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Probabilistic Metric Space, Menger Space and Some Common Fixed Point Theorem

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Abstract- In this note we use the concept of semi compatible pair of reciprocal continuous maps in Probabilistic metric space and in Menger Space. We define existing definitions in this literature, and prove a common fixed point theorem in these spaces. Our results improve many well-known existing results in this literature.

***Keywords*:** Probabilistic metric space, Menger space, Semi compatible maps, Reciprocal continuous maps.
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I. INTRODUCTION

Banach [1] introduced the concept of Banach contraction principle in metric spaces in 1922. This contraction mapping principle played a pivotal role in the development of fixed point of mappings in many branches of mathematical analysis. It also has vast applications in many branches of modern mathematics, economics, physics, biology etc. In the last few decades, the concept of metric space has been extended. Menger [14] has introduced the theory of probabilistic metric space. Sehgal [19] initiated to study the contraction mapping theorem in PM space. Sehgal and Bharucha-Reid [20] have generalized the Banach contraction mapping principle in probabilistic metric (PM) spaces. They originated the study of contraction mappings in the development of fixed point theorems. Probabilistic metric spaces are probabilistic simplification of metric spaces. In these spaces, instead of a non-negative real number, every pair of elements is provided to a distribution function. The inherent flexibility of these spaces permits us to spread the contraction mapping principle in numerous in-equivalent methods. Menger space is a specific type of probabilistic metric space in which the triangular inequality is proposed with the help of a t-norm. The hypothesis of Menger spaces is an important piece of stochastic research. Schweizer and Sklar [18] have given a broad explanation of numerous types of such spaces. Schweizer and Sklar [18] played key role in the improvement of fixed point theory in PM space. Singh et. al. [22] presented the notion of weakly commuting mapping in PM- space. Kumar and Chugh [11] showed some common fixed point theorem with the notion of reciprocal continuous of mappings. Many researchers have extended numerous results of metric fixed point theory to these spaces. Bouhadjera and Godet-thobie [2] presented two new thoughts as sub-sequential continuity and sub-compatibility which are weaker than reciprocal continuity and compatibility respectively.

The purpose of this paper is to present some common fixed point theorem in Menger Space using the thought of semi compatible pairs of reciprocal continuous maps. Our results are also extension of the widespread results of Malviyaet.al. [13] and those of many others.

II. DEFINITIONS AND MATHEMATICAL PRELIMINARIES

In this section we discuss some important definitions and mathematical preliminaries which we use in our main results.

Definition 2.1. (Probabilistic metric space [3], [18]). A probabilistic metric space (PM-space) is an ordered pair (X, F), where X is a non-empty set and F is a mapping from X ×X into the set of all distribution functions. The function $F_{x,y}$ is assumed to satisfy the following conditions for all $x,y,z \in X$,

(i) $F_{x,y}(0) = 0$,

(ii) $F_{x,y}(t) = 1$ for all $t > 0$ if and only if $x = y$,
(iii) $F_{x,y}(t) = F_{y,x}(t)$ for all $t > 0$,
(iv) if $F_{x,y}(t_1) = 1$ and $F_{y,z}(t_2) = 1$ then $F_{x,z}(t_1 + t_2) = 1$ for all $t_1,t_2> 0$, where $F_{x,y}$ are distribution functions, that is, each $F_{x,y}$, $x,y\in X$ is non-decreasing and left continuous with $\inf (t\in R)F_{x,y}(t) = 0$ and $\sup (t\in R)F_{x,y}(t) = 1$, where R is the set of real numbers and R_+ is the set of non-negative real numbers.

Shi, Ren and Wang [21] give the following definition of n-th order t-norm.

