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A Study of Kahlerian Einstein Manifold with Bochner Curvature Tensor

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Abstract—The main purpose of the present paper is to study of Kahlerian Einstein manifold with Bochner curvature tensor. Few interesting results for Kahlerian Einstein manifold with Bochner curvature tensor have been obtained. Further we discussed about the theory of Kahlerian Einstein manifold with Bochner curvature tensor with R is non-zero. If a non-affine holomorphically projective transformation satisfying the condition $\nabla_h R_{ji} = 0$, .then Kahlerian manifold is terns into Kahlerian Einstein manifold. Necessary and sufficient condition that a holomorphically projective transformation is analytic in a Kahlerian Einstein manifold with non–vanishing scalar curvature tensor then its associated vector is analytic. An Kahlerian Einstein manifold, the associated vector of an analytic holomorphically projective transformation is analytic. An Einstein manifold with $R \neq 0$, any infinitesimal affine transformation is a killing vector. If the associated vector ρ^i of an analytic holomorphically projective transformation satisfying the condition $\nabla_i \rho^i = 0$, then a Kahlerian manifold satisfying the condition $\nabla_h R_{ji} = 0$ is not an Einstein manifold.

Keywords— *Kahlerian manifolds, Einstein manifold, Bochner Curvature Tensor, Kahlerian Einstein manifold, Killing vector, holomorphically projective transformation, Lie derivative, Lie Algebra, contravariant almost analytic vector, Covariant almost analytic vector*

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I. INTRODUCTION

In the Present paper firstly we defined Kahlerian manifold and Bochner curvature tensor Further, we have shown that the existence of a non-trivial analytic holomorphically projective transformations under Bochner curvature tensor in Kahlerian manifolds satisfying the condition $\nabla_h R_{ji} = 0$ reduced to Einstein manifolds. This paper is devoted to the study of Kahlerian Einstein manifold with Bochner curvature tensor. Few interesting results for Einstein Kahlerian manifold with Bochner curvature tensor has been obtained. In section 2 we discuss about Projective transformation with Bochner curvature tensor. In section 3 we have studied a transformation in a Kahlerian Einstein manifold with Bochner curvature tensor with R is non zero. In year 1978, C. Shibata gave the concept of Finsler manifold of non-vanishing scalar curvature with vanishing curvature tensor. S. Tachibana, S. Ishihara [16] have studied of infinitesimal holomorphically projective transformations in Kahlerian manifolds. S. Tachibana [15] has defined and discussed on the Bochner curvature tensor. N. Cengiz, O. Tarakc, A. Salirnov [11] has discussed about Kahlerian manifolds. T. Sumitomo [20] and K. Yano, T. Nagano [8] have studied infinitesimal projective transformation in a Riemannian manifold. The concept of Kahlerian Einstein manifold of Lie algebra of contravariant almost analytic vector is given by S. Tachibana [17]. He obtained the analytic form of the scalar curvature of such a manifold. P. Bhardwaj, N. Kumar, M. Chandra [13] have studied of infinitesimal holomorphically projective transformations (IHPT) in Kahlerian sub-manifolds with Bochner curvature tensor. P. Bhardwaj, N. Kumar, M. Chandra [12] have studied of Generalized ricci 3-recurrent space in Kahlerian manifolds with Bochner curvature tensor. M. Matsumoto [10] has studied Kaehlerian space with parallel or vanishing Bochner curvature tensor.

1.1. KAHLERIAN MANIFOLDS

An $n = 2m$ dimensional Kahlerian space K^n is a Riemannian space which admits a tensor field φ_λ^μ satisfying

$$\varphi_\alpha^\lambda \varphi_\mu^\alpha = -\delta_\mu^\lambda, \varphi_{\lambda\mu} = -\varphi_{\mu\lambda}, \left(\varphi_{\lambda\mu} = g_{\mu\alpha}\varphi_\lambda^\alpha\right) \text{ and } \nabla_\nu \varphi_\lambda^\mu = 0$$

Where ∇_ν means the operator of covariant differentiation.

