International Journal of Scientific Research in Mathematical and Statistical Sciences
Research Paper
Vol.6, Issue.2, pp.68-72, April (2019)
E-ISSN: 2348-4519
DOI: https://doi.org/10.26438/ijsrmss/v6i2.6872

Wave Forces Due To Interaction of Water Wave with A Cylindrical Hollow Structure in Water of Finite Depth

M. Hassan[1*], P. Borah[1], M. Viswakarma[1]

[1]Dept. of Mathematics, North Eastern Regional Institute of Science and Technology, Itanagar, India

[*]*Corresponding Author: mdhassan000@gmail.com,*

Available online at: www.isroset.org

Received: 09/Mar/2019, Accepted: 13/Apr/2019, Online: 30/Apr/2019

***Abstract*—** We consider a single floating hollow cylinder in water of finite depth and investigated associated diffraction problem due to interaction of water wave with hollow cylinderical structure. This device can be considered as wave energy device because a single hollow cylindrical structure represents a particular form of wave energy device (oscillating water column). We used the method of separation of variables to obtain the analytical expressions for the diffracted velocity potentials in clearly identified regions. By using the appropriate matching conditions along the virtual and physical boundaries between the regions, we obtained and then solved a system of linear equations for the unknown coefficients. We obtained wave forces which play a significant part for a floating structure. It is observed that the changes in radius and draft of the cylinder have significant effect on exciting forces. For higher range of frequencies, the exciting forces diminished and tends to zero. The values of exciting forces with various parameters are depicted graphically and compared with available results.

Keywords *— Diffraction, Wave force, finite depth, virtual boundary, frequency*

I. INTRODUCTION

Diffraction of water waves by floating structure has long been investigated by many researchers under the assumption of linearized water wave theory. The present investigation is related to the diffraction of water waves by a single hollow cylindrical structure. This model can be referred as one kind of wave energy device (Oscillating Water Column). By this system of energy device, the power of ocean waves can be converted to an electrical energy. Proper positioning of the device will allow the device to capture waves as large as possible. This type of energy converter assumes immense significance for offshore structures also. It is known, in general, that corresponding to a coordinate system OXYZ, a floating structure undergoes six degrees of freedom: the translational motions in the x-, y- and z-directions are referred to as surge, sway and heave, respectively and the rotational motions about x, y and z axes are referred to as roll, pitch and yaw, respectively.

Various theoretical investigations have been carried out to analyze the wave motion and wave force on a structure. Garrett [1] presented the results for the horizontal and vertical forces and torque on a dock. He used Galerkin′s method to solve the problem numerically. Bhatta and Rahman [2] calculated the wave loading due to scattering and radiation for a floating cylinder in water of finite depth. They decomposed the total velocity potential in to four: one due to scattering and the other three due to radiation. For each case, they derived the velocity potential by considering interior and exterior regions. Wu et. al. [3, 4] investigated the problem of diffraction and radiation for two solid cylinders under different considerations. They obtained the expression for the velocity potential by using the separation of variables method and matched eigenfunction expansion method and investigated the effect of the caisson, approximated by a solid cylinder, on the floating cylinder. Hydrodynamic coefficients and exciting forces were presented for some ratios of the radius of the submerged cylinder to that of the riding one. Mavrakos [5] investigated the diffraction problem of the interaction between regular sinusoidal incident wave and a bottomless cylindrical floating body with a vertical symmetry axis and finite wall thickness. Newman [6] presented the hydrodynamic coefficients of a special toroidal body under linear water wave theory. He investigated hydrodynamic coefficient and elevation to the free surface for a range of wavenumbers in the moon pool along with singular results. Hassan and Bora [7, 8] considered a pair of co-axial hollow cylinder and solid cylinder in water of finite depth and presented sets of exciting forces for different radii of the cylinders and for different gaps between the cylinders.