Definition 2.2. (n-th order t-norm [21]). A mapping $T: \prod_{i=1}^{n}[0,1]\rightarrow [0,1]$ is called a n-th order t-norm if the following conditions are satisfied:
(i) $T(0,0,...,0) = 0, T(a,1,1,...,1) = a$ for all $a \in [0,1]$,
(ii) $T(a_1,a_2,a_3,...,a_n) = T(a_2,a_1, a_3...,a_n) =T(a_2,a_3, a_1...,a_n)$
(iii) $a_i>b_i$, $i=1,2,3,...,n$ implies $T(a_1,a_2,a_3,...,a_n) \geq T(b_1,b_2,b_3,...,b_n)$
(iv) $T(T(a_1,a_2,,a_3,...,a_n),b_2,,b_3,...,b_n)=T(a_{1,}T(a_2,a_3,...,a_n,b_2),b_3,...,b_n)=$
$T(a_{1,} a_2,T(a_3,...,a_n,b_{2,} b_3),b_4,...,b_n)= ... = T(a_1,a_2,a_3,...,a_{n-1,} T(a_n,b_2,b_3,...,b_n))$

When n = 2, we have a binary t-norm, which is commonly known as t-norm.

Definition 2.3. (Menger space [3],[13], [18]). A Menger space is a triplets (X,F,Δ), where X is a nonempty set, F is a function from $X\times X$ to the set of all distribution functions and Δ is a second order t-norm, such that the following conditions are satisfied:
(i) $F_{x,y}(0) = 0$ for all $x,y\in X$,
(ii) $F_{x,y}(s) = 1$ for all $s > 0$ if and only if $x = y$,
(iii) $F_{x,y}(s) = F_{x,y}(s)$ for all $x,y\in X$, $s > 0$ and
(iv) $F_{x,y}(u + v) > \Delta(F_{x,z}(u),F_{z,y}(v))$ for all $u,v> 0$ and $x,y,z \in X$.
(v) If $F_{x,y}(u) = 1$ and $F_{y,z}(v) = 1$, then $F_{x,z}(u+v) = 1$, for all x,y,z in X ,u,v> 0.

Definition 2.4. [15] A t-norm is a binary operation on the interval [0, 1] such that for all a, b, c, d ∈ [0,1] the following conditions are satisfied
(i) $a*1 = a$;
(ii) $a*b = b*a$;
(iii) $a*b \leq c*d$, whenever $a\leq c$ and $b\leq d$;
(iv) $a *(b*c) = (a*b)*c$.

Definition 2.5. [15] A mapping $F : R \rightarrow R$, is called a distribution if it is non-decreasing left continuous with $\inf\{F(t) : t \in R\}= 0$ and $\sup\{F(t) : t \in R\}= 1$.

Definition 2.6. [15] A mapping $t:[0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if it is satisfies the following conditions :
(i) t is commutative and associative ;
(ii) $t(a,1) = a$, for all $a\in [0,1]$;
(iii) $t(a,b) \leq t(c,d)$, for $a\leq c$ and $b\leq d$.

Definition 2.7. {[13],[15]} A sequence $\{x_n\}$ in a Menger space (X,F,t) is said to be converges to a point x in X if and only if for each $\varepsilon> 0$ and $t > 0$, there is an integer $M(\varepsilon) \in N$ such that
$Fx_{n,}x_m (\varepsilon)> 1- t$, for all $n, m \geq M(\varepsilon)$.

Definition 2.8. [15] A Menger PM-space (X,F,t) is said to be complete if every Cauchy sequence in X converges to a point in X.

Definition 2.9. [10] Self mappings P and S of a Menger space (X,F,t) are said to be compatible if $F_{PS}x_{n,\,SP}x_n (x) \rightarrow 1$, for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $PSx_n, SPx_n \rightarrow u$, for some u in X, as $n\rightarrow \infty$.

Definition 2.10. [13] Two maps P and S are said to be weakly compatible if they commute at a coincidence point.

Definition 2.11. [12] Two self maps P and S of a Menger space (X,F,t) are said to be reciprocally continuous if $PSx_n\rightarrow Pz$ and $SPx_n\rightarrow Sz$, Whenever $\{x_n\}$ is a sequence in X such that $Px_n, Sx_n \rightarrow z$, for some z in X as $n \rightarrow \infty$.