We define Riemannian curvature tensor $R_{\lambda\mu\nu}^\kappa$ is

$$R_{\lambda\mu\nu}^\kappa = \partial_\lambda \begin{Bmatrix} \kappa \\ \mu\nu \end{Bmatrix} - \partial_\mu \begin{Bmatrix} \kappa \\ \lambda\nu \end{Bmatrix} + \begin{Bmatrix} \kappa \\ \lambda\alpha \end{Bmatrix}\begin{Bmatrix} \alpha \\ \mu\nu \end{Bmatrix} - \begin{Bmatrix} \kappa \\ \mu\alpha \end{Bmatrix}\begin{Bmatrix} \alpha \\ \lambda\nu \end{Bmatrix}$$

and $R_{\mu\nu} = R_{\alpha\mu\nu}^\alpha$, $R = g^{\lambda\mu} R_{\lambda\mu}$ are Ricci tensor and the scalar curvature respectively.

It is well known that these tensors satisfy the following identities: $R_{\alpha\mu\nu}^{\kappa}\varphi_{\lambda}^{\alpha}=-R_{\lambda\alpha\nu}^{\kappa}\varphi_{\mu}^{\alpha}$, $R_{\lambda\mu\alpha}^{\kappa}\varphi_{\nu}^{\alpha}=R_{\lambda\mu\nu}^{\alpha}\varphi_{\alpha}^{\kappa}$, $\varphi_{\lambda}^{\alpha}R_{\alpha\mu}=-R_{\lambda\alpha}\varphi_{\mu}^{\alpha}$, $\varphi_{\lambda}^{\alpha}R_{\alpha}^{\kappa}=R_{\lambda}^{\alpha}\varphi_{\alpha}^{\kappa}$,

$\nabla_{\alpha}R_{\lambda\mu\nu}^{\alpha}=\nabla_{\lambda}R_{\mu\nu}-\nabla_{\mu}R_{\lambda\nu}$ and $\nabla_{\lambda}R=2\nabla_{\alpha}R_{\lambda}^{\alpha}$.

If we define a tensor $S_{\mu\nu}$ by $S_{\mu\nu}=\varphi_{\mu}^{\alpha}R_{\alpha\nu}$

then we have

$S_{\mu\nu}=-S_{\nu\mu}, \varphi_{\lambda}^{\alpha}S_{\alpha\nu}=-S_{\lambda\alpha}\varphi_{\nu}^{\alpha}$,

$S_{\mu\nu}=-(1/2)\varphi^{\alpha\beta}R_{\alpha\beta\mu\nu}$ and

$2\nabla_{\alpha}S_{\lambda}^{\alpha}=\varphi_{\lambda}^{\alpha}\nabla_{\alpha}R$

The differential form $S=(1/2)S_{\lambda\mu}dx^{\lambda}\wedge dx^{\mu}$ is closed.

It follows that $\varphi_{\lambda}^{\alpha}\nabla_{\alpha}S_{\mu\nu}=-\nabla_{\mu}R_{\nu\lambda}+\nabla_{\nu}R_{\mu\lambda}$

It is also known as 2-form S is harmonic, where R is a constant.

1.2. EINSTEIN MANIFOLDS

If a non-affine holomorphically projective transformation satisfying the condition $\nabla_{h}R_{ji}=0,$ then the Kahlerian manifold is termed as an Einstein manifold.

1.3. BOCHNER CURVATURE TENSOR

A tensor $K_{\lambda\mu\nu}^{\kappa}$ is defined by

$$K_{\lambda\mu\nu}^{\kappa}=R_{\lambda\mu\nu}^{\kappa}+\frac{1}{n+4}\big(R_{\lambda\nu}\delta_{\mu}^{\kappa}-R_{\mu\nu}\delta_{\lambda}^{\kappa}+g_{\lambda\nu}R_{\mu}^{\kappa}-g_{\mu\nu}R_{\lambda}^{\kappa}$$
$$+S_{\lambda\nu}\varphi_{\mu}^{\kappa}-S_{\mu\nu}\varphi_{\lambda}^{\kappa}+\varphi_{\lambda\nu}S_{\mu}^{\kappa}-\varphi_{\mu\nu}S_{\lambda}^{\kappa}+2S_{\lambda\mu}\varphi_{\nu}^{\kappa}+2\varphi_{\lambda\mu}S_{\nu}^{\kappa}\big)$$
$$-\frac{R}{(n+2)(n+4)}\big(g_{\lambda\mu}\delta_{\mu}^{\kappa}-g_{\mu\nu}\delta_{\lambda}^{\kappa}+\varphi_{\lambda\mu}\varphi_{\mu}^{\kappa}-\varphi_{\mu\nu}\varphi_{\lambda}^{\kappa}+2\varphi_{\lambda\mu}\varphi_{\nu}^{\kappa}\big)$$

