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Computation of leap hyper-Zagreb indices of certain windmill graphs

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Abstract— In this paper, we establish the expressions for the first and second leap hyper-Zagreb indices of certain windmill graphs such as French windmill graph $\boldsymbol{F_n^m}$, Dutch windmill graph $\boldsymbol{D_n^m}$, Kulli cycle windmill graph $\boldsymbol{C_{n+1}^m}$ and Kulli path windmill graph $\boldsymbol{P_{n+1}^m}$.

Keywords— *French Windmill graph; Dutch Windmill graph; Kulli cycle windmill graph; Kulli path windmill graph; leap hyper-Zagreb index.*

I. INTRODUCTION

Throughout this paper, we consider only finite, connected, undirected graphs without loops and multiple edges. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For a vertex v, the degree $d(v)$ is the number of vertices adjacent to v. The distance $d(u,v)$ between any two vertices u and v is the length of shortest path connecting u and v. For a positive integer k, the open k-neighbourhood $N_k(v)$ of a vertex v in a graph G is defined as $N_k(v/G) = \{u \in V(G): d(u,v) = k\}$. The k-distance degree $d_k(v)$ of a vertex v in G is defined as the number of k neighbours of v in G. We denote $d_k(v) = |N_k(v)|$. We refer to [10] for undefined terminologies and notations from graph theory.

A topological index is a numerical parameter mathematically derived from the graph structure. It is a graph invariant. The topological indices have their applications in various disciplines of science and technology. The first and second Zagreb indices are amongst the oldest and best known topological indices defined in 1972 by Gutman [7] as follows:

$$M_1(G) = \sum_{v \in V(G)} d(v)^2$$

and

$$M_2(G) = \sum_{uv \in E(G)} d(u) \cdot d(v),$$

respectively.

In 2017, Naji et al. [8] defined first and second leap Zagreb indices by taking 2-distance degree of vertices instead of taking vertex degree and they are defined as follows:

$$LM_1(G) = \sum_{v \in V(G)} d_2(v)^2$$

and

$$LM_2(G) = \sum_{uv \in E(G)} d_2(u) \cdot d_2(v),$$

respectively. For more details one can refer to [1,2,4,5,9].

A new version of the first leap Zagreb index was proposed by Kulli in [13], defined as

$$LM_1^*(G) = \sum_{uv \in E(G)} [d_2(u) + d_2(v)]. \tag{1.1}$$

The first and second leap hyper-Zagreb indices were introduced in [13] and they are defined as

$$HLM_1(G) = \sum_{uv \in E(G)} [d_2(u) + d_2(v)]^2, \tag{1.2}$$

and

$$HLM_2(G) = \sum_{uv \in E(G)} [d_2(u) \cdot d_2(v)]^2 \tag{1.3}$$

respectively. For applications of first and second leap hyper-Zagreb indices one can refer [3].

The rest of the paper is organized as follows. In sections II, III, IV and V, we obtain the expressions for first, second leap hyper-Zagreb indices of french windmill graphs, dutch windmill graphs, Kulli cycle windmill graphs and Kulli path windmill graphs, respectively.

II. FRENCH WINDMILL GRAPHS

The French windmill graph [6] F_n^m is the graph obtained by taking $m \geq 2$ copies of $K_n, n \geq 2$ with a vertex in common. The graph F_n^m is shown in Figure 1. The French windmill graph F_2^m is called a star graph, the French windmill graph F_3^m is called a friendship graph and the French windmill graph F_4^2 is called a butterfly graph.

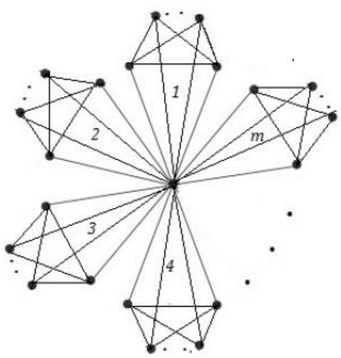

Figure 1: French windmill graph F_n^m

Let F be a French windmill graph F_n^m. The graph has $1 + m(n-1)$ vertices and $\frac{1}{2}mn(n-1)$ edges, $m \geq 2, n \geq 2$. Then there are two types of the 2-distance degree of edges as given in Table 1.

