International Journal of Scientific Research in Mathematical and Statistical Sciences
Vol.6, Issue.4, pp.77-83, August (2019)
DOI: https://doi.org/10.26438/ijsrmss/v6i4.7783

Research Paper
E-ISSN: 2348-4519

Modified Ratio-Cum-Product Estimator for Finite Population Mean

Rajesh Tailor[1*], Priya Mehta[2]

[1,2]School of Studies in Statistics, Vikram University, Ujjain, M.P. India

Available online at: www.isroset.org

Received: 18/Jul/2019, Accepted: 20/Aug/2019, Online: 31/Aug/2019

Abstract- This paper develops two modified RCP estimators for finite PM. Infact, dual to two RCP estimators have been developed. The BIAS and MSqER of the developed estimators have been obtained upto the first degree of approximation. The developed estimators have been compared with existing estimators and conditions under which the developed estimators are more efficient have been obtained. An empirical study also has been conducted.

***Keywords*:** RCP estimator, Finite PM, Bias, MSqER.

I. INTRODUCTION

Let us consider a finite population U={U_1, U_2,....,U_N}. A sample of size n is drawn using SRSWOR. Let y be the study variate and x_1 and x_2 be the AVs such that x_1 is positively correlated and x_2 is negatively correlated with the SV y.

Cochran (1940) and Robson (1957) invented RP estimators for PM $\bar{Y}$ respectively as

$$\hat{\bar{Y}}_R = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right) \qquad (1.1)$$

and

$$\hat{\bar{Y}}_P = \bar{y}\left(\frac{\bar{x}_2}{\bar{X}_2}\right) \qquad (1.2)$$

respectively.

Where $\bar{y} = \sum_{i=1}^{n}\frac{y_i}{n}$, $\bar{x}_1 = \sum_{i=1}^{n}\frac{x_i}{n}$ and $\bar{x}_2 = \sum_{i=1}^{n}\frac{x_i}{n}$ are SM of y, x_1 and x_2 and unbiased estimators of PMs $\bar{Y} = \sum_{i=1}^{N}\frac{y_i}{N}$, $\bar{X}_1 = \sum_{i=1}^{N}\frac{x_{1i}}{N}$ and $\bar{X}_2 = \sum_{i=1}^{N}\frac{x_{2i}}{N}$ respectively.

With the assumption of known PM of AVs x_1 and x_2, [Singh (1967)] suggested a RCP estimator for PM $\bar{Y}$ as

$$\hat{\bar{Y}}_{RP} = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{X}_2}\right). \qquad (1.3)$$

MSqERs of ratio estimator $\hat{\bar{Y}}_R$, product estimator $\hat{\bar{Y}}_P$ and RCP estimator $\hat{\bar{Y}}_{RP}$ respectively are

$$MSE\left(\hat{\bar{Y}}_R\right) = \gamma\bar{Y}^2\left[C_y^2 + C_{x_1}^2\left(1-2K_{yx_1}\right)\right], \qquad (1.4)$$

$$MSE\left(\hat{\bar{Y}}_P\right) = \gamma\bar{Y}^2\left[C_y^2 + C_{x_2}^2\left(1+2K_{yx_2}\right)\right], \qquad (1.5)$$

and

$$MSE\left(\hat{\bar{Y}}_{RP}\right) = \gamma\bar{Y}^2\left[C_y^2 + C_{x_1}^2\left(1-2K_{yx_1}\right) + C_{x_2}^2\left\{1+2\left(K_{yx_2}-K_{x_1x_2}\right)\right\}\right] \qquad (1.6)$$

where

$\gamma=\left(\frac{1}{n}-\frac{1}{N}\right)$, $C_y=\frac{S_y}{\bar{Y}}$, $C_{x_1}=\frac{S_{x_1}}{\bar{X}_1}$, $C_{x_2}=\frac{S_{x_2}}{\bar{X}_2}$, $K_{yx_1}=\rho_{yx_1}\frac{C_y}{C_{x_1}}$,

