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Union of Fuzzy Normal Semi- "N" Groups and Anti- Fuzzy Normal Semi- "N" Groups (for n= 1,2,….n)

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Abstract-In this paper, definition of Fuzzy normal semigroup, definition of Fuzzy normal semi bi-group, definition of Fuzzy normal semi tri-group, definition of Fuzzy normal semi – Quadratic group, definition of fuzzy normal semi- Pentant group and definition of Fuzzy normal semi –N groups are invented. Moreover, some properties and theorems of union in fuzzy normal based on these have been invented.

Keywords - Union of Fuzzy subsets, Fuzzy Normal semigroup, Fuzzy Normal semi-bigroup, Fuzzy Normal semi-trigroup , Fuzzy Normal semi – Quadratic group, Fuzzy Normal semi - Pendant group and Fuzzy Normal semi –N groups.

I. INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld gave the idea of fuzzy subgroups Since the paper fuzzy set theory has been considerably developed by zadeh himself and some researchers. The original concept of fuzzy sets was introduced as an extension of crisps (usual) sets, by enlarging the truth value set of "grade of members"from the two value set {0,1} to unit interval {0,1} of real numbers. The study of fuzzy group was started by Rosenfeld. It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets. Rosenfield introduced the notion of fuzzy group and showed that many group theory results can be extended in an elementary manner to develop the theory of fuzzy group. The underlying logic of the theory of fuzzy group is to provide a strict fuzzy algebraic structure where level subset of a fuzzy group of a group G is a subgroup of the group.

The notion of bigroup was first introduced by P.L. Maggu in 1994. W.B. Vasanthakandasamy introduced fuzzy sub-bigroup with respect to +‟‟ and .‟‟and illustrate it with example. W.B. Vasanthakandasamy was the first one to introduce the notion of bigroups in the year 1994 [8].

II .PRELIMINARIES

In this section contain some definitions, examples and some results**.**

Concept of a Fuzzy set: 2 .1.
The concept of a fuzzy set is an extension of the concept of a crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1} , a fuzzy set on a domain U is defined by its membership function from U to [0,1] .

Let U be a non-empty set, to be called the **Universal set** (or) **Universe of discourse or simply a domain**. Then, by a fuzzy set on U is meant a function **A: U →[0 ,1]. A** is called **the membership function; A (x)** is called **the membership grade** of x in A. We also write
A= {(x, A (x)): x∈U}.

Examples:
Consider U={a ,b ,c ,d} and A:U→**1** defined by A(a)=0,A(b)=0.7,A(c)=0.4,and A(d)=1.Then A is a fuzzy set can also be written as follows:
A= {(a, 0), (b,0.7),(c,0.4),(d,1)}.

Relation between Fuzzy sets: 2.2.
Let U be a domain and A, B be fuzzy sets on U.
Inclusion (or) Containment: A is said to be included (or) contained in B if and only if A(x)≤B(x) for all x in U. In symbols, we write, A⊆B. We also say that A is a subset of B.

Definition: 2.3
Let S be a set. A fuzzy subset A of S is a function A: S → [0,1].
Definition of Union of Fuzzy sets: 2.4. [14].

The union of two fuzzy subsets μ_1,μ_2 is defined by
$(\mu_1 \cup \mu_2)(x) = \max\{\mu_1(x),\mu_2(x)\}$ for every x in U.

Definition of Fuzzy Union of the fuzzy sets μ_1 and μ_2: 2.5.

Let μ_1 be a fuzzy subset of a set x_1 and μ_2 be a fuzzy subset of a set x_2 , then the fuzzy union of the fuzzy sets μ_1 and μ_2 is defined as a function.

$\mu_1 \cup \mu_2 : x_1 \cup x_2 \rightarrow [0,1]$ given by

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} \max(\mu_1(x),\mu_2(x)) & \text{if } x \in x_1 \cap x_2. \\ \mu_1(x) & \text{if } x \in x_1 \,\&\, x \notin x_2. \\ \mu_2(x) & \text{if } x \in x_2 \,\&\, x \notin x_1. \end{cases}$$

NOTE:

For Union of Anti-Fuzzy sets Replace min in the place of max in definitions 2.4 and 2.5. For differentiation may use "A" instead of μ .