Table 1: 2-distance degree edge partition of F

$d_2(u), d_2(v) \mid uv \in E(F)$	$(0, (n-1)(m-1))$	$((n-1)(m-1), (n-1)(m-1))$
Number of edges	$m(n-1)$	$\frac{1}{2}m(n-1)(n-2)$

Theorem 2.1. *The new version of first leap Zagreb index of a French windmill graph F_n^m is given by*

$$LM_1^*(F_n^m) = m(m-1)(n-1)^3.$$

Proof. Let $F = F_n^m$. From Eq. (1.1) and by using Table 1, we obtain

$$\begin{aligned} LM_1^*(F_n^m) &= \sum_{uv \in E(F)} [d_2(u) + d_2(v)] \\ &= [0 + (n-1)(m-1)]m(n-1) \\ &\quad + \frac{1}{2}[(n-1)(m-1) + (n-1)(m-1)]m(n-1)(n-2) \\ &= m(m-1)(n-1)^3. \end{aligned}$$

Theorem 2.2. *The first leap hyper-Zagreb index of a French windmill graph F_n^m is*

$$HLM_1(F_n^m) = m(m-1)^2(n-1)^3(2n-3).$$

Proof. Let F be the graph of a French windmill graph F_n^m. By using Eq. (1.2) and by using Table 1, we deduce

$$\begin{aligned} HLM_1(F_n^m) &= \sum_{uv \in E(F)} [d_2(u) + d_2(v)]^2 \\ &= [0 + (n-1)(m-1)]^2 m(n-1) \\ &\quad + \frac{1}{2}[(n-1)(m-1) + (n-1)(m-1)]^2 m(n-1)(n-2) \\ &= m(m-1)^2 (n-1)^3(2n-3). \end{aligned}$$

Theorem 2.3. *The second leap hyper-Zagreb index of F_n^m is*

$$HLM_2(F_n^m) = \frac{1}{2}m(m-1)^4(n-1)^5(n-2).$$

Proof. Let $F = F_n^m$. From Eq. (1.3), we have

$$HLM_2(F_n^m) = \sum_{uv \in E(F)} [d_2(u)d_2(v)]^2$$

By using Table 1, we derive

$$\begin{aligned} HLM_2(F_n^m) &= [0 \times (n-1)(m-1)]^2 m(n-1) \\ &\quad + \frac{1}{2}[(n-1)(m-1) \times (n-1)(m-1)]^2 m(n-1)(n-2) \\ &= \frac{1}{2}m(m-1)^4(n-1)^5(n-2). \end{aligned}$$

III. DUTCH WINDMILL GRAPHS

The Dutch windmill graph, denoted by $D_n^m, m \geq 2, n \geq 5$, is the graph obtained by taking m copies of C_n with a vertex in common. The graph D_n^m is presented in Figure 2. The dutch windmill graph D_3^m is called a friendship graph. For more details on windmill graph, see [6].

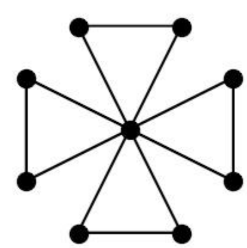

Figure 2: Dutch windmill graph D_3^4

Let D be a Dutch windmill graph D_n^m. The graph has $1 + m(n-1)$ vertices and mn edges, $m \geq 2, n \geq 5$. Then the edge partitions of 2-distance degree of edges as given in Table 2.

Table 2: 2-distance degree edge partition of D

$d_2(u), d_2(v) \mid uv \in E(D)$	$(2m, 2m)$	$(2m, 2)$	$(2,2)$
Number of edges	$2m$	$2m$	$m(n-4)$

Theorem 3.1. *Let D be the graph of a Dutch windmill graph D_n^m. Then*

$$LM_1^*(D_n^m) = 4mn + 12m^2 - 12m.$$

Proof. By using Eq. (1.1) and Table 2, we obtain

$$\begin{aligned} LM_1^*(D_n^m) &= \sum_{uv \in E(D)} [d_2(u) + d_2(v)] \\ &= (2m+2m)2m + (2m+2)2m + (2+2)m(n-4) \\ &= 4mn + 12m^2 - 12m. \end{aligned}$$