$K_{yx_2}=\rho_{yx_2}\frac{C_y}{C_{x_2}}$, $K_{x_1x_2}=\rho_{x_1x_2}\frac{C_{x_1}}{C_{x_2}}$, $S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}^2\right)$, $S_{x_1}^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_{1i}-\bar{X}_1^{\,2}\right)$,

$S_{x_2}^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_{2i}-\bar{X}_2^{\,2}\right)$, $\rho_{yx_1}=\frac{S_{yx_1}}{S_yS_{x_1}}$, $\rho_{yx_2}=\frac{S_{yx_2}}{S_yS_{x_2}}$, $\rho_{x_1x_2}=\frac{S_{x_1x_2}}{S_{x_1}S_{x_2}}$, $S_{yx_1}=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_{1i}-\bar{X}_1\right)$,

$S_{yx_2}=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_{2i}-\bar{X}_2\right)$, and $S_{x_1x_2}=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_{1i}-\bar{X}_1\right)\left(x_{2i}-\bar{X}_2\right)$.

Srivenkataramana (1980) applied a transformation $x_i^*=\frac{N\bar{X}-n\bar{x}_i}{N-n}$ on AVs x_1 and x_2 and suggested dual to RP estimators $\hat{\bar{Y}}_R^*$ and $\hat{\bar{Y}}_P^*$ as

$$\hat{\bar{Y}}_R^*=\bar{y}\left(\frac{\bar{x}_1^*}{\bar{X}_1}\right) \tag{1.7}$$

and

$$\hat{\bar{Y}}_P^*=\bar{y}\left(\frac{\bar{X}_2}{\bar{x}_2^*}\right) \tag{1.8}$$

where $\bar{x}_1^*=(1+g)\bar{X}-g\bar{x}_1$ and $\bar{x}_2^*=(1+g)\bar{X}_2-g\bar{x}_2$ and $g=\frac{n}{N-n}$.

Singh et al. (2005) determined dual to Singh (1967) RCP estimator $\hat{\bar{Y}}_{RP}$ as

$$\hat{\bar{Y}}_{RP}^*=\bar{y}\left(\frac{\bar{x}_1^*}{\bar{X}_1}\right)\left(\frac{\bar{X}_2}{\bar{x}_2^*}\right) \tag{1.9}$$

MSqER of $\hat{\bar{Y}}_R^*$, $\hat{\bar{Y}}_P^*$ and $\hat{\bar{Y}}_{RP}^*$ are

$$MSE\left(\hat{\bar{Y}}_R^*\right)=\gamma\bar{Y}^2\left[C_y^2+gC_{x_1}^2\left(g-2K_{yx_1}\right)\right], \tag{1.10}$$

$$MSE\left(\hat{\bar{Y}}_P^*\right)=\gamma\bar{Y}^2\left[C_y^2+gC_{x_2}^2\left(g-2K_{yx_2}\right)\right], \tag{1.11}$$

$$MSE\left(\hat{\bar{Y}}_{RP}^*\right)=\gamma\bar{Y}^2\left[C_y^2+gC_{x_1}^2\left(g-2K_{yx_1}\right)+gC_{x_2}^2\left\{g+2\left(K_{yx_2}-gK_{x_1x_2}\right)\right\}\right]. \tag{1.12}$$

[Upadhyaya and Singh (1999)] suggested a RP type estimators using CV and CK as.

$$\hat{\bar{Y}}_{RUS1}=\bar{y}\left(\frac{\bar{X}_1C_{x_1}+\beta_2(x_1)}{\bar{x}_1C_{x_1}+\beta_2(x_1)}\right) \tag{1.13}$$

and

$$\hat{\bar{Y}}_{PUS1}=\bar{y}\left(\frac{\bar{x}_2C_{x_2}+\beta_2(x_2)}{\bar{X}_2C_{x_2}+\beta_2(x_2)}\right) \tag{1.14}$$