Definition of Fuzzy Semigroup : 2.6. [14].

Let (G, ·) be a Semigroup . A map μ: G → [0,1] is called a fuzzy semigroup.
If $\mu(x . y) = \min\{\mu(x),\mu(y)\}$ for every x, y ∈ G.

Definition of Fuzzy normal semigroup : 2.7 [3].

Let G be a semigroup. A fuzzy semigroup μ of a group G is called a **Fuzzy normal semigroup** of G. if for all x, y ∈ G,
$\mu(xyx^{-1}) = \mu(y)$ (or) $\mu(xy) = \mu(yx)$.

Definition of Fuzzy Normal Semi-bi group of the bigroup G : 2.8. [3].

Let (G , .) be a semi - bigroup with binary operation . (multiplications) .Then μ : G → [0,1] is said to be a **fuzzy normal semi-bigroup of the semi - bigroup G** under " . " operation defined on G. if there exists two proper fuzzy subsets μ_1 of G_1 and μ_2 of G_2 such that

(i) $(\mu_1,.)$ is a fuzzy normal semigroup of $(G_1, .)$.
(ii) $(\mu_2,.)$ is a fuzzy normal semigroup of $(G_2,.)$.

Definition of Fuzzy Normal Semi-tri group of the tri group G (Anti-Fuzzy) : 2.9. [3].

Let (G, .) be a semi - trigroup with binary operation . (multiplications) . Then μ : G → [0,1] is said to be a **fuzzy normal semi-tri group** of the semi- tri group G under ". "operations defined on G. if there exists three proper fuzzy subsets μ_1 of G_1 and μ_2 of G_2, μ_3 of G_3 such that

(i) $(\mu_1, .)$ is a fuzzy normal semigroup of $(G_1, .)$
(ii) $(\mu_2, .)$ is a fuzzy normal semigroup of $(G_2, .)$
(iii) $(\mu_3, .)$ is a fuzzy normal semigroup of $(G_3 , .)$

Definition of Fuzzy Normal Semi-quadratic group of the Quadratic group G : 2.10. [3].

Let (G, .) be a semi- Quadratic group with binary operations. (multiplications) . Then μ : G → [0,1] is said to be a **fuzzy normal semi- quadratic group** of the Quadratic group G under ". " operations defined on G. if there exists four proper fuzzy subsets μ_1 of G_1 and μ_2 of G_2, μ_3 of G_3 , μ_4 of G_4 such that

(i) $(\mu_1, .)$ is a fuzzy normal semigroup of $(G_1, .)$
(ii) $(\mu_2, .)$ is a fuzzy normal semigroup of $(G_2, .)$
(iii) $(\mu_3, .)$ is a fuzzy normal semigroup of $(G_3 , .)$.
(iv) $(\mu_4, .)$ is a fuzzy normal semigroup of $(G_4 , .)$

Definition of Fuzzy Normal Semi- pentant group of the pentant group G : 2.11. [3].

Let (G , .) be a Semi- Pendant group with binary operations . (multiplications) . Then μ : G → [0,1] is said to be a **fuzzy normal semi - pentant group** of the Pendant group G under " . " operations defined on G. if there exists five proper fuzzy subsets μ_1 of G_1 and μ_2 of G_2, μ_3 of G_3 , μ_4 of G_4 , μ_5 of G_5 such that

(i) $(\mu_1,.)$ is a fuzzy normal semigroup of $(G_1, .)$
(ii) $(\mu_2,.)$ is a fuzzy normal semigroup of $(G_2, .)$
(iii) $(\mu_3,.)$ is a fuzzy normal semigroup of $(G_3 ,.)$
(iv) $(\mu_4, .)$ is a fuzzy normal semigroup of $(G_4 , .)$
(v) $(\mu_5, .)$ is a fuzzy normal semigroup of $(G_5 , .)$

Definition of Fuzzy Normal Semi- "n "group of the "n " group G : 2.12. [3].