Nonlinear water wave theory was employed by Rahman and Bhatta [9] in which they derived second order wave forces acting on a pair of cylinders. Shen et. al. [10] investigated the influence of a bottom sill on the added mass and damping coefficient, wave force to a rectangular structure floating on

the free surface. Siddorn and Taylor [11] considered array of truncated cylinders in water of finite depth and investigated the diffraction and radiation problems under linear water waves for this structure. Zhu and Mitchell [12] derived a first order analytical solution for the diffraction problem around a hollow cylinder. They used a new approach to analyze the dependence of the solution upon various parameters, as well as the rate of convergence of the series solution. Zhang et. al. [13] considered two vertical truncated cylinders in water of finite depth. They presented sets of hydrodynamic coefficient and wave forces for various parameters. Kumar and Sharma [15] discussed about the flow between annular space surrounded by a rotating coaxial cylinder with co-axial cylindrical porous medium.
Remaining part of the paper is organized as follows: Section 2 describes the mathematical formulation of the problem. Methodology of the paper is given in Section 3. The governing equation and related physical boundary conditions are shown in Section 4. With the help of diffracted velocity potential, we derived the expressions of exciting force in Section 5. The numerical results and discussion is given in Section 6. The last Section 7 describes the conclusion and future work of this paper.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider linear water wave propagation in an ocean of uniform depth h_1. We consider single hollow cylinder of radius R. Some part of the hollow cylinder is above the free surface as shown in Figure 1. A right-handed Cartesian coordinate system $Oxyz$ is defined with the origin O in the undisturbed free surface and z-axis measured positive upwards, direction of propagation of waves is considered along x-axis. The hollow cylinder, occupies the region defined by $r \le R,\ 0 \le \theta \le 2\pi,\ -e_1 \le z \le 0$. Since we consider the motion is irrotational, fluid is incompressible and amplitude is small so that we can apply the theory of linear water wave.

III. METHODOLOGY

We can introduce the total velocity potential

$$\Phi(r,\theta,z,t) = \mathrm{Re}[\phi(r,\theta,z)e^{-i\omega t}], \tag{1}$$

where Re denotes the real part of the quantity in bracket, ω is the angular frequency of the incident wave and $\phi(r,\theta,z)$ is the spatial part of the velocity potential. Therefore $\phi(r,\theta,z)$ satisfy Laplace's equation

$$\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2} = 0, \tag{2}$$

Since we divide the whole fluid region into to sub regions one is exterior and the other is interior regions. The solutions for the boundary value problem are obtained in interior and exterior regions. Therefore, the velocity potential ϕ is decomposed into two potentials defined on $r \le R$ and $r \ge R$, respectively:

$$\phi = \begin{cases} \phi^{\mathrm{int}}, & r \le R \\ \phi^{ext}, & r \ge R \end{cases} \tag{3}$$

Where the velocity potentials ϕ^{int} and ϕ^{ext} denote the velocity potential in the interior and exterior regions, respectively. The incident velocity potential with unit amplitude and angular frequency ω, propagating along the positive x-direction is given by (MacCamy and Fuch, [14])

$$\phi_i = -\frac{ig}{\omega}\frac{\cosh[k(z+h_1)]}{\cosh(kh_1)}\sum_{m=0}^{\infty}\varepsilon_m J_m(kr)\cos m\theta, \tag{4}$$

where $i=\sqrt{-1}$, g is the gravitational acceleration and the wave number k can be determined from the dispersion relation $\omega^2 = gk\tanh(kh_1)$ and $J_m(.)$ is the Bessel of first kind of order m and ε_m is given by