MSqER of $\hat{\bar{Y}}_{RUS1}$ and $\hat{\bar{Y}}_{PUS1}$ are respectively obtained as

$$MSE\left(\hat{\bar{Y}}_{RUS1}\right)=\gamma\bar{Y}^2\left[C_y^2+\lambda_1C_{x_1}^2\left(\lambda_1-2K_{yx_1}\right)\right], \tag{1.15}$$

and

$$MSE\left(\hat{\bar{Y}}_{PUS1}\right)=\gamma\bar{Y}^2\left[C_y^2+\lambda_2 C_{x_2}^2\left(\lambda_2+2K_{yx_2}\right)\right]. \qquad (1.16)$$

Dual to $\hat{\bar{Y}}_{RUS1}$ and $\hat{\bar{Y}}_{PUS1}$ are determined as

$$\hat{\bar{Y}}_{RUS1}^{*}=\bar{y}\left(\frac{\bar{x}_1^{*}C_{x_1}+\beta_2(x_1)}{\bar{X}_1 C_{x_1}+\beta_2(x_1)}\right) \qquad (1.17)$$

and

$$\hat{\bar{Y}}_{PUS1}^{*}=\bar{y}\left(\frac{\bar{X}_2 C_{x_2}+\beta_2(x_2)}{\bar{x}_2^{*}C_{x_2}+\beta_2(x_2)}\right). \qquad (1.18)$$

MSqER of $\hat{\bar{Y}}_{RUS1}^{*}$ and $\hat{\bar{Y}}_{PUS1}^{*}$ are

$$MSE\left(\hat{\bar{Y}}_{RUS1}^{*}\right)=\gamma\bar{Y}^2\left[C_y^2+g\lambda_1 C_{x_1}^2\left(g\lambda_1-2K_{yx_1}\right)\right] \qquad (1.19)$$

and

$$MSE\left(\hat{\bar{Y}}_{PUS1}^{*}\right)=\gamma\bar{Y}^2\left[C_y^2+g\lambda_2 C_{x_2}^2\left(g\lambda_2+2K_{yx_2}\right)\right]. \qquad (1.20)$$

where $\lambda_i=\dfrac{\bar{X}_i C_{x_{i1}}}{\bar{X}_i C_{x_i}+\beta_2(x_i)}$, $\gamma_i=\dfrac{\bar{X}_i\beta_2(x_i)}{\bar{X}_i\beta_2(x_i)+C_{x_i}}$; $i=1,2.$

II. DEVELOPED ESTIMATORS

Assuming that the information on CV (C_{x_1} and C_{x_2}) and CK ($\beta_2(x_1)$ and $\beta_2(x_2)$) of AVs x_1 and x_2 are available. Parmar (2013) suggested two RCP estimators for PM as

$$\hat{\bar{Y}}_1=\bar{y}\left(\frac{\bar{X}_1 C_{x_1}+\beta_2(x_1)}{\bar{x}_1 C_{x_1}+\beta_2(x_1)}\right)\left(\frac{\bar{x}_2 C_{x_2}+\beta_2(x_2)}{\bar{X}_2 C_{x_2}+\beta_2(x_2)}\right) \qquad (2.1)$$

and

$$\hat{\bar{Y}}_2=\bar{y}\left(\frac{\bar{X}_1\beta_2(x_1)+C_{x_1}}{\bar{x}_1\beta_2(x_1)+C_{x_1}}\right)\left(\frac{\bar{x}_2\beta_2(x_2)+C_{x_2}}{\bar{X}_2\beta_2(x_2)+C_{x_2}}\right). \qquad (2.2)$$

MSqER of the estimators $\hat{\bar{Y}}_1$ and $\hat{\bar{Y}}_2$ are obtained as

$$MSE\left(\hat{\bar{Y}}_1\right)=\gamma\bar{Y}^2\left[C_y^2+\lambda_1 C_{x_1}^2\left(\lambda_1-2K_{yx_1}\right)+\lambda_2 C_{x_2}^2\left\{\lambda_2+2\left(K_{yx_2}-\lambda_1 K_{x_1x_2}\right)\right\}\right] \qquad (2.3)$$

and

$$MSE\left(\hat{\bar{Y}}_2\right)=\gamma\bar{Y}^2\left[C_y^2+\gamma_1 C_{x_1}^2\left(\gamma_1-2K_{yx_1}\right)+\gamma_2 C_{x_2}^2\left\{\gamma_2+2\left(K_{yx_2}-\gamma_1 K_{x_1x_2}\right)\right\}\right] \qquad (2.4)$$