Let (G , .) be a semi - n group with binary operations "." (multiplications). Then μ : G → [0,1] is said to be a **fuzzy normal semi – n group** of the " n " group G under " ." operations defined on G. if there exists "n" proper fuzzy subsets μ_1 of G_1 and μ_2 of G_2, μ_3 of G_3 , μ_4 of G_4 , μ_5 of G_5 ……. , μ_n of G_n such that

(i) $(\mu_1, .)$ is a fuzzy normal semigroup of $(G_1, .)$
(ii) $(\mu_2,.)$ is a fuzzy normal semigroup of $(G_2, .)$
(iii) $(\mu_3, .)$ is a fuzzy normal semigroup of $(G_3 , .)$
(iv) $(\mu_4, .)$ is a fuzzy normal semigroup of $(G_4 , .)$
(v) $(\mu_5, .)$ is a fuzzy normal semigroup of $(G_5 , .)$
(vi) ………………………………………….
(vii) $(\mu_n, .)$ is a fuzzy normal semigroup of $(G_n , .)$

NOTE:

For Definition of Anti- Fuzzy semi –"n" groups for n = 1,2,3,…n . Replace Anti- fuzzy in the place of fuzzy in definitions 2.6 , 2.7, 2.8, 2.9, 2.10, 2.11 and 2.12. For differentiation may use "A" instead of μ .

III.THEOREMS

Main Theorem: 3.1.
Every Fuzzy Normal Semi – bi group of a group G is a Fuzzy Normal Semigroup of the group G but not conversely[3].

Proof:
It follows from the definition of a fuzzy normal semi – bigroup of a group G that every fuzzy normal semi – bigroup of a group G is a fuzzy normal semigroup of the group G. Converse part is not true .

Corollary : 3.1[3].
Every Anti - Fuzzy Normal Semi – bi group of a group G is an Anti - Fuzzy Normal Semigroup of the group G but not conversely .

Proof:
It follows from the definition of an Anti - fuzzy normal semi – bigroup of a group G that every an Anti - fuzzy normal semi – bigroup of a group G is an Anti - fuzzy normal semigroup of the group G. Converse part is not true.

Main Theorem: 3.2.
Every Fuzzy Normal Semi – Tri group of a group G is a Fuzzy Normal Semigroup of the group G but not conversely .

Proof:
It follows from the definition of a fuzzy normal semi – trigroup of a group G that every fuzzy normal semi– trigroup of a group G is a fuzzy normal semigroup of the group G . Converse part is not true .

Corollary : 3.2.
Every Anti - Fuzzy Normal
Semi – tri group of a group G is an Anti - Fuzzy Normal Semigroup of the group G but not conversely[3]. .

Proof:
It follows from the definition of an Anti - fuzzy normal semi – trigroup of a group G that every an Anti - fuzzy normal semi – trigroup of a group G is an Anti - fuzzy normal semigroup of the group G. Converse part is not true.

Main Theorem: 3.3.
Every Fuzzy Normal
Semi – Quadratic group of a group G is a Fuzzy Normal Semigroup of the group G but not conversely[3].

Proof:
It follows from the definition of a fuzzy normal semi – Quadratic group of a group G that every fuzzy normal semi – Quadratic group of a group G is a fuzzy normal semigroup of the group G . Converse part is not true.

Corollary: 3.3.
Every Anti - Fuzzy Normal
Semi – Quadratic group of a group G is an Anti - Fuzzy Normal Semigroup of the group G but not conversely[3].

Proof:
It follows from the definition of an Anti - fuzzy normal semi – Quadratic group of a group G that every an Anti - fuzzy normal semi – Quadratic group of a group G is an Anti - fuzzy normal semigroup of the group G. Converse part is not true .

Main Theorem: 3.4.
Every Fuzzy Normal Semi – Pentant group of a group G is a Fuzzy Normal Semigroup of the group G but not conversely [3].

Proof:
It follows from the definition of a fuzzy normal semi– pentant group of a group G that every fuzzy normal semi – pentant group of a group G is a fuzzy normal subgroup of the group G . Converse part is not true .

Corollary : 3.4.
Every Anti - Fuzzy Normal
Semi – Pentant group of a group G is an Anti - Fuzzy Normal Semigroup of the group G but not conversely[3]. .

Proof:
It follows from the definition of an Anti - fuzzy normal semi – pentant group of a group G that every an Anti - fuzzy normal semi – pentant group of a group G is an Anti - fuzzy normal semigroup of the group G. Converse part is not true .