$$\varepsilon_m = \begin{cases} 2i^m, & m>0 \\ 1, & m=0 \end{cases}$$

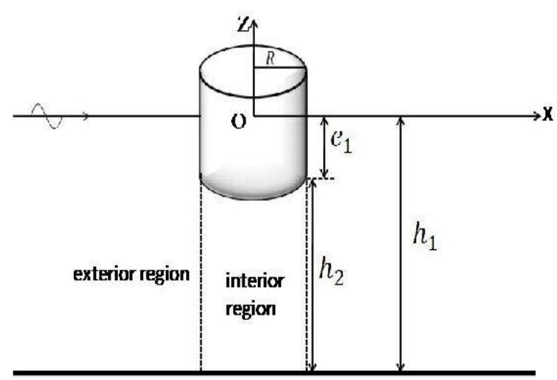

Figure: 1: Schematics diagram of the device

IV. BOUNDARY VALUE PROBLEM

The governing equation and boundary conditions:
The diffracted velocity potential Φ_d can be written as

$\Phi_d(r,\theta,z,t) = \mathrm{Re}[\phi_d(r,\theta,z)e^{-i\omega t}]$, where the spatial part ϕ_d satisfies the following governing equation and boundary conditions:

$$\nabla^2\phi_d = 0, \qquad \text{in the respective regions} \tag{5}$$

$$\frac{\partial \phi_d}{\partial z} - \frac{\omega^2}{g}\phi_d = 0, \quad (z=0) \tag{6}$$

$$\frac{\partial \phi_d}{\partial z} = 0, \quad (z=-h_1) \tag{7}$$

$$\frac{\partial(\phi_d + \phi_i)}{\partial r} = 0 \quad (-e_1 < z < 0, r = R) \tag{8}$$

and the radiation condition is given by

$$\lim_{r\to\infty}\sqrt{kr}\left(\frac{\partial \phi_d}{\partial r} - ik\phi_d\right) = 0 \tag{9}$$

Solution to the problem

The fluid domain is divided into two subdomains namely interior and exterior regions as indicated in Figure 1. We apply the separation of variables method in each subdomain in order to obtain expressions for the velocity potential. The analytical expression for the diffracted velocity potentials in the exterior and interior regions can be obtained as,

$$\phi_d^{ext} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[A_{m,n}\frac{R_m(\lambda_n r)}{R_m(\lambda_n R)}\cos[\lambda_n(z+h_1)]\right]\cos m\theta, \tag{10}$$

$$\phi_d^{\text{int}} = -\phi_i + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left[B_{m,n}\frac{U_m(\lambda_n r)}{U_m(\lambda_n R)}\cos[\lambda_n(z+h_1)]\right]\cos m\theta, \tag{11}$$

where $A_{m,n}$ and $B_{m,n}$ are the unknown constants and λ_n can be determined from the dispersion relation:

$$\begin{cases} \lambda_n = -ik & \omega^2 = gk\tanh(kh_1),\ n=0 \\ \omega^2 = -g\lambda_n \tan(\lambda_n h_1) & n = 1, 2, \ldots \end{cases}$$

The radial functions $R_m(.)$ and $U_m(.)$ are given by

$$R_m(\lambda_n r) = H_m^{(1)}(kr) = H_m^{(1)}(i\lambda_0 r), \quad n=0$$
$$R_m(\lambda_n r) = K_m(\lambda_n r), \quad n = 1, 2, \ldots$$
$$U_m(\lambda_n r) = J_m(kr), \quad n=0$$
$$U_m(\lambda_n r) = I_m(\lambda_n r), \quad n = 1, 2, \ldots$$

where $H_m^{(1)}(.)$ and $J_m(.)$ are the first kind Hankel functions of order m and first kind Bessel function of order m, respectively, whereas $I_m(.)$ and $K_m(.)$ are the first and second kind modified Bessel functions of order m, respectively.