Using the transformation $\bar{x}_1^{*}$ and $\bar{x}_2^{*}$, modified to the estimators are expressed as

$$\hat{\bar{Y}}_1^{*}=\bar{y}\left(\frac{\bar{x}_1^{*}C_{x_1}+\beta_2(x_1)}{\bar{X}_1 C_{x_1}+\beta_2(x_1)}\right)\left(\frac{\bar{X}_2 C_{x_2}+\beta_2(x_2)}{\bar{x}_2^{*}C_{x_2}+\beta_2(x_2)}\right) \qquad (2.5)$$

and

$$\hat{\bar{Y}}_2^{*}=\bar{y}\left(\frac{\bar{x}_1^{*}\beta_2(x_1)+C_{x_1}}{\bar{X}_1\beta_2(x_1)+C_{x_1}}\right)\left(\frac{\bar{X}_2\beta_2(x_2)+C_{x_2}}{\bar{x}_2^{*}\beta_2(x_2)+C_{x_2}}\right). \qquad (2.6)$$

To obtain the BIAS and MSqER of the developed estimators, it is assumed that $\bar{y}=\bar{Y}(1+e_0)$, $\bar{x}_1=\bar{X}_1(1+e_1)$, $\bar{x}_2=\bar{X}_2(1+e_2)$ such that $E(e_0)=E(e_1)=E(e_2)=0$ and $E(e_0^2)=\gamma C_y^2$, $E(e_1^2)=\gamma C_{x_1}^2$, $E(e_2^2)=\gamma C_{x_2}^2$, $E(e_0e_1)=\gamma\rho_{yx_1}C_yC_{x_1}$, $E(e_0e_1)=\gamma\rho_{yx_2}C_yC_{x_2}$.

MSqER of the developed estimators are

$$MSE\left(\hat{\bar{Y}}_1^*\right)=\gamma\bar{Y}^2\left[C_y^2+g\lambda_1C_{x_1}^2\left(g\lambda_1-2K_{yx_1}\right)+g\lambda_2C_{x_2}^2\left(g\lambda_2+2\left\{g\lambda_1K_{x_1x_2}-K_{yx_2}\right\}\right)\right] \quad (2.7)$$

$$MSE\left(\hat{\bar{Y}}_2^*\right)=\gamma\bar{Y}^2\left[C_y^2+g\gamma_1C_{x_1}^2\left(g\gamma_1-2K_{yx_1}\right)+g\gamma_2C_{x_2}^2\left(g\gamma_2-2g\gamma_1K_{x_1x_2}+2K_{yx_2}\right)\right]. \quad (2.8)$$

III. EFFICIENCY COMPARISONS

We known that the variance of SM $\bar{y}$ in simple SRSWOR is determined as

$$V(\bar{y})=\gamma S_y^2 \quad (3.1)$$

From (1.4), (1.5), (1.6), (1.10), (1.11), (1.12), (1.15), (1.16), (1.19), (1.20), (2.7) and (3.1) it follows that

(i) $MSE\left(\hat{\bar{Y}}_1^*\right)<V(\bar{y})$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2}<\frac{g\lambda_2\left(2\lambda_1K_{x_1x_2}-\lambda_2\right)-2\lambda_2K_{yx_2}}{\lambda_1\left(g\lambda_1-2K_{yx_1}\right)}, \quad (3.2)$$

(ii) $MSE\left(\hat{\bar{Y}}_1^*\right)<MSE\left(\hat{\bar{Y}}_R\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2}<\frac{g^2\lambda_2\left(\lambda_1K_{x_1x_2}-\lambda_2\right)-2g\lambda_2K_{yx_2}}{\left(g^2\lambda_1^2-1+2K_{yx_1}(1-g)\right)}, \quad (3.3)$$