Main Theorem: 3.5.
Every Fuzzy Normal Semi – "n" group of a group G is a Fuzzy Normal Semigroup of the group G but not conversely [3].

Proof:
It follows from the definition of a fuzzy normal semi – "n " group of a group G that every fuzzy normal semi – " n" group of a group G is a fuzzy normal semigroup of the group G . Converse part is not true .

Corollary : 3.5.
Every Anti - Fuzzy Normal
Semi – "n" group of a group G is an Anti - Fuzzy Normal Semigroup of the group G but not conversely [3].

Proof:
It follows from the definition of an Anti - fuzzy normal semi – "n" group of a group G that every an Anti - fuzzy normal semi – "n" group of a group G is an Anti - fuzzy normal semigroup of the group G. Converse part is not true.

Main theorem : 3.6.
The union of two fuzzy normal semigroups of a group G is a fuzzy normal semigroup if and only if one is contained in the other[3].

Proof:

Necessary part:

Let μ_1 and μ_2 be two fuzzy normal semigroups of G such that one is contained in the other.

To Prove:

$\mu_1 \cup \mu_2$ is a fuzzy normal semigroup of G.

Let $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$. Since μ_1 and μ_2 are fuzzy normal semigroups of G, which implies if for all x, y $\in$ G,

$$\left.\begin{array}{l} \mu_1(xy) = \mu_1(yx) \\ \mu_2(xy) = \mu_2(yx). \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Let the union of two fuzzy subsets μ_1 , μ_2 is defined by

$$\mu_1 \cup \mu_2(xy) = \max\{\mu_1(xy), \mu_2(xy)\}\ldots\ldots(2).$$

By (1) and (2)

$$\left.\begin{array}{l} \mu_1 \cup \mu_2(xy) = \mu_1(xy) = \mu_1(yx) = \mu_1 \cup \mu_2(yx) \\ \mu_1 \cup \mu_2(xy) = \mu_2(xy) = \mu_2(yx) = \mu_1 \cup \mu_2(yx) \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(3).$$

From (3) ,

$\rightarrow$ $\mu_1 \cup \mu_2(xy) = \mu_1 \cup \mu_2(yx)$.

Hence $\mu_1 \cup \mu_2$ is fuzzy normal semigroup of G.

Sufficient Part:

Suppose $\mu_1 \cup \mu_2$ is a fuzzy normal semigroup of G.

To Claim: $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$.

Since μ_1 and μ_2 are fuzzy normal semigroup of G is obviously a fuzzy semigroup of G and $\mu_1 \cup \mu_2$ is a fuzzy normal semigroup of G.

By using the following theorem:

The union of two fuzzy semigroups of a group G is a fuzzy semigroup if and only if one is contained in the other.

Which implies , $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$.

Hence, The union of two fuzzy normal semigroups of a group G is a fuzzy normal semigroup if and only if one is contained in the other.

Corollary: 3.6.

The union of two Anti - fuzzy normal semigroups of a group G is an Anti - fuzzy normal semigroup if and only if one is contained in the other[3].

Proof:

In this, Main theorem replace "A" in the place of μ and "min" in the place of "max" and Anti-Fuzzy in the place of fuzzy respectively.

Main theorem : 3.7.

The union of two fuzzy normal semi-bigroups of a group G is a fuzzy normal semi-bigroup if and only if one is contained in the other[3].

Proof:

Necessary part:

Let μ_1 and μ_2 be two fuzzy normal semi-bigroups of G such that one is contained in the other .

To Prove:

$\mu_1 \cup \mu_2$ is a fuzzy normal semi-bigroup of G.

. Let $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$.

Since μ_1 and μ_2 are fuzzy normal semi-bigroups of G, which implies ,

if for all x, y $\in$ G,

$$\left.\begin{array}{l} \mu_1(xy) = \mu_1(yx) \\ \mu_2(xy) = \mu_2(yx). \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Let the union of two fuzzy subsets μ_1 , μ_2 is defined by

$$\mu_1 \cup \mu_2(xy) = \max\{\mu_1(xy), \mu_2(xy)\}\ldots\ldots(2).$$

By (1) and (2),

$$\left.\begin{array}{l} \mu_1 \cup \mu_2(xy) = \mu_1(xy) \\ \qquad = \mu_1(yx) = \mu_1 \cup \mu_2(yx) \\ \mu_1 \cup \mu_2(xy) = \mu_2(xy) \\ \qquad = \mu_2(yx) = \mu_1 \cup \mu_2(yx) \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(3).$$

From (3)

$\rightarrow$ $\mu_1 \cup \mu_2(xy) = \mu_1 \cup \mu_2(yx)$.