Matching conditions:

We can have the appropriate matching conditions by means of continuity of pressure and that of velocity along the virtual boundaries as depicted in Figure 1. At , $r = R$ i.e., along the curved surface of cylinder, extended up to the bottom, we have

$$\phi_d^{ext} = \phi_d^{\text{int}} \quad (-h_1 \le z \le -e_1) \tag{12}$$

$$\frac{\partial \phi_d^{ext}}{\partial r} = \begin{cases} \dfrac{\partial \phi_d^{\text{int}}}{\partial r} & (-h_1 \le z \le e_1) \\ -\dfrac{\partial \phi_i}{\partial r} & (-e_1 \le z \le 0) \end{cases} \tag{13}$$

In order to find the unknown coefficients which are present in the expression of the diffracted velocity potential, we apply these matching conditions.

V. WAVE FORCE

Now we proceed to find the wave exciting forces acting on the cylinder due to the diffraction taking place on their surfaces. Exciting forces are generally due to the combined action of an incident velocity potential and a diffracted velocity potential on the structure. Let us assume that F_i is the horizontal exciting force for an incident velocity potential and F_d is the diffraction force due to diffracted velocity potential.

Therefore, the total horizontal exciting force acting on the hollow cylinder can be written as

$$F_h = F_i + F_d = -i\rho\omega\left(\int_W \phi_i n_x ds + \int_W \phi_d n_x ds\right) \tag{14}$$

where $\vec{n} = n_x\hat{i} + n_y\hat{j} + n_z\hat{k}$ is the outward unit normal vector on the surface of cylinder, W is the wetted surface of the cylinder and ds is the small surface element and F_d is the horizontal diffraction force due to the diffracted velocity potential ϕ^{ext} .

Now by using equations (4), (10) and (14), we get the total horizontal exciting force acting on the hollow cylinder as

$$F_h = \frac{-2i\pi\rho g R J_1(kR)\{\sinh k(h_1) - \sinh[k(h_1 - e_1)]\}}{k\cosh(kh_1)} - i\pi\rho\omega R\sum_{n=0}^{\infty} A_{1,n}\frac{\sin\lambda_n(h_1) - \sin[\lambda_n(h_1 - e_1)]}{\lambda_n}. \tag{15}$$

The dimensionless horizontal exciting force F_h / w_0, where $w_o = \rho g \pi R^2$, is given by

$$F_h / w_0 = \frac{-2iJ_1(kR)\{\sinh k(h_1) - \sinh[k(h_1 - e_1)]\}}{k\cosh(kh_1)} - \frac{i\omega}{gR}\sum_{n=0}^{\infty} A_{1,n} \frac{\sin\lambda_n(h_1) - \sin[\lambda_n(h_1 - e_1)]}{\lambda_n}. \tag{16}$$

VI. NUMERICAL RESULTS AND DISCUSSION

Since the expression of the diffracted velocity potentials are in the form of an infinite series, therefore, it is need to truncate each series suitably to compute the values of the coefficients. Hence, all infinite series are truncated after a finite number of terms, say, N=30. Subsequently, we arrived at a linear system of algebraic equations and then solved this system of linear equations with the help of MATLAB programme. Once we have the values of all unknowns coefficients which are present in the expression of diffracted velocity potentials. This allow us to evaluate the exciting force acting on the cylinder.

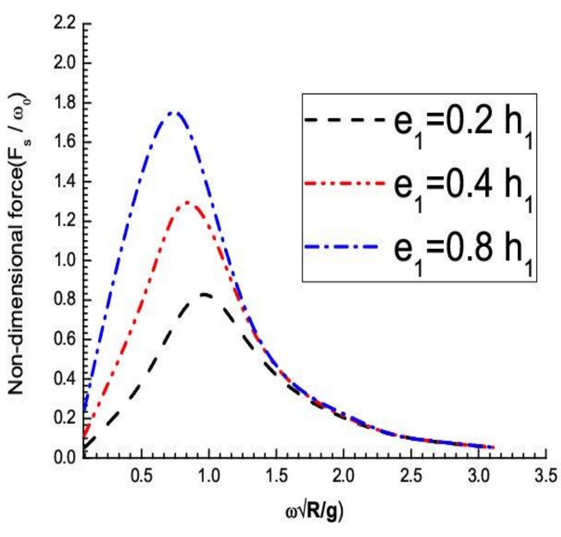

Figure: 2: Non-dimensional horizontal exciting force F_h / w_0 acting on the cylinder versus non-dimensional frequency $\omega\sqrt{R/g}$ for different values of draft of the cylinder with $R/h_1 = 0.2$

