_{Psys}$ if

$$Var(\hat{\overline{Y}}_{RPe}^{sys})-Var(\hat{\overline{Y}}_{Psys})\prec 0,$$

$$\Rightarrow \left[\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*c_y^2+\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]-\right.$$
$$\left.\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*c_y^2+\rho_z^*c_z^2(1+2k\sqrt{\rho_2^{**}})\right]\right]\prec 0$$

$$\Rightarrow\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)-\rho_z^*c_z^2(1+2k\sqrt{\rho_2^{**}})+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]\prec 0 \quad (4.3.3)$$

(v) $\hat{\overline{Y}}_{Pesys}$ if

$$Var(\overline{Y}_{RPe}^{sys})-Var(\overline{Y}_{Pesys})\prec 0,$$

$$\Rightarrow \frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*c_y^2+\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]-$$
$$\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_y^*c_y^2+\rho_z^*(c_z^2/4)(1-4k\sqrt{\rho_2^{**}})\right]\prec 0$$

$$\Rightarrow \frac{(N-1)}{nN}\overline{Y}^2\left[\rho_x^*(c_x^2/4)+\rho_z^* c_z^2 k\sqrt{\rho_2^{**}}+k^* c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]$$

$$\Rightarrow \frac{(N-1)}{nN}\overline{Y}^2\left[\rho_x^*(c_x^2/4)+\rho_z^* c_z^2 k\sqrt{\rho^{**}}+k^* c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]\prec 0 \quad (4.3.4)$$

IV. EMPIRICAL STUDY

This section compares considerd estimators empirically. For this purpose an artificial population is being considered. Parameters of this artificial population are given below.

$\overline{X}=44.47$, $\overline{Y}=80$, $\overline{Z}=44.47$, $C_x=0.28$, $C_y=0.56$, $C_z=0.43$, $S_x^2=149.55$, $S_y^2=2000$, $S_z^2=427.83$, $S_{xz}=-241.06$, $S_{yz}=-902.86$, $S_{xy}=-241.06$, $\rho_{xy}=0.9848$, $\rho_{yz}=-0.9760$ $\rho_{zx}=-0.9530$, $\rho_x=0.707$, $\rho_y=0.6652$, $\rho_z=0.5487$, N=15, n=3

Table (4.1) Percent Relative Efficiencies of $\hat{\overline{Y}}_{Rsys}, \hat{\overline{Y}}_{Psys}, \hat{\overline{Y}}_{Resys}, \hat{\overline{Y}}_{Pesys}$ and $\hat{\overline{Y}}_{RPesy}^{sys}$ with respect to $\overline{y}_{sys}$.

Estimator	PRE$(.\,,\overline{y}_{sys})$
$\overline{y}_{sys}$	100.00
$\hat{\overline{Y}}_{Rsys}$	389.620
$\hat{\overline{Y}}_{Psys}$	189.452
$\hat{\overline{Y}}_{Resys}$	177.434
$\hat{\overline{Y}}_{pesys}$	139.318
$\hat{\overline{Y}}_{RPe}^{sys}$	617.606

Table 4.2 Empirical Exhibition of Theoretical Conditions Obtained in Section-(4.3)

$Var(\overline{Y}_{RPe}^{sys})-Var(\overline{y}_{sys})\prec 0$	$\frac{(N-1)}{nN}\overline{Y}^2\left[\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\right]\prec 0$	$1219.52\prec 0$
$Var(\hat{\overline{Y}}_{RPe}^{sys})-Var(\hat{\overline{Y}}_{Rsys})\prec 0$	$\frac{(N-1)}{nN}\overline{Y}^2\left[\begin{array}{l}\rho_x^*(c_x^2/4)-\rho_x^*C_x^2(1-2K\sqrt{\rho^{**}})+\rho_z^*(c_z^2/4)+\\ k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\end{array}\right]\prec 0$	$-137.86\prec 0$
$Var(\hat{\overline{Y}}_{RPe}^{sys})-Var(\hat{\overline{Y}}_{Resys})\prec 0$	$\frac{(N-1)}{nN}\overline{Y}^2\left[\begin{array}{l}\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)-\rho_z^*c_z^2(1+2k\sqrt{\rho_2^{**}})\\ +k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\end{array}\right]\prec 0$	$-2042.9\prec 0$

$Var(\hat{\hat{Y}}_{RPe}^{sys})-Var(\hat{\hat{Y}}_{Psys})\prec 0$	$\frac{(N-1)}{nN}\overline{Y}^2\left[\begin{array}{l}\rho_x^*(c_x^2/4)+\rho_z^*(c_z^2/4)-\rho_z^*c_z^2(1+2k\sqrt{\rho_2^{**}})\\+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\end{array}\right]\prec 0$	$-5197.93\prec 0$
$Var(\hat{\hat{Y}}_{RPe}^{sys})-Var(\hat{\hat{Y}}_{Pesys})\prec 0$	$\frac{(N-1)}{nN}\overline{Y}^2\left[\begin{array}{l}\rho_x^*(c_x^2/4)+\rho_z^*c_z^2k\sqrt{\rho^{**}}+k^*c_z^2\sqrt{\rho_y^*}\rho_z^*\\-kc_x^2\sqrt{\rho_y^*}\rho_x^*-\frac{1}{2}k^{**}c_z^2\sqrt{\rho_x^*\rho_z^*}\end{array}\right]\prec 0$	-2646.2<0

V. CONCLUSIONS

In this paper, a ratio-cum-product type exponential estimator for population mean has been developed using exponential function. Theoretical comparsion has been done in section 3 where conditions have been obtained under which developed estimator has less mean squared error as compared to other considered estimators. Table 4.1 shows that the developed ratio-cum-product estimator has maximum PRE among all other considered estimators. In addition ,Table 4.2 exhibits that theoretical conditions are satisfied for the given empirical illustration. Thus it can be conclude that estimator developed in this work is more efficient then $\overline{y}_{sys}$ $\hat{\hat{Y}}_{Rsys}, \hat{\hat{Y}}_{Psys}, \hat{\hat{Y}}_{Resys}, \hat{\hat{Y}}_{Pesys}$. and recommended for estimation of population mean if conditions obtained in section 3 are satisfied.

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