Hence $\mu_1 \cup \mu_2$ is a fuzzy normal semi-bigroup of G.

Sufficient Part:

Suppose $\mu_1 \cup \mu_2$ is a fuzzy normal semi-bigroup of G.

To Claim: $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$.

Since μ_1, and μ_2 are fuzzy normal semi-bigroup of G is obviously a fuzzy semi-bigroup of G and $\mu_1 \cup \mu_2$ is a fuzzy normal semi-bigroup of G is also a fuzzy semi-bigroup of G.

By using the following theorem:

The union of two fuzzy semi - bigroups of a group G is a fuzzy semi-bigroup if and only if one is contained in the other.

Which implies , $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$.

Hence, The union of two fuzzy normal semi-bigroups of a group G is a fuzzy normal semi-bigroup if and only if one is contained in the other.

Corollary: 3.7.
The union of two fuzzy normal semi-bigroups of a group G is a fuzzy normal semi-bigroup if and only if one is contained in the other[3].

Proof:
In this, Main theorem replace "A" in the place of μ and "min" in the place of "max" and Anti-Fuzzy in the place of fuzzy respectively.

Main theorem : 3.8.
The union of three fuzzy normal semi-trigroups of a group G is a fuzzy normal semi-trigroup if and only if one is contained in the other [3].

Proof:
Necessary part:
Let μ_1 , μ_2 and μ_3 are three fuzzy normal semi-trigroups of G such that one is contained in the other .
To Prove:
$\mu_1 \cup \mu_2 \cup \mu_3$ is a fuzzy normal semi - trigroup of G.
Let $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$.
Since μ_1 and μ_2 , μ_3 are fuzzy normal semi-trigroups of G, which implies

if for all x, y $\in$ G,

$$\left.\begin{aligned} \mu_1(xy) &= \mu_1(yx) \\ \mu_2(xy) &= \mu_2(yx). \\ \mu_3(xy) &= \mu_3(yx). \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots(1).$$

Let the union of three fuzzy subsets μ_1 , μ_2, μ_3 is defined by

$$\mu_1 \cup \mu_2 \cup \mu_3\ (xy) = \max\{\mu_1(xy), \mu_2(xy), \mu_3(xy)\} \ldots\ldots\ldots\ldots (2).$$

By (1) and (2),

$$\left.\begin{aligned} \mu_1 \cup \mu_2 \cup \mu_3\ (xy) &= \mu_1(xy) = \mu_1(yx) \\ &= \mu_1 \cup \mu_2 \cup \mu_3\ (yx) \\ \mu_1 \cup \mu_2 \cup \mu_3\ (xy) &= \mu_2(xy) = \mu_2(yx) \\ &= \mu_1 \cup \mu_2 \cup \mu_3\ (yx) \\ \mu_1 \cup \mu_2 \cup \mu_3\ (xy) &= \mu_3(xy) = \mu_3(yx) \\ &= \mu_1 \cup \mu_2 \cup \mu_3\ (yx) \end{aligned}\right\} \ldots\ldots\ldots\ldots(3).$$

From (3) ,
$\rightarrow \mu_1 \cup \mu_2 \cup \mu_3\ (xy) = \mu_1 \cup \mu_2 \cup \mu_3\ (yx)$.
Hence $\mu_1 \cup \mu_2 \cup \mu_3$ is a fuzzy normal semi-trigroup of G.

Sufficient Part:
Suppose $\mu_1 \cup \mu_2 \cup \mu_3$ is a fuzzy normal semi-trigroup of G.

To Claim: $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$.
Since μ_1, and μ_2 , μ_3 are fuzzy normal semi - trigroup of G is obviously a fuzzy semi-trigroup of G and $\mu_1 \cup \mu_2 \cup \mu_3$ is also a fuzzy normal semi-trigroup of G is also a fuzzy semi-trigroup of G.
By using the following theorem:
The union of three fuzzy semi -trigroups of a group G is a fuzzy semi-trigroup if and only if one is contained in the other.
Which implies , $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$.