(iii) $MSE\left(\hat{\bar{Y}}_1^*\right)<MSE\left(\hat{\bar{Y}}_P\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2}<\frac{g^2\lambda_2\left(2\lambda_1K_{x_1x_2}-\lambda_2\right)+1+2K_{yx_2}\left(1-g\lambda_2\right)}{g\lambda_1\left(g\lambda_1-2K_{yx_1}\right)}, \quad (3.4)$$

(iv) $MSE\left(\hat{\bar{Y}}_1^*\right)<MSE\left(\hat{\bar{Y}}_{RP}\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2}<\frac{1-g^2\lambda_2^2+2K_{x_1x_2}\left(g^2\lambda_1\lambda_2-1\right)-2K_{yx_2}\left(g\lambda_2-1\right)}{\left(g\lambda_1-1\right)\left[\left(g\lambda_1+1\right)-2K_{yx_1}\right]}, \quad (3.5)$$

(v) $MSE\left(\hat{\bar{Y}}_1^*\right)<MSE\left(\hat{\bar{Y}}_R^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2}<\frac{g\lambda_2\left(2g\lambda_1K_{x_1x_2}-2K_{yx_2}-1\right)}{\left(\lambda_1-1\right)\left[g^2\left(\lambda_1+1\right)-2gK_{yx_1}\right]}, \quad (3.6)$$

(vi) $MSE\left(\hat{\bar{Y}}_1^*\right)<MSE\left(\hat{\bar{Y}}_P^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2}<\frac{\left(1-\lambda_2\right)\left[g^2\left(1+\lambda_2\right)+2gK_{yx_2}\right]+2g^2\lambda_1\lambda_2K_{x_1x_2}}{g\lambda_1\left(g\lambda_1-2K_{yx_1}\right)}, \quad (3.7)$$

(vii) $MSE\left(\hat{\bar{Y}}_1^*\right)<MSE\left(\hat{\bar{Y}}_{RP}^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2}<\frac{2g^2K_{x_1x_2}\left(\lambda_1\lambda_2-1\right)-\left(\lambda_2-1\right)\left\{g^2\left(\lambda_2+1\right)+2gK_{yx_2}\right\}}{\left(\lambda_1-1\right)\left\{g^2\left(\lambda_1+1\right)-2gK_{yx_1}\right\}}, \quad (3.8)$$

(viii) $MSE\left(\hat{\bar{Y}}_1^*\right) < MSE\left(\hat{\bar{Y}}_{RUS1}\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{g^2\lambda_2\left(\lambda_1 K_{x_1x_2} - \lambda_2\right) - 2g\lambda_2 K_{yx_2}}{(g+1)\left\{\lambda_1^2(g-1) + 2\lambda_1 K_{yx_1}\right\}}, \qquad (3.9)$$

(ix) $MSE\left(\hat{\bar{Y}}_1^*\right) < MSE\left(\hat{\bar{Y}}_{PUS1}\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{(1-g)\left\{\lambda_2^2(1+g) + 2\lambda_2 K_{yx_2}\right\} + g^2\lambda_1\lambda_2 K_{x_1x_2}}{g\lambda_1\left(g\lambda_1 - 2K_{yx_1}\right)}, \qquad (3.10)$$

(x) $MSE\left(\hat{\bar{Y}}_1^*\right) < MSE\left(\hat{\bar{Y}}_{RUS1}^*\right)$ if

$$\frac{2K_{yx_2}}{\left(\lambda_1 K_{x_1x_2} - \lambda_2\right)} < g \qquad (3.11)$$

and

(xi) $MSE\left(\hat{\bar{Y}}_1^*\right) < MSE\left(\hat{\bar{Y}}_{PUS1}^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{\lambda_2\left(g\lambda_1 K_{x_1x_2} - 4K_{yx_2}\right)}{\lambda_1\left(g\lambda_1 - 2K_{yx_1}\right)}. \qquad (3.12)$$