Theorem 3.2. *The first leap hyper-Zagreb index of Dutch windmill graph D_n^m is given by*

$$HLM_1(D_n^m) = 16mn + 40m^3 + 16m^2 - 56m.$$

Proof. Let D be the graph of a Dutch windmill graph D_n^m. By using Eq. (1.2) and by using Table 2, we derive

$$\begin{aligned} HLM_1(D_n^m) &= \sum_{uv \in E(D)} [d_2(u) + d_2(v)]^2 \\ &= (2m+2m)^2 2m + (2m+2)^2 2m + (2+2)^2 m(n-4) \\ &= 16mn + 40m^3 + 16m^2 - 56m. \end{aligned}$$

Theorem 3.3. *The second leap hyper-Zagreb index of Dutch windmill graph D_n^m is*

$$HLM_2(D_n^m) = 16mn + 36m^5 + 32m^3 - 64m.$$

Proof. Let $D = D_n^m$. By using Eq. (1.3), we have

$$HLM_2(D_n^m) = \sum_{uv \in E(D)} [d_2(u) d_2(v)]^2$$

By using Table 2, we deduce

$$\begin{aligned} HLM_2(D_n^m) &= (2m+2m)^2 2m + (2m+2)^2 2m \\ &\quad + (2+2)^2 m(n-4) \\ &= 16mn + 36m^5 + 32m^3 - 64m. \end{aligned}$$

IV. KULLI CYCLE WINDMILL GRAPHS

The Kulli cycle windmill graph [11] is the graph obtained by taking m copies of the graph $K_1 + C_n$ for $n \geq 3$ with a vertex K_1 in common and it is denoted by C_{n+1}^m This graph is depicted in Figure 3.

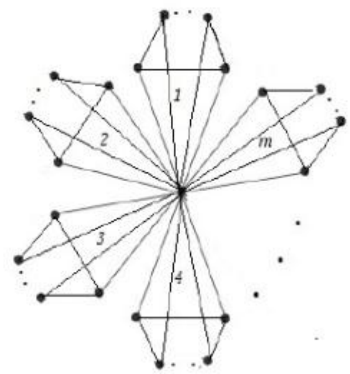

Figure 3: Kulli cycle windmill graph C_{n+1}^m.

Let $C = C_{n+1}^m$ be a Kulli cycle windmill graph with $mn+1$ vertices and $2mn$ edges, $m \geq 2, n \geq 5$. Then C has two types of 2-distance degree of edges as given in Table 3.

Table 3: 2-distance degree edge partition of C

$d_2(u), d_2(v) \mid uv \in E(C)$	$(0, mn-3)$	$(mn-3, mn-3)$
Number of edges	mn	mn

Theorem 4.1. *Let $C = C_{n+1}^m, m \geq 2, n \geq 5$. Then*

$$LM_1^*(C_{n+1}^m) = 3mn(mn-3).$$

Proof. From Eq. (1.1) and using Table 3, we obtain

$$\begin{aligned} LM_1^*(C_{n+1}^m) &= \sum_{uv \in E(C)} [d_2(u) + d_2(v)] \\ &= (0 + mn - 3)mn + (mn - 3 + mn - 3)mn \\ &= 3mn(mn-3). \end{aligned}$$

Theorem 4.2. *The first leap hyper-Zagreb index of C_{n+1}^m is given by*

$$HLM_1(C_{n+1}^m) = 5mn(mn-3)^2.$$

Proof. By using Eq. (1.2) and by using Table 3, we derive

$$\begin{aligned} HLM_1(C_{n+1}^m) &= \sum_{uv \in E(C)} [d_2(u) + d_2(v)]^2 \\ &= (0 + mn - 3)^2 mn + (mn - 3 + mn - 3)^2 mn \\ &= 5mn(mn-3)^2. \end{aligned}$$