Hence, The union of three fuzzy normal semi-trigroups of a group G is a fuzzy normal semi - trigroup if and only if one is contained in the other.

Corollary: 3.8.
The union of three Anti - fuzzy normal semi-trigroups of a group G is an Anti - fuzzy normal semi-trigroup if and only if one is contained in the other[3].

Proof:
In this, Main theorem replace "A" in the place of μ and "min" in the place of "max" and Anti-Fuzzy in the place of fuzzy respectively.

Main theorem : 3.9.
The union of four fuzzy normal semi- Quadratic groups of a group G is a fuzzy normal semi- Quadratic group if and only if one is contained in the other[3].

Proof:
Necessary part:
Let μ_1 , μ_2 and μ_3 , μ_4 are four fuzzy normal semi-Quadratic groups of G such that one is contained in the other .
To Prove:
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ is a fuzzy normal semi-Quadratic group of G.
Let $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$

Since μ_1 and μ_2 , μ_3, μ_4 are fuzzy normal semi - Quadratic groups of G, which implies if for all x, y $\in$ G,

$\mu_1 (xy) = \mu_1(yx)$
$\mu_2 (xy) = \mu_2(yx)$ ………………..……(1).
$\mu_3 (xy) = \mu_3(yx)$.
$\mu_4 (xy) = \mu_4(yx)$.

Let the union of four fuzzy subsets μ_1, μ_2, μ_3, μ_4 is defined by

$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (xy) = \max \{\mu_1(xy), \mu_2 (xy), \mu_3 (xy), \mu_4 (xy)\}$………. … (2)

By (1) and (2),

$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (xy) = \mu_1(xy) = \mu_1(yx) = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (yx)$
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (xy) = \mu_2(xy) = \mu_2(yx) = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (yx)$
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (xy) = \mu_3(xy) = \mu_3(yx) = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (yx)$
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (xy) = \mu_4(xy) = \mu_4(yx) = \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (yx)$
………………………………..(3).

From (3)
$\rightarrow \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (xy)$
$= \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 (yx)$.

Hence $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ is a fuzzy normal semi- Quadratic group of G.

Sufficient Part:

Suppose $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ is a fuzzy normal semi - Quadratic group of G.

To Claim:

$\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$ $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$.

Since μ_1, and μ_2, μ_3, μ_4 are fuzzy normal semi - Quadratic group of G is obviously a fuzzy semi - Quadratic group of G and $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ is also a fuzzy normal semi - Quadratic group of G is also a fuzzy semi - Quadratic group of G.

By using the following theorem:
The union of four fuzzy semi- Quadratic groups of a group G is a fuzzy
semi - Quadratic group if and only if one is contained in the other. Which implies , $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$.

Hence, The union of four fuzzy normal semi - Quadratic groups of a group G is a fuzzy normal semi - Quadratic group if and only if one is contained in the other.

Corollary: 3.9.
The union of four Anti - fuzzy normal semi- Quadratic groups of a group G is an Anti - fuzzy normal semi- Quadratic group if and only if one is contained in the other[3].

Proof:
In this, Main theorem replace "A" in the place of μ and "min" in the place of "max" and Anti-Fuzzy in the place of fuzzy respectively.

Main theorem : 3.10.
The union of five fuzzy normal semi - pentant groups of a group G is a fuzzy normal semi - pentant group if and only if one is contained in the other[3].

Proof:
Necessary part:
Let μ_1 , μ_2 and μ_3 , μ_4 , μ_5 are five fuzzy normal semi - pentant groups of G such that one is contained in the other .

To Prove:
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ is a fuzzy normal semi - pentant group of G.

Let $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$, $\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$, $\mu_5 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_5$, $\mu_5 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_5$, $\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$

Since μ_1 and μ_2 , μ_3, μ_4 , μ_5 are fuzzy normal semi - pentant groups of G, which implies

if for all x, y $\in$ G,

$\mu_1 (xy) = \mu_1(yx)$
$\mu_2 (xy) = \mu_2(yx)$.
$\mu_3 (xy) = \mu_3(yx)$.
$\mu_4 (xy) = \mu_4(yx)$.
$\mu_5 (xy) = \mu_5(yx)$. …………………(1).