Similarly from (1.4), (1.5), (1.6), (1.10), (1.11), (1.12), (1.15), (1.16), (1.19), (1.20), (2.8) and (3.1) it follows that

(i) $MSE\left(\hat{\bar{Y}}_2^*\right) < V(\bar{y})$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{g^2\gamma_2\left(2\gamma_1 K_{x_1x_2} - \gamma_2\right) - 2\gamma_2 g K_{yx_2}}{g\gamma_1\left(g\gamma_1 - 2K_{yx_1}\right)}, \qquad (3.13)$$

(ii) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_R\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{g^2\gamma_2\left(\gamma_1 K_{x_1x_2} - \gamma_2\right) - 2K_{yx_2}}{(g\gamma_1 - 1)\left[(g\gamma_1 + 1) - 2K_{yx_1}\right]}, \qquad (3.14)$$

(iii) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_P\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{1 - g\gamma_2\left(g - 2\gamma_1 K_{x_1x_2}\right) - 2K_{yx_2}(g\gamma_2 - 1)}{g\gamma_1\left(g\gamma_1 - 2K_{yx_2}\right)}, \qquad (3.15)$$

(iv) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_{RP}\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{1 - g^2\gamma_2^2 - 2K_{yx_2}(\gamma_2 - 1) - 2K_{x_1x_2}\left(1 - g^2\gamma_1\gamma_2\right)}{(g\gamma_1 - 1)\left\{(g\gamma_1 + 1) - 2K_{yx_1}\right\}}, \qquad (3.16)$$

(v) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_R^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{g^2\gamma_2\left(2\gamma_1 K_{x_1x_2} - \gamma_2\right) - 2g\gamma_2 K_{yx_2}}{(\gamma_1 - 1)\left\{g^2(\gamma_1 + 1) - 2gK_{yx_1}\right\}}, \qquad (3.17)$$

(vi) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_P^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{(1-\gamma_2)\left\{g^2(1+\gamma_2) + 2gK_{yx_2}\right\} + 2g^2\gamma_1\gamma_2 K_{x_1x_2}}{g\gamma_1\left(g\gamma_1 - 2K_{yx_1}\right)}, \qquad (3.18)$$

(vii) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_{RP}^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{(\gamma_2 - 1)\left\{2gK_{yx_2} - g^2(\gamma_2 + 1)\right\} + 2g^2K_{x_1x_2}(\gamma_1\gamma_2 - 1)}{(\gamma_1 - 1)\left\{g^2(\gamma_1 + 1) - 2gK_{yx_1}\right\}}, \quad (3.19)$$

(viii) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_{RUS1}\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{2g^2\gamma_1\gamma_2K_{x_1x_2} - g\gamma_2\left(g\gamma_2 + 2K_{yx_2}\right)}{\left(g^2\gamma_1^2 - \lambda_1^2\right) - 2K_{yx_1}\left(g\gamma_1 + \lambda_1\right)}, \quad (3.20)$$

(ix) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_{PUS1}\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{\lambda_2^2 - g^2\gamma_2\left(2\gamma_1K_{x_1x_2} - \gamma_2\right) - 2K_{yx_2}\left(g\gamma_2 - \lambda_2\right)}{g\gamma_1\left(g\gamma_1 - 2K_{yx_1}\right)}, \quad (3.21)$$

(x) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_{RUS1}^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{g^2\gamma_2\left(2\gamma_1K_{x_1x_2} - \gamma_2\right) - 2g\gamma_2K_{yx_2}}{(\gamma_1 - \lambda_1)\left\{g^2(\gamma_1 + \lambda_1) - 2gK_{yx_1}\right\}}, \quad (3.22)$$

and

(xi) $MSE\left(\hat{\bar{Y}}_2^*\right) < MSE\left(\hat{\bar{Y}}_{PUS1}^*\right)$ if

$$\frac{C_{x_1}^2}{C_{x_2}^2} < \frac{2g^2\gamma_1\gamma_2K_{x_1x_2} - (\gamma_2 - \lambda_2)\left\{g^2(\gamma_2 + \lambda_2) + 2gK_{yx_2}\right\}}{g\gamma_1\left(g\gamma_1 - 2K_{yx_1}\right)}. \quad (3.23)$$

IV. EMPIRICAL STUDY

To see the performance of the proposed estimators we are considering a natural population data sets.