Definition 2.12. [13,23] Two self maps P and S of a Menger space (X,F,t) are said to be semi compatible if $F_{PS}x_{n,\,S}x_n\,(x) \to 1$, for all x > 0, whenever $\{x_n\}$ is a sequence in X such that
Px_n, $Sx_n \to u$ for some u in X as $n \to \infty$.
Lemma 2.1. [13,23] Let (X,F,*) be a Menger space with continuous t- norm* , if there exists a constant $h \in (0,1)$ such that $F_{x,y}(ht) \geq F_{x,y}(t)$, for all $x,y\in X$, and t > 0 then x = y.
Note: Any two mapping are reciprocally continuous mappings but they are not continuous in usual metric.[13]
Lemma 2.2. [13,23] Let $\{x_n\}$ be a sequence in a Menger space (X,F,t), where t is continuous and satisfies $t(x,y) \geq x$, for all $x \in [0,1]$. If there exists a constant $k\in (0,1)$ such that
$Fx_{n,}x_{n+1}\,(kx) \geq Fx_{n-1},x_n\,(x)$, n= 1,2,3…
then $\{x_n\}$ is a Cauchy sequence in X.

III. MAIN RESULTS

Theorem 3.1. Let A,B,C,D,K,M,P and V be self mappings on a complete Menger space (X,F,t) with continuous t-norm such that $t(c,c) \geq c$, for some $c \in [0,1]$ satisfying:
(3.1) $V(X) \subseteq ABC(X)$, $P(X) \subseteq DKM(X)$,
(3.2) (V,DKM) is weakly compatible,
(3.3) For all $x,y\in X$, and h >1,
$F_{Px,Vy}(hx)$
$\geq \min[F_{ABCx,\,DKMy}(x),\{F_{ABCx,\,Px}(x).\,F_{DKMy,\,Px}(x)\},\,F_{ABCx,\,Px}(x),\,F_{DKMy,\,Vy}(x),\,F_{DKMy,\,Px}(x)]$
If (P,ABC) is semi compatible pairs of reciprocal continuous maps then A,B,C,D,K,M,P and V have a unique common fixed point.
Proof: Let $x_0\in X$, be any arbitrary point. Then we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_{2n} = Px_{2n} = DKMx_{2n+1}$, and $y_{2n+1}=Vx_{2n+1}= ABCx_{2n+2}$, for n = 0,1,2,…
First, we will prove that $\{y_n\}$ is a Cauchy sequence in X.
Now, by inequality (3.3), we have
$Fy_{2n,}\,y_{2n+1}(hx)= F_Px_{2n,V}x_{2n+1}\,(hx) \geq$
$\min [\, F_{ABC}x_{2n,\,DKM}x_{2n+1}(x),\{F_{ABC}x_{2n,\,P}x_{2n}\,(x).\,F_{DKM}x_{2n+1,\,P}x_{2n}\,(x)\},\,F_{ABC}x_{2n,\,P}x_{2n}\,(x),$
$F_{DKM}x_{2n+1,\,V}x_{2n+1}\,(x),\,F_{DKM}x_{2n+1,\,P}x_{2n}\,(x)]$
$= \min [Fy_{2n-1,}\,y_{2n}(x),\{Fy_{2n-1,}\,y_{2n}(x).Fy_{2n,}\,y_{2n}(x)\},\,Fy_{2n-1,}\,y_{2n}(x),Fy_{2n,}\,y_{2n+1}(x),Fy_{2n,}\,y_{2n}(x)]$
$= Fy_{2n-1,}\,y_{2n}(x)$ (As F is a non- decreasing function).
So, we get, $Fy_{2n,}\,y_{2n+1}(hx) \geq Fy_{2n-1,}\,y_{2n}(x)$,
Similarly, we get, $Fy_{2n+1,}\,y_{2n+2}(hx) \geq Fy_{2n,}\,y_{2n+1}(x)$,
In general, we have $Fy_{n,}\,y_{n+1}(hx) \geq Fy_{n-1,}y_n(x)$
Then by Lemma 2.2, $\{y_n\}$ is a Cauchy sequence and it convergent to some point z in X.
Hence the subsequences convergent as follows:
$\{Px_{2n}\}\to z$, $\{Sx2n\} \to z$, $\{Vx_{2n+1}\}\to z$, $\{DKMx_{2n+1}\}\to z$ and $\{ABCx_{2n+2}\} \to z$.
Now, since P and ABC are reciprocal continuous and semi- compatible then we have
$\lim_{n\to\infty} PABCx_{2n+2} =Pz$, $\lim_{n\to\infty} ABCPx_{2n}= ABCz$ and $\lim_{n\to\infty} M(PABCx_{2n+2}\,,ABCz,t)=1$
Therefore we get Pz=ABCz.
Now we will show that Pz = z.
By inequality (3.3), putting x = z, y = x_{2n+1} we get,
$F_{Pz,V}x_{2n+1}\,(hx) \geq$
$\min[F_{ABCz,\,DKM}x_{2n+1}(x),\{F_{ABCz,\,Pz}(x).\,F_{DKM}x_{2n+1,\,Pz}(x)\},\,F_{ABCz,\,Pz}(x),$
$F_{DKM}x_{2n+1,\,V}x_{2n+1}\,(x),\,F_{DKM}x_{2n+1,\,Pz}(x)]$
Taking $\lim_{n\to\infty}$ on both side, we get
$F_{Pz,\,z}\,(hx) \geq \min[F_{Pz,\,z}(x),\{F_{Pz,\,Pz}(x).\,F_{z,\,Pz}(x)\},\,F_{Pz,\,Pz}(x),\,F_{z,\,z}\,(x),\,F_{z,\,Pz}(x)] = F_{z,\,Pz}(x)$
i.e., $F_{Pz,\,z}\,(hx) \geq F_{z,\,Pz}(x)$
then by lemma 2.1 we get that, Pz=z.
also we have Pz=ABCz, so, z=Pz=ABCz.
Now, $P(X) \subseteq DKM(X)$, therefore there exists a point $u\in X$ such that z= Pz = DKMu.
Putting x= x_{2n}, y = u in inequality (3.3), we get,
$F_Px_{2n,Vu}(hx) \geq$
$\min[F_{ABC}x_{2n,\,DKMu}(x),\{F_{ABC}x_{2n,\,P}x_{2n}\,(x).\,F_{DKMu,\,P}x_{2n}\,(x)\},\,F_{ABC}x_{2n,\,P}x_{2n}\,(x),$
$F_{DKMu,\,Vu}\,(x),\,F_{DKMu,\,P}x_{2n}\,(x)]$