Let the union of four fuzzy subsets μ_1 , μ_2, μ_3, μ_4 , μ_5 is defined by
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5 (xy)$
$=\max\{\mu_1(xy),\mu_2(xy),\mu_3(xy), \mu_4(xy), \mu_5(xy)\}$ ……….
……………………..…... (2)

By (1) and (2),

$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (xy) = μ_1(xy)
= μ_1(yx)
= $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (yx)
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (xy) = μ_2(xy)
= μ_2(yx)
= $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (yx)
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (xy) = μ_3(xy)
= μ_3(yx)
= $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (yx) $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (xy) = μ_4(xy)
= μ_4(yx)
= $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (yx)
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (xy) = μ_5(xy)
= μ_5(yx)
= $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (yx)
……………………………………..(3).

From (3) ,
$\rightarrow \mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (xy)
= $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ (yx) .

Hence $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ is a fuzzy normal semi- pentant group of G.

Sufficient Part:

Suppose $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ is a fuzzy normal semi - pentant group of G.

To Claim:

$\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$ $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$,
$\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$, $\mu_5 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_5$, $\mu_5 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_5$, $\mu_4 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_4$.

Since μ_1 and μ_2 , μ_3 , μ_4 , μ_5 are fuzzy normal semi - pentant group of G is obviously a fuzzy semi - pentant group of G and $\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ is also a fuzzy normal semi - pentant group of G is also a fuzzy semi - pentant group of G.

By using the following theorem:

The union of five fuzzy semi - pentant groups of a group G is a fuzzy semi- pentant group if and only if one is contained in the other. Which implies , $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$, $\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$, $\mu_5 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_5$, $\mu_5 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_5$, $\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$.

Hence, The union of five fuzzy normal semi - pentant groups of a group G is a fuzzy normal semi - pentant group if and only if one is contained in the other.

Corollary: 3.10.
The union of five Anti - fuzzy normal semi - pentant groups of a group G is an Anti - fuzzy normal semi - pentant group if and only if one is contained in the other [3]..

Proof:
In this, Main theorem replace "A" in the place of μ and "min" in the place of "max" for and Anti-Fuzzy in the place of fuzzy respectively.
Similarly for "n",

Main theorem : 3.11.
The union of "n" fuzzy normal semi - "n" groups of a group G is a fuzzy normal semi - "n" group if and only if one is contained in the other[3].

Proof:
Necessary part:
Let μ_1 , μ_2 and μ_3 , μ_4 , μ_5 …. μ_n are "n" fuzzy normal semi - "n" groups of G such that one is contained in the other .
To Prove:
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4 \cup \mu_5$ ……$\cup \mu_n$ is a fuzzy normal semi - "n" group of G.

Let $\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$, $\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$, $\mu_5 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_5$, $\mu_5 \subseteq \mu_2$ $\mu_2 \subseteq \mu_5$, $\mu_4 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_4$,… … … … … … … . $\mu_1 \subseteq \mu_n$ and $\mu_n \subseteq \mu_1$, $\mu_n \subseteq \mu_3$ and $\mu_3 \subseteq \mu_n$, $\mu_n \subseteq \mu_2$ and $\mu_2 \subseteq \mu_n$.

Since μ_1 and μ_2 , μ_3, μ_4 , μ_5 … … . μ_n are fuzzy normal semi - "n" groups of G, Which implies, if for all x, y $\in$ G,

μ_1 (xy) = μ_1(yx)
μ_2 (xy) = μ_2(yx).
μ_3 (xy) = μ_3(yx).
μ_4 (xy) = μ_4(yx).
……………………
……………………
μ_n (xy) = μ_n(yx)……………………………(1)

Let the union of "n" fuzzy subsets μ_1 , μ_2, μ_3, μ_4 , μ_5 ,…… , μ_n is defined by
$\mu_1 \cup \mu_2 \cup \mu_3 \cup \mu_4$ ……$\cup \mu_n$ (xy)

$= \max\{\mu_1(xy),\mu_2(xy),\mu_3(xy),\ldots, \mu_n(xy)\}$ ………. ………………………….. (2)