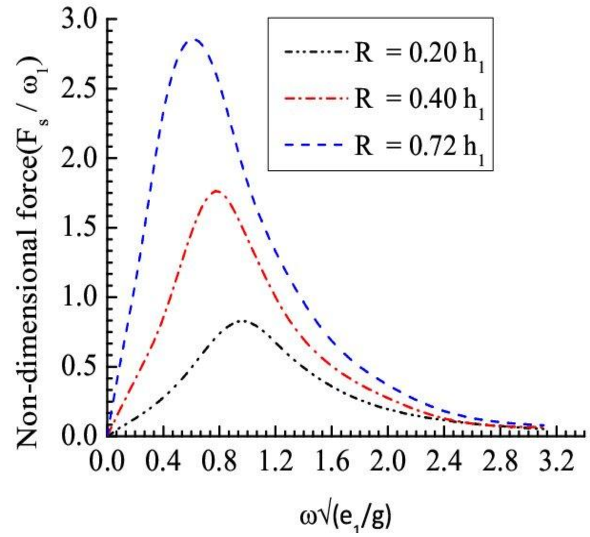

Figure: 3: Non-dimensional horizontal exciting force F_h / w_1 acting on the cylinder versus non-dimensional frequency $\omega\sqrt{e_1/g}$ for different values of radius of the cylinder with $h_1 = 3m$ $e_1/h_1 = 0.2$

Figure 2 represents the non-dimensional horizontal exciting force F_h / w_0 versus non-dimensional frequency $\omega\sqrt{R/g}$ with different values of draft of the cylinder and it is observed that higher values of the force are attained corresponding to the higher values of the draft. The forces have higher values corresponding to the lower values of $\omega\sqrt{R/g}$. The peak values occurred for smaller values of the frequencies. Figure 3 represents the non-dimensional horizontal exciting force F_h / w_1 where $w_1 = \rho g \pi e_1^2$ versus non-dimensional frequency $\omega\sqrt{e_1/g}$ with different radii of the cylinder for a fixed draft e_1. In this figure, the main observations are that for larger values of radius ratios, the concerned force attains higher values. The exciting forces are decreasing as frequencies increasing.

VI. CONCLUSION AND FUTURE SCOPE

We derived the analytical solution of the diffraction problem of water wave by considering a floating single hollow cylindrical structure in uniform water depth. We used matched eigen function expansion and separation of variables methods to solve the problem completely. This structure may be considered as one kind of wave energy device (Oscillating water column). We have presented the influence of various parameters on the exciting forces. Our results may give the useful information for engineer to design the device in order to extract maximum energy. One can extend this work in two layer fluid.

ACKNOWLEDGMENT

The first author wishes to thank north eastren regional institute of science and technology, itanagar and department of science and technology, serb (yss/14/000884), govt. Of india for providing necessary facilities and support in order to complete this work. Moreover, i would like to thank prof. S. N. Bora, department of mathematics, iit guwahati for their valuable suggestions.

REFERENCES

[1] C. J. R. Garrett, "*Wave forces on a circular dock*", Journal of Fluid Mechanics, Vol. **46**, No. **19**, pp.**129-139**, **1971**.

[2] D. D._Bhatta, M. Rahman, "*On the scattering and radiation problem for a cylinder in water of finite depth*", International Journal of Engineering Science, Vol. **41**, pp. **931-967**, **2003**.