Taking $\lim_{n\to\infty}$ on both side, we get

$F_{z,Vu}(hx) \geq \min[F_{z,z}(x),\{F_{z,z}(x). F_{z,z}(x)\}, F_{z,z}(x), F_{z,Vu}(x), F_{z,z}(x)] = F_{z,Vu}(x)$

then by lemma 2.1 we get that, Vu=z.

i.e., DKMu=Vu=z.

As (V,DKM) weakly compatible, then we have DKM.Vu=V.DKMu

i.e., DKMz=Vz.

Now, we will show that Vz=z.

To Prove this put, $x= x_{2n}$, y = z in inequality (3.3), we get,

$F_{Px_{2n},Vz}(hx) \geq$

$\min[F_{ABCx_{2n},DKMz}(x),\{F_{ABCx_{2n},Px_{2n}}(x). F_{DKMz,Px_{2n}}(x)\}, F_{ABCx_{2n},Px_{2n}}(x),$

$F_{DKMz,Vz}(x), F_{DKMz,Px_{2n}}(x)]$

Taking $\lim_{n\to\infty}$ on both side, we get

$F_{z,Vz}(hx) \geq \min[F_{z,Vz}(x),\{F_{z,z}(x). F_{Vz,z}(x)\}, F_{z,z}(x), F_{Vz,Vz}(x), F_{Vz,z}(x)] = F_{z,Vz}(x)$

Then by lemma 2.1 we get that, Vz=z.

so, we have z=Vz=Pz=ABCz=DKMz.

If we put, x=Cz and y=z, then from (3.1) we can easily prove that Cz=z.

Again, If we put, x=Bz and y=z, then from (3.1) we can easily prove that Bz=z.

So, finally we get, ABCz=z i.e., ABz=z, i.e., Az=z.

So, we have z=Az=Bz=Cz.

Similarly, If we put, x=z and y=Mz, then from (3.1) we can easily prove that Mz=z.

Again, If we put, x=z and y=Kz, then from (3.1) we can easily prove that Kz=z.