Which is constructed formally from $C_{\lambda\mu\nu}^{\kappa}$ by taking account of the form arisen balance between $W_{\lambda\mu\nu}^{\kappa}$ and $P_{\lambda\mu\nu}^{\kappa}$. Then we can prove that the tensor $K_{\lambda\mu\nu\varpi}=g_{\kappa\varpi}K_{\lambda\mu\nu}^{\kappa}$ has components of the tensor given by S. Bochner with respect to complex local coordinates. Hence it is known as Bochner curvature tensor.

Remark-1: If we put $L_{\lambda\mu}=R_{\lambda\mu}-\frac{R}{2(n+2)}g_{\lambda\mu}$, $M_{\lambda\mu}=\varphi_{\lambda}^{\alpha}L_{\alpha\mu}=S_{\lambda\mu}-\frac{R}{2(n+2)}\varphi_{\lambda\mu}$ and $K_{\lambda\mu\nu}^{\kappa}$ has the following form

$$K_{\lambda\mu\nu}^{\kappa}=R_{\lambda\mu\nu}^{\kappa}+\frac{1}{n+4}\big(L_{\lambda\mu}\delta_{\mu}^{\kappa}-L_{\mu\nu}\delta_{\lambda}^{\kappa}+g_{\lambda\nu}L_{\mu}^{\kappa}-g_{\mu\nu}L_{\lambda}^{\kappa}$$
$$+M_{\lambda\nu}\varphi_{\mu}^{\kappa}-M_{\mu\nu}\varphi_{\lambda}^{\kappa}+\varphi_{\lambda\nu}M_{\mu}^{\kappa}-\varphi_{\mu\nu}M_{\lambda}^{\kappa}+2M_{\lambda\mu}\varphi_{\nu}^{\kappa}+2\varphi_{\lambda\mu}M_{\nu}^{\kappa}\big)$$

The following identities are obtained by the straight forward computations

$K_{\lambda\mu\nu}^{\kappa}=-K_{\mu\lambda\nu}^{\kappa}$, $K_{\lambda\mu\nu\varpi}=-K_{\lambda\mu\varpi\nu}$,

$K_{\lambda\mu\nu}^{\kappa}+K_{\mu\nu\lambda}^{\kappa}+K_{\nu\lambda\mu}^{\kappa}=0$,

$K_{\alpha\mu\nu}^{\alpha}=0$, $K_{\lambda\mu\alpha}^{\alpha}=0$, $K_{\lambda\mu\nu}^{\alpha}\varphi_{\alpha}^{\kappa}=K_{\lambda\mu\alpha}^{\kappa}\varphi_{\nu}^{\alpha}$,

$K_{\alpha\mu\nu}^{\kappa}\varphi_{\lambda}^{\alpha}=-K_{\lambda\alpha\nu}^{\kappa}\varphi_{\mu}^{\alpha}$, $K_{\lambda\mu\alpha}^{\beta}\varphi_{\beta}^{\alpha}=0$ and $K_{\alpha\mu\nu}^{\beta}\varphi_{\beta}^{\alpha}=0$

Next we introduce a tensor $K_{\lambda\mu\nu}$ is given by

$$K_{\lambda\mu\nu}=\nabla_{\lambda}R_{\mu\nu}-\nabla_{\mu}R_{\lambda\nu}$$
$$+\frac{1}{2(n+2)}\big(g_{\lambda\nu}\delta_{\mu}^{\varepsilon}-g_{\mu\nu}\delta_{\lambda}^{\varepsilon}+\varphi_{\lambda\nu}\varphi_{\mu}^{\varepsilon}-\varphi_{\mu\nu}\varphi_{\lambda}^{\varepsilon}+2\varphi_{\lambda\mu}\varphi_{\nu}^{\varepsilon}\big)\nabla_{\varepsilon}R$$