Population [Source: Bhuyan (2005) P. No. 77]

$\bar{Y} = 34.826$, $\bar{X}_1 = 774.666$, $\bar{X}_2 = 753.666$, $C_y^2 = 0005$, $C_{x_1}^2 = 0.007$, $C_{x_2}^2 = 0.009$, $\rho_{yx_1} = -0.363$, $\rho_{yx_2} = -0.409$, $\rho_{x_1x_2} = 0.901$, $N = 15$, $n = 6$, $\beta_2(x_1) = 1.761$, $\beta_2(x_2) = 1.828$.

Estimators	Population
$\bar{y}$	100
$\hat{\bar{Y}}_R$	33.607
$\hat{\bar{Y}}_P$	64.331
$\hat{\bar{Y}}_{RP}$	99.482
$\hat{\bar{Y}}_R^*$	49.900
$\hat{\bar{Y}}_P^*$	102.519
$\hat{\bar{Y}}_{RP}^*$	108.511
$\hat{\bar{Y}}_{RUS1}$	34.625
$\hat{\bar{Y}}_{PUS1}$	66.668
$\hat{\bar{Y}}_{RUS1}^*$	51.007

$\hat{\bar{Y}}^*_{PUS1}$	104.535
$\hat{\bar{Y}}_1$	83.284
$\hat{\bar{Y}}_2$	102.939
$\hat{\bar{Y}}^*_1$	110.933
$\hat{\bar{Y}}^*_2$	110.423

V. CONCLUSION

Table 4.1 shows that the developed estimators $\hat{\bar{Y}}^*_1$ and $\hat{\bar{Y}}^*_2$ have highest PRE than usual unbiased estimator $\bar{y}$, classical ratio estimator $\hat{\bar{Y}}_R$, classical product estimator $\hat{\bar{Y}}_P$, Singh (1967) ratio-cum-product estimator $\hat{\bar{Y}}_{RP}$, dual to ratio estimator $\hat{\bar{Y}}^*_R$, dual to product estimator $\hat{\bar{Y}}^*_P$, dual to ratio-cum-product estimator $\hat{\bar{Y}}^*_{RP}$, Upadhyaya and Singh (1999) estimators $\hat{\bar{Y}}_{RUS1}$ & $\hat{\bar{Y}}_{PUS1}$ and dual to Upadhyaya and Singh (1999) estimators $\hat{\bar{Y}}^*_{RUS1}$ & $\hat{\bar{Y}}^*_{PUS1}$. Thus if coefficient of variation and coefficient of kurtosis of the two AVs are known and conditions obtained in sections 3 are fulfilled.

REFERENCES

[1]. Cochran, W. G. (1940). The estimation of yields of cereal experiments by sampling for ratio of grain to total produce. J. Agril. Sci., 30, 262-275.
[2]. Parmar, R. (2013). Use of auxiliary information for estimation of some population parameters in sample surveys. Ph.D. thesis, Vikram University, Ujjain (M.P.), India.
[3]. Robson, D. S. (1957). Application of multivariable polykays to the theory of unbiased ratio type estimation. J. Amer. Statist. Assoc., 50, 1225-1226.
[4]. Singh, M. P. (1967). Ratio-cum-product method of estimation. Metrika, 12, 1, 34-43.
[5]. Singh, H. P., Singh, R., Espejo, M. R., Pineda, M. D. and Nadarajah (2005). On the efficiency of a dual to ratio-cum-product estimator in sample surveys. Math. Proc. R. Ir. Acad., 105 A, 2, 51-56.
[6]. Srivenkataramana, T. (1980). A dual of ratio estimator in sample surveys. Biom. J., 67, 1, 199-204.
[7]. Upadhyaya, L. N. and Singh, H. P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. Biom. J., 41, 5, 627-636.