Theorem 4.3. *The second leap hyper-Zagreb index of Dutch windmill graph C_{n+1}^m is*

$$HLM_2(C_{n+1}^m) = mn(mn-3)^4.$$

Proof. By using Eq. (1.3), we have

$$HLM_2(C_{n+1}^m) = \sum_{uv\in E(C)} [d_2(u)d_2(v)]^2$$

By using Table 3, we obtain

$$HLM_2(C_{n+1}^m) = (0\times(mn-3))^2 mn + ((mn-3)(mn-3))^2 mn = mn(mn-3)^4.$$

V. KULLI PATH WINDMILL GRAPHS

The Kulli path windmill graph [12] is the graph obtained by taking m copies of the graph K_1+P_n with a vertex K_1 in common and it is symbolized by P_{n+1}^m This graph is presented in Figure 4. The Kulli path windmill graph P_3^m is a friendship graph. Let $P=P_{n+1}^m, m\ge 2, n\ge 5$. Then P has $mn+1$ vertices and $2mn-m$ edges. The graph P has four types of 2-distance degree of edges as given in Table 4.

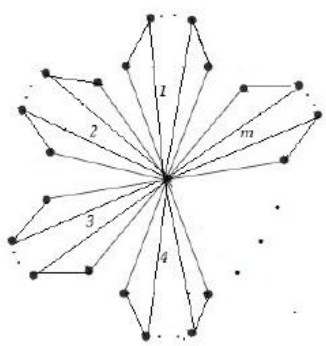

Figure 4: Kulli path windmill graph P_{n+1}^m.

Table 4: 2-distance degree edge partition of P

$d_2(u), d_2(v) \| uv \in E(P)$	$(0, mn-2)$	$(0, mn-3)$	$(mn-2, mn-3)$	$(mn-3, mn-3)$
Number of edges	$2m$	$mn-2m$	$2m$	$mn-3m$

Theorem 5.1. *Let* $P=P_{n+1}^m, m\ge 2, n\ge 5$. *Then*

$$LM_1^*(P_{n+1}^m) = 3m^2n^2-2m^2n-9mn+10m.$$

Proof. By using Eq. (1.1) and Table 4, we obtain

$$LM_1^*(P_{n+1}^m) = \sum_{uv\in E(P)} [d_2(u)+d_2(v)]$$
$$= (0+mn-2)2m+(0+mn-3)(mn-2m) +(mn-2+mn-3)2m +(mn-3+mn-3)(mn-3m)$$
$$= 3m^2n^2-2m^2n-9mn+10m.$$

Theorem 5.2. *Let* $P_{n+1}^m, m\ge 2, n\ge 5$. *Then*

$$HLM_1(P_{n+1}^m) = 5m^3n^3-4m^3n^2-30m^2n^2+36m^2n + 45mn-68m.$$

Proof. From Eq. (1.2) and by using Table 4, we deduce

$$HLM_1(P_{n+1}^m) = \sum_{uv\in E(P)} [d_2(u)+d_2(v)]^2$$
$$= (0+mn-2)^2 2m+(0+mn-3)^2(mn-2m) +(mn-2+mn-3)^2 2m +(mn-3+mn-3)^2(mn-3m)$$
$$= 5m^3n^3-4m^3n^2-30m^2n^2+36m^2n+45mn -68m.$$

Theorem 5.3. *Let* $P_{n+1}^m, m\ge 2, n\ge 5$. *Then*

$$HLM_2(P_{n+1}^m) = (mn-3)^2(m^3n^2+2m^3n^2-6m^2n^2 - 3m^3n+10m^2n+9mn-19m).$$

Proof. By using Eq. (1.3), we have

$$HLM_2(P_{n+1}^m) = \sum_{uv\in E(P)} [d_2(u)d_2(v)]^2$$

By using Table 4, we obtain

$$HLM_2(P_{n+1}^m) = (0\times(mn-2))^2 2m+(0\times(mn-3))^2 +[(mn-2)(mn-3)]^2 2m) +[(mn-3)(mn-3)]^2(mn-3m$$
$$= (mn-3)^2(m^3n^2+2m^3n^2-6m^2n^2 -3m^3n+10m^2n+9mn-19m).$$

VII. CONCLUSION

In this paper, we have obtained the expressions for the first and second leap hyper-Zagreb indices of french windmill graphs, dutch windmill graphs, Kulli cycle windmill graphs and Kulli path windmill graphs.

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