Then we can get the following identity

$$\nabla_{\alpha}K_{\lambda\mu\nu}^{\alpha}=\frac{n}{n+4}K_{\lambda\mu\nu}$$

Now consider a tensor $U_{\lambda\mu\nu}^{\kappa}$ is given by

$$U_{\lambda\mu\nu}^{\kappa}=R_{\lambda\mu\nu}^{\kappa}+\frac{R}{n(n+2)}\big(g_{\lambda\nu}\delta_{\mu}^{\kappa}-g_{\mu\nu}\delta_{\lambda}^{\kappa}+\varphi_{\lambda\nu}\varphi_{\mu}^{\kappa}-\varphi_{\mu\nu}\varphi_{\lambda}^{\kappa}+2\varphi_{\lambda\mu}\varphi_{\nu}^{\kappa}\big)$$

II. PROJECTIVE TRANSFORMATION WITH BOCHNER CURVATURE TENSOR

Let a Kahlerian manifold admit infinitesimal projective transformations with regard to the vector field ξ^{i} . Then denoting by L_{V} the Lie derivative with regard to the vector field ξ^{i}, we have

(2.1) $$L_{V}\left\{ {i \atop j\;\;k} \right\}=\delta_{j}^{i}B_{k}+\delta_{k}^{i}B_{j},$$

(2.2) $$L_{V}R_{j\gamma h}^{k}=\delta_{\gamma}^{k}B_{j,h}-\delta_{h}^{k}B_{j,\gamma}$$

(2.3) $$L_{V}P_{jih}^{k}=0$$

Wherein $\left\{ {i \atop j\;\;k} \right\}$ the Christoffel symbol of g_{ij}, B_{i} is the gradient vector field, and P_{jih}^{k} is a Projective curvature tensor, and is defined as

(2.4) $$P_{jih}^{k}=R_{jih}^{k}-\{1/(n-1)\}\left(\delta_{h}^{k}R_{ji}-\delta_{i}^{k}R_{jh}\right)$$

Wherein $B_{i}=0,$ the infinitesimal projective transformation is an affine one.

Tensor P_{jih}^{k} satisfies the relation.

(2.5) $$P_{jih,\alpha}^{k}=v_{\alpha}P_{jih}^{k},$$

Theorem 2.1:
For $n>2$, the recurrent Einstein space admitting infinitesimal projective transformation is of constant curvature or this transformation is an affine one.

III. KAHLERIAN EINSTEIN MANIFOLD WITH BOCHNER CURVATURE TENSOR

Definition 3.1:
If a Kahlerian manifold have a definite Ricci's form, then any infinitestimal affine transformation is analytic, so it is known as Killing vector field.

Definition 3.2:
If a non-affine holomorphically projective transformation satisfying the condition $\nabla_h R_{ji} = 0,$ then the Kahlerian manifold is termed as an Einsteinian manifold.
We have $L_V R_{ji} = 0$ for an infinitesimal affine transformation v^i.
If a Kahlerian manifold reduced to Einsteinian then

(3.1a) $$L_V\, g_{ji} = 0$$

(3.1b) $$(n/R) L_V\, R_{ji} = 0$$

In this regard, we have a theorem:

Theorem 3.1:
Necessary and sufficient condition that a holomorphically projective transformation is analytic in a Kahlerian Einstein manifold with non-vanishing scalar curvature then its associated vector is analytic.
In an Einstein Kahlerian manifold, the associated vector of an analytic holomorphically projective transformation is analytic.
Let us assume that the associated vector ρ_i of holomorphically projective transformation v^i is analytic. we obtain

(3.2) $$L_V\, R_{ji} = -(n+2)\nabla_j \rho_i,$$

If the Kahlerian manifold is an Einstein manifold then

(3.3) $$L_V g_{ji} = (1/h)\nabla_j \rho_i,$$

Wherein

(3.4) $$K = -\{1/n(n+2)\}R,$$

By virtue of equations (3.3) and (3.4), then a vector field p_i is defined in such a way

(3.5) $$p_i \xrightarrow{def} v_i - (1/2h)\rho_i.$$

We have

(3.6) $$\nabla_j\, p_i + \nabla_i\, p_j = 0,$$

This means that the vector p_i is a Killing vector.

Remark 3.1:
It is to be noted that in an Einstein manifold with $R \neq 0,$ any infinitesimal affine transformation is a Killing vector.

