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Some Non-Extendable Diophantine Triples Involving Centered Square Numbers

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***Abstract*—**In this paper, we accomplish three non-extendable Diophantine triples of the form {a, b, 2(a+b-Square of the difference of the ranks)} involving centered square numbers with property D(-Square of the difference of the ranks).

***Keywords*—** *Diophantine Triples, Centered Square Number, Non-extendability.*

I. INTRODUCTION

Numerous mathematicians considered the problem of the existence of Diophantine quadruples with the property D(n) for any integer n and furthermore for any direct polynomial in n [1-5]. In this unique situation, one may allude for a comprehensive review of various problems on Diophantine triples [6-10]. In [11], some non-extendable triples were analyzed. These outcomes spurred us to search for non-extendable Diophantine triples with components spoken to by centered square numbers.

In this paper, we exhibit three non-extendable Diophantine triples of the form {a, b, 2(a+b-Square of the difference of the ranks)}involving centered square numbers with property D(-Square of the difference of the ranks).

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be a Diophantine triple with property $D(n)$ if $a_i * a_j + n$ is a perfect square for all $1 \le i < j \le 3$, where n may be non-zero integer or polynomial with integer coefficients.

III. METHOD OF ANALYSIS

Section A:

Construction of Diophantine triples for centered square numbers of rank n and $n+1$:

Let $a = \mathrm{CS}_n$ and $b = \mathrm{CS}_{n+1}$ be Centered Square numbers of rank n and $n+1$ respectively such that $ab + (-\text{Square of the difference of the ranks}) = ab - 1$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac - 1 = \beta^2 \quad (1)$$

$$bc - 1 = \gamma^2 \quad (2)$$

Eliminating 'c' from (1) and (2), we obtain

$$(2n^2 + 2n + 1)\beta^2 - (2n^2 - 2n + 1)\gamma^2 = -4n \quad (3)$$

Using the linear transformations

$$\beta = x + (2n^2 - 2n + 1)y \ \ \& \gamma = x + (2n^2 + 2n + 1)y \quad (4)$$

in (3), it leads to the pell's equation

$$x^2 = (4n^4 + 1)y^2 - 1 \quad (5)$$

Let $y_0 = 1$ & $x_0 = 2n^2$ be the initial solution of (4)

Thus (4) yields $\beta_0 = 4n^2 - 2n + 1$

And using (1), we get $c = 8n^2 + 2 = 2(\mathrm{CS}_n + \mathrm{CS}_{n+1} - 1)$

Hence, $\{a, b, c\} = \{\mathrm{CS}_n, \mathrm{CS}_{n+1}, 2(\mathrm{CS}_n + \mathrm{CS}_{n+1} - 1)\}$ is a Diophantine triple.

Some numerical examples are given below in the following table1.

Table 1

n	Diophantine Triples	$D(-1)$
1	(1, 5, 10)	-1
2	(5, 13, 34)	-1
3	(13, 25, 74)	-1

Hence, $\{a,b,c\}=\{CS_n, CS_{n+1}, 2(CS_n + CS_{n+1} - \text{Square of the difference of the ranks})\}$ is a Diophantine triple with property $D(-\text{Square of the difference of the ranks})$.

Non-Extendability:
We show that the above triple cannot be extended to quadruple.
Let 'd' be any non-zero integer such that

$$ad-1=p^2 \quad (6)$$

$$bd-1=q^2 \quad (7)$$

$$cd-1=r^2 \quad (8)$$

Eliminating 'd' from (7) and (8), we obtain

$$(8n^2+2)q^2-(2n^2+2n+1)r^2=-6n^2+2n-1 \quad (9)$$

Using the linear transformations

$$q=x+(2n^2+2n+1)y \;\; \& \; r=x+(8n^2+2)y \quad (10)$$

in (9), it leads to the pell's equation

$$x^2=(16n^4+16n^3+12n^2+4n+2)y^2-1 \quad (11)$$

Let $y_0=1$ & $x_0=4n^2+2n+1$ be the initial solution of (11).

Thus (10) yields $q_0=6n^2+4n+2$

And using (7), we get $d=18n^2+6n+5$

Verify Quadruple:
Substituting the above value of 'd' in L.H.S of (6), we have

$ad-1=36n^4-24n^3+16n^2-4n+4$

Note that the R.H.S is not a perfect square.
Hence the triple, $\{a,b,c\}=\{CS_n, CS_{n+1}, 2(CS_n + CS_{n+1} - \text{Square of the difference of the ranks})\}$ cannot be extended to a quadruple with property $D(-\text{Square of the difference of the ranks})$.