[3] B. J. Wu, Y. H. Zhang, Y. G. You, D. S. Jie, Y. Chen, "*On diffraction and radiation problem for two cylinders in water of finite depth*", Ocean Engineering, Vol. **33**, pp.**679-704**, **2006**.

[4] B. J. Wu, Y. M. Zhang, Y. G. You, X. Y. Sun, Y. Chen, "*On diffraction and radiation problem for a cylinder over a caisson in water of finite depth*", International Journal of Engineering Science, Vol. **42**, pp.**1193-1213**, **2004**.

[5] S. A. Mavrakos, "Wave loads on a stationary floating bottomless cylindrical body with finite wall thickness", Applied Ocean Research, Vol. **7**, Iissue. **4**, pp.**213-224**, **1985**.

[6] J. N. Newman, "*Radiation and diffraction analysis of the McIver toroid*", Journal Engineering Mathematics, Vol. **35**, pp.**135—147**, **1999**.

[7] M. Hassan, S. N. Bora, "*Exciting Forces for a Pair of Coaxial Hollow Cylinder and Bottom-mounted Cylinder in Water of Finite Depth*", Ocean Engineering, Vol. **50**, pp. **38-43**, **2012**.

[8] M. Hassan, S. N. Bora, "*Exciting forces for a wave energy device consisting of a pair of coaxial cylinders in water of finite depth*", Journal of Marine Science and Application}, Vol. **12**, pp. **315-324**, **2013**.

[9] M. Rahman, D. D. Bhatta,"*Second order wave forces on a pair of cylinders*", Canadian Application of Mathematics Quarterly, Vol. **1**, Issue. **3**, pp. **343-382, 1993.**

[10] Y. M. Shen, Y. H. Zhang, Y. G. You, "*On the radiation and diffraction of linear water waves by a rectangular structure over a sill. Part I. Infinite domain of finite water depth*", Ocean Engineering, Vol. **32**, pp. **1073-1097**, **2005**.

[11] P. Siddorn, R. E. Taylor, "*Diffraction and independent radiation by an array of floating cylinders*", Ocean Engineering, Vol. **35**, pp.**1289-1303**, **2008**.

[12] S. P. Zhu, L. Mitchell, "*Diffraction of ocean waves around a hollow cylindrical shell structure*", Wave Motion, Vol. **46**, pp.**78-88**, **2009**.

[13] Y. H. Zhang, Y. M. Shen, Y. G. You, B. J. Wu, L. Rong,, "*Hydrodynamic properties of two vertical truncated cylinders in waves*", Ocean Engineering, Vol. **32**, pp.**241-271**, **2005.**

[14] R. C. MacCamy, R. A. Fuchs, "*Wave Forces on Piles: A Diffraction Theory Technical*", Memo,No. 69 US Army Beach Erosion Board, pp.**17**, **1954**.

[15] A. Kumar and G. C. Sharma, "*Performance and analysis of flow between Annular Space Surrounded by a Rotating Coaxial cylinder with Co-axial Cylindrical Porous Medium*", International Journal of Scientific Research in Computer Sciences and Engineering, Vol. **5**, No.**3**, pp. **1-6**, **2017.**

AUTHORS PROFILE

Dr. M. Hassan received his Ph.D. degree in Mathematics from Indian Institute of Technology (I I T) Guwahati in the year 2014. He is presently working as a faculty in the Department of Mathematics, North Eastern Regional Institute of Science and Technology, Itanagar, India. He has got NBHM Postdoctoral fellowship. He is pursuing young scientist research project of SERB DST Govt. of India. He has published more than 10 research papers in reputed international journals including SCI journals also. His area of research focus on fluid-structure interaction, channel problems, partial differential equations.

P. Borah completed his M.Sc. in Mathematics from Tezpur University, Assam, in the year 2016. He is currently pursuing Ph.D. in Mathematics from North Eastern Regional Institute of Science and Technology, Itanagar, India. He has published two research paper in reputed international journal.