Remark 3.2:
It is noteworthy that an infinitesimal affine transformation v^i is an analytic holomorphically projective transformation in a Kahlerian Einstein manifold.
Then the relation (3.5) reduces to equation (3.3).
If we take,

(3.7) $$q^i = (1/2h)\rho_i = -(1/2)\xi^i_\alpha \rho^\alpha_i,$$

then we obtain

(3.8) $$q^i = -(1/2h)\rho^i,$$

(3.9) $$v^i = p^i + \xi^i_\alpha\, q^\alpha$$

In this regard we have the following theorem.

Theorem 3.2:
If $R \neq 0,$ in a Kahlerian Einstein manifold then an analytic holomorphically projective transformation v^i is uniquely decomposed in the form of the relation (3.9).
The equation

(3.10) $$L_V \xi^k_i = 0$$

is equivalent to

(3.11) $$\nabla_j u_i - \nabla_i u_j = \xi^\alpha_j \left(\nabla_\alpha u_i + \nabla_i u_\alpha\right)$$

If q^i is analytic and vector Killing field, so q^i is also analytic and vector Killing. If we take $u^i = q^i$ then we get

(3.12) $$\nabla_j\, q_i = \nabla_i\, q_j,$$

Thus $\xi^i_\alpha\, q^\alpha$ is gradient analytic.
By virtue of equation (3.9), we have

(3.13) $$L_V \left\{ \begin{matrix} k \\ j \quad i \end{matrix} \right\} = -L_V \left\{ \begin{matrix} k \\ i \quad j \end{matrix} \right\}$$

Inserting equation (5-4.1) and equation (3.8) into the equation (3.13), we get

(3.14) $$\nabla_j \nabla_i \rho^k + R^k_{\alpha ji}\, \rho^\alpha = 2h\left(\rho_j\, \delta^k_i + \rho_i\, \delta^k_j - \rho_j \xi^k_i - \rho_i\, \xi^k_j\right)$$

In this regard, we have the following theorem.

Theorem 3.3:
If $R \neq 0,$ in a Kahlerian Einstein manifold then the associated vector of an analytic holomorphically projective transformation is a gradient analytic holomorphically projective transformation.
Let L is the Lie algebra consisting of analytic holomorphically projective transformation and L_1 is the Lie algebra consisting of all Killing vector and L_2 is the vector space of all analytic gradient holomorphically projective transformation. Now constract a relation between L, L_1 and L_2 in such a way that

(3.15) $$L = L_1 + L_2.$$

By virtue of equation (3.4), we have

(3.16) $$\nabla_j \nabla_i \rho_k + R_{\gamma jik} \rho^\gamma = 2h\left(g_{jk}\rho_\gamma + g_{ik}\rho_j - \xi_{jk}\rho_i - \xi_{ik}\rho_j\right)$$

If we take the alternating part of equation (3.16) with regard to i and k, we obtain

(3.17) $$R_{\gamma jik}\rho^\gamma = h\left(g_{\gamma i}g_{jk} - g_{ji}g_{\gamma k} + \xi_{\gamma i}\xi_{jk} - \xi_{ji}\xi_{\gamma k} + 2\xi_{\gamma j}\xi_{jk}\right)\rho^i.$$

Theorem 3.4:

If $R \neq 0$, in a Kahlerian Einstein manifold and the vector space consisting to all analytic gradient holomorphically projective transformation is transitive at each point, then the manifold is a space of constant holomorphic curvature.

Proof: Let us consider a Kahlerian Einstein manifold then it holds

$$P_{hji} = 0,$$

Let v^i be a non affine analytic holomorphically projective transformation and ρ_i be its associated vector then, from equation

(3.18) $$L_V P_{hji} = P^\alpha_{hji}\rho_\alpha,$$

it follows that

(3.19) $$P^\alpha_{hji}\rho_\alpha = 0.$$

If the vector space L_2 is transitive at each point of the manifold, then we have

(3.20) $$P^k_{hji} = 0.$$

This shows that the manifold has constant holomorphic curvature.

If the tensor field H_{hj} is defined in such a way

$$H_{ji} = R_{ji},$$

Then the Kahlerian space of scalar curvature turns into a space of constant curvature.

In this regard, we have the following theorem:

Theorem 3.5:

If the tensor $H_{hj}(x)$ is independent of y^i, then the Kahlerian Einstein manifold of scalar curvature turns into a Kahlerian manifold of constant curvature.

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