Section B:
Construction of Diophantine triples for centered square numbers of rank n and $n+2$:

Let $a=CS_n$ and $b=CS_{n+2}$ be centered square numbers of rank n and $n+2$ respectively such that $ab+(-\text{Square of the difference of the ranks})=ab-4$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac-4=\beta^2 \quad (12)$$

$$bc-4=\gamma^2 \quad (13)$$

Applying the procedure as mentioned in section A, we have

$$c=8n^2+8n+4=2(CS_n+CS_{n+2}-4)$$

Hence, $\{a,b,c\}=\{CS_n, CS_{n+2}, 2(CS_n+CS_{n+2}-4)\}$ is a Diophantine triple with property $D(-4)$.

Some numerical examples are given below in the following table 2.

Table 2

n	Diophantine Triples	$D(-4)$
1	(1, 13, 20)	-4
2	(5, 25, 52)	-4
3	(13, 41, 100)	-4

Hence, $\{a,b,c\}=\{CS_n, CS_{n+2}, 2(CS_n + CS_{n+2} - \text{Square of the difference of the ranks})\}$ is a Diophantine triple with property $D(-\text{Square of the difference of the ranks})$.

Non-Extendability:
We show that the above triple cannot be extended to quadruple.

Let 'd' be any non-zero integer such that

$$ad-4=p^2 \quad (14)$$

$$bd-4=q^2 \quad (15)$$

$$cd-4=r^2 \quad (16)$$

Proceeding as in Section A, we get $d=18n^2+30n+17$

Verify Quadruple:
Substituting the above value of 'd' in L.H.S of (14), we have

$ad-4=$ not a perfect square.

Hence the triple, $\{a,b,c\}=\{CS_n, CS_{n+2}, 2(CS_n + CS_{n+2} - \text{Square of the difference of the ranks})\}$ cannot be extended to a quadruple with property $D(-\text{Square of the difference of the ranks})$.

Section C:
Construction of Special dio 3-tuples for centered square numbers of rank n and $n+3$:

Let $a=CS_n$ and $b=CS_{n+3}$ be centered square numbers of rank n and $n+3$ respectively such that $ab+(-\text{Square of the difference of the ranks})=ab-9$ is a perfect square say α^2.

Proceeding as in earlier cases,

$$c=8n^2+16n+10=2(CS_n+CS_{n+3}-9)$$

Hence, $\{a,b,c\}=\{CS_n, CS_{n+3}, 2(CS_n+CS_{n+3}-9)\}$ is a Diophantine triple with property $D(-9)$.

Some numerical examples are given below in the following table 3.

Table 3

n	Diophantine Triples	$D(-9)$
1	(1, 25, 34)	-9
2	(5, 41, 74)	-9
3	(13, 61, 130)	-9

Hence,
$\{a,b,c\} = \{CS_n, CS_{n+3}, 2(CS_n + CS_{n+3} - \text{Square of the difference of the ranks})\}$
is a Diophantine triple with property
$D(-\text{Square of the difference of the ranks})$.

Non-Extendability:
We show that the above triple cannot be extended to quadruple.

Let 'd' be any non-zero integer such that

$$ad - 9 = p^2 \quad (17)$$

$$bd - 9 = q^2 \quad (18)$$

$$cd - 9 = r^2 \quad (19)$$

Proceeding as in earlier cases, we get
$d = 18n^2 + 54n + 45$

Verify Quadruple:

Substituting the above value of 'd' in L.H.S of (17), we have

$ad - 9$ is not a perfect square.

Hence the triple,
$\{a,b,c\} = \{CS_n, CS_{n+3}, 2(CS_n + CS_{n+3} - \text{Square of the difference of the ranks})\}$
cannot be extended to a quadruple with property $D(-\text{Square of the difference of the ranks})$.

IV. REMARKABLE NOTE

In general, {a, b, 2(a+b-Square of the difference of the ranks)} is a non-extendable Diophantine triple for centered square numbers with property D(-Square of the difference of the ranks).

V. CONCLUSION

In this paper, we have presented Diophantine triples for centered square numbers of different ranks with the property D(-Square of the difference of the ranks). To conclude one may search for Diophantine triples for other numbers with their corresponding properties.

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