By (1) and (2),

$\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n (xy) = \mu_1(xy)$
$= \mu_1(yx)$
$= \mu_1 \cup \mu_2 \cup \mu_3 \ldots\ldots\ldots\ldots \cup \mu_n (yx)$

$\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n (xy) = \mu_2(xy)$
$= \mu_2(yx)$
$= \mu_1 \cup \mu_2 \cup \mu_3 \ldots\ldots\ldots\ldots \cup \mu_n (yx)$

$\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n (xy) = \mu_3(xy)$
$= \mu_3(yx)$
$= \mu_1 \cup \mu_2 \cup \mu_3 \ldots\ldots\ldots \cup \mu_n (yx)$

…………………………………………………… ………………………………….

…………………………………………………… ………………………………….

$\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n (xy) = \mu_n(xy)$
$= \mu_n(yx)$
$= \mu_1 \cup \mu_2 \cup \mu_3 \ldots\ldots \cup \mu_n (yx)$

…………………………………..(3).

From (3)

→ $\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n (xy)$
$= \mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n (yx)$.

Hence $\mu_1 \cup \mu_2 \cup \ldots \cup \mu_n$ is a fuzzy normal semi - "n" group of G.

Sufficient Part:

Suppose $\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n$ is a fuzzy normal semi - "n" group of G.

To Claim:

$\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$

Since μ_1, and μ_2 , μ_3 , μ_4 , μ_5 are fuzzy normal semi - "n" group of G is obviously a fuzzy semi - "n" group of G and $\mu_1 \cup \mu_2 \cup \mu_3 \ldots \cup \mu_n$ is also a fuzzy normal semi -"n" group of G is also a fuzzy semi -"n" group of G.

By using the following theorem:

The union of four fuzzy semi - "n" groups of a group G is a fuzzy semi - "n" group if and only if one is contained in the other.

Which implies ,

$\mu_1 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_1$, $\mu_1 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_1$, $\mu_3 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_3$, $\mu_1 \subseteq \mu_4$ and $\mu_4 \subseteq \mu_1$, $\mu_4 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_4$, $\mu_4 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_4$, $\mu_1 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_1$, $\mu_5 \subseteq \mu_3$ and $\mu_3 \subseteq \mu_5$, $\mu_5 \subseteq \mu_2$ and $\mu_2 \subseteq \mu_5$, $\mu_4 \subseteq \mu_5$ and $\mu_5 \subseteq \mu_4$,… ………………… . $\mu_1 \subseteq \mu_n$ and $\mu_n \subseteq \mu_1$, $\mu_n \subseteq \mu_3$ and $\mu_3 \subseteq \mu_n$, $\mu_n \subseteq \mu_2$ and $\mu_2 \subseteq \mu_n$.

Hence, The union of four fuzzy normal semi - "n" groups of a group G is a fuzzy normal semi - "n" group if and only if one is contained in the other.

Corollary:3.11.

The union of "n" Anti - fuzzy normal semi - "n" groups of a group G is an Anti - fuzzy normal semi - "n" group if and only if one is contained in the other[3].

Proof:

In this, Main theorem replace "A" in the place of μ and "min" in the place of "max" and Anti-Fuzzy in the place of fuzzy respectively.

IV. CONCLUSION

In this paper, I have invented the definitions of Fuzzy normal semigroup, Fuzzy normal semi -bigroup, Fuzzy normal semi- trigroup, Fuzzy normal semi – Quadratic group, fuzzy normal semi - Pentant group and definition of Fuzzy normal semi –"n" groups . Shows that Every Fuzzy normal Semi- Bigroups , Fuzzy normal Semi -Trigroups and soon Fuzzy normal Semi- "n" groups are Fuzzy normal Semi group . Definition of Anti - Fuzzy normal semi –"n" groups for n = 1,2,3,…n . Shows that Every Anti-Fuzzy Normal Semi-Bigroups, Anti-Fuzzy Normal Semi - Trigroups and so on Anti-Fuzzy Normal Semi- "n" groups are Anti-Fuzzy Normal Semi group are also have been derived as replace "A" in the place of μ and "min" in the place of "max" for Anti-Fuzzy Normal in Main theorems and in definitions respectively.

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