So, finally we get, DKMz=z i.e., DKz=z, i.e., Dz=z.

i.e., z=Az=Bz=Cz=Dz=Kz=Mz=Pz=Vz.

Which shows that, z is a common fixed point of A,B,C,D,K,M,P and V.

To prove the uniqueness of z, let us assume if possible there exists another common fixed point w(≠z) of A,B,C,D,K,M,P and V.

Then, from (3.1) we get,

$F_{z,w}(hx) = F_{Pz,Vw}(hx)$

$\geq \min[F_{ABCz,DKMw}(x),\{F_{ABCz,Pz}(x). F_{DKMw,Pz}(x)\}, F_{ABCz,Pz}(x), F_{DKMw,Vw}(x), F_{DKMw,Pz}(x)]$

$= \min[F_{z,w}(x), \{F_{z,z}(x).F_{z,w}(x)\}, F_{z,z}(x), F_{w,w}(x), F_{z,w}(x)]$

$= F_{z,w}(x)$

From lemma (2.1) we get that z=w.

So the common fixed point z is unique.

Corollary 3.2. Let A,B,D,K,P and V be self mappings on a complete Menger space (X,F,t) with continuous t-norm such that $t(c,c) \geq c$, for some $c \in [0,1]$ satisfying:

(3.1) $V(X) \subseteq AB(X)$, $P(X) \subseteq DK(X)$,

(3.2) (V, DK) is weakly compatible,

(3.3) For all $x,y \in X$, and h >1,

$F_{Px,Vy}(hx) \geq \min[F_{ABx,DKy}(x),\{F_{ABx,Px}(x). F_{DKy,Px}(x)\}, F_{ABx,Px}(x), F_{DKy,Vy}(x), F_{DKy,Px}(x)]$

If (P,AB) is semi compatible pairs of reciprocal continuous maps then A,B,D,K,P and V have a unique common fixed point.

Corollary 3.3. Let A,D,P and V be self mappings on a complete Menger space (X,F,t) with continuous t-norm such that $t(c,c) \geq c$, for some $c \in [0,1]$ satisfying:

(3.1) $V(X) \subseteq A(X)$, $P(X) \subseteq D(X)$,

(3.2) (V, D) is weakly compatible,

(3.3) For all $x,y \in X$, and h >1,

$F_{Px,Vy}(hx) \geq \min[F_{Ax,Dy}(x),\{F_{Ax,Px}(x). F_{Dy,Px}(x)\}, F_{Ax,Px}(x), F_{Dy,Vy}(x), F_{Dy,Px}(x)]$

If (P,A) is semi compatible pairs of reciprocal continuous maps then A,D,P and V have a unique common fixed point, which is the generalization of Malviya et al [13]. Anyone can see the Corollary 3.2, Corollary 3.3 of [13] for more corollaries of our main theorem.

IV. RELATED RESULTS IN CONE BANACH SPACES.

In this section we apply our main theorem to derive the corresponding common fixed point theorem in Cone Banach Spaces.

Theorem 4.1.

Let, $(X,\|.\|)$ be a Cone Banach Space and d: $X\times X\rightarrow E$ with $d(x,y)=\|x-y\|$.

Let, A, B, C, D, K, M, P and V be eight self mappings on X that satisfy the conditions:

(a) $V(X) \subseteq ABC(X)$ and $P(X) \subseteq DKM(X)$.

(b) $a\|Px - Vy\| + b\,\{\|ABCx - Px\| + \|DKMy - Vy\|\} + c\{\|DKMy - Px\| + \|ABCx - Vy\|\} \leq r\,\|ABCx - DKMy\|$; for all $x,y\in X$, $0 \leq r < a+2b+3c$, $a+b+c\neq 0$, $r\neq a+2c$. …(1)

(c) (P,ABC) and (V,DKM) are weakly compatible.

(d) If one of P(X), ABC(X), V(X), DKM(X) is a complete subspace of X then,

1. P and ABC have a coincidence point and
2. V and DKM have a coincidence point in X.

Then, A, B, C, D, K, M, P and V have a unique common fixed point in X.

Proof: The result follows from Sarkar and Tiwary [17].

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