International Journal of Scientific Research in Mathematical and Statistical Sciences **Research Paper**
Volume-5, Issue-5, pp.104-107, October (2018) **E-ISSN:** 2348-4519

Discriminatiing Between Degum Distribution and Burr-III Distribution

B. Srinivasa Rao[1], P. Sricharani[2*]

[1]Department of Mathematics and Humanities, R.V.R & J.C College of Engineering, Guntur, India
[2]Department of Basic science, Shri Vishnu Engineering College for Women(Autonomous), Bhimavaram, India

[*]*Corresponding author: charani.ravi657@gmail.com*

Available online at: www.isroset.org

Received: 12/Sept/2018, Accepted: 03/Oct/2018, Online: 31/Oct/2018

Abstract - Test statistics based on Likelihood function , population Quantiles are suggested to discriminate between Dagum distribution and Burr-III distribution. Because of the non tractability of their exact sampling distributions, the percentiles of the proposed test statistics are tabulated with the help of simulated sampling distributions of the test statistics . The power of the test statistics are also tabulated and a comparative study w.r.t the powers for a given sample and the level of significance are worked out.

Key words and phrases- Dagum distribution(DD), Burr-III distribution(BD), Likelihood function , Quantiles , power of the test.

I. INTRODUCTION

Having experienced with the lack of analytical expression for the classical maximum likelihood estimation (MLE) of parameters in Dagum distribution, we propose to study whether any other standard model be an alternative to Dagum distribution with a reasonably admissible risk. Accordingly, we have chosen Burr type III model to test whether it can be an alternative to Dagum distribution. This aspect is viewed as the problem of discriminating between Dagum and Burr III models where Dagum is null population. We have considered the principle of Likelihood ratio criterion in a practically usable way. Hence we may call our procedure as likelihood ratio type procedure. The distinction is- in the likelihood procedure to the classical MLE is used for both null and alternative populations. In the likelihood ratio type procedure we use any admissible estimators of concerned models as given by [1,2, 3].

The Probability density function (pdf) of Dagum distribution is given by

$$f(x,b,a,p)=\frac{ap}{x}\left(\frac{\left(\frac{x}{b}\right)^{ap}}{\left(\left(\frac{x}{b}\right)^{a}+1\right)^{p+1}}\right), x>0, a>0, b>0, p>0. \tag{1.1}$$

Its cumulative distribution function (cdf) is

$$F(x,b,a,p)=\left(1+\left(\frac{x}{b}\right)^{-a}\right)^{-p}, x>0, a>0, b>0, p>0. \tag{1.2}$$

The likelihood function of Dagum distribution is

$$L(x,a,p,b)=\frac{(ap)^{n}\left(\prod_{i=1}^{n}x_i^{(ap-1)}\right)}{(b)^{nap}\left(\prod_{i=1}^{n}\left(1+\left(\frac{x_i}{b}\right)^{a}\right)^{p+1}\right)} \tag{1.3}$$

The Dagum distribution is a skewed, uni-modal distribution on the positive real line.

The probability density function (pdf) of Burr-III distribution is given by

$$f(x;c,k)=ck\frac{x^{-(c+1)}}{\left(1+x^{-c}\right)^{k+1}} \tag{1.4}$$

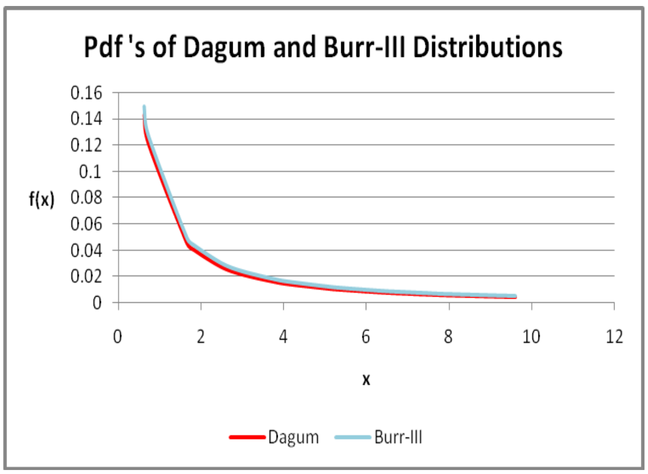
The cumulative distribution function(cdf) is

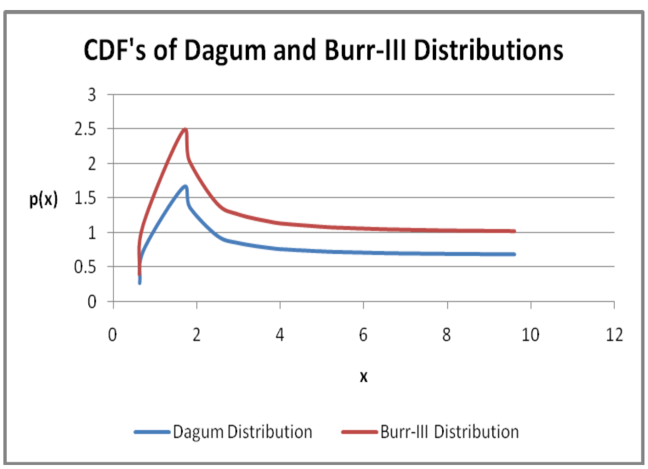
$$F(x;c,k)=\left(1+x^{-c}\right)^{-k} \tag{1.5}$$

The likelihood function of Burr-III distribution is

$$L(x,c,k)=(ck)^{n}\left(\prod_{i=1}^{n}\left(x_i^{-(c+1)}\right)\left(1+(x_i)^{-c}\right)^{-(k+1)}\right) \tag{1.6}$$

Many people have studied the importance of these two distributions as life testing models. The related works include [4&5]. In fact inferential properties of Dagum distribution are simple, efficient and mathematically tractable, where as it is not the same case with Burr-III distribution. We are therefore motivated to study whether Dagum distribution is a reasonable alternative to Bur-III distribution at least for the sake of adopting the analytical, powerful inferential characteristics of the data following Burr-III distribution. With this backdrop we suggested two different test statistics to discriminate Dagum distribution(with a=0.5,p=0.5 and b=2/3) and Burr-III distribution(with c=0.5 and k=0.5). The pdfs and cdfs plotted for the mentioned values of both the distributions are given below





The basic distribution characteristics of Dagum distribution(DD) and its Properties are Presented in Section I. The rest of the paper is organized as follows. The proposed test statistics based on ratio of likelihood functions along with their percentiles and power values are explained in section II and Population quantiles explained in section III along with their percentiles and power values. Summary and Conclusions are given in section IV.

II .TEST STATISTIC BASED ON LIKELIHOOD RATIO

Let us consider Dagum distribution as the null population say (P_0), the Bur-III distribution is considered as the alternative population say (P_1). If a random sample of $x_1, x_2, .. x_n$ is drawn from P_0 evaluate the estimation of the parameters of P_0 using the given sample of $x_1, x_2, .. x_n$.

Let $\hat{P}_0$ denote the value of the likelihood function of the sample $x_1, x_2, .. x_n$ with respect to null population , at the estimates of its parameters using $x_1, x_2, .. x_n$. Let $\hat{P}_1$ denote the value of likelihood function w.r.t to alternative population , the estimates of P_1 using the sample $x_1, x_2, .. x_n$ The ratio $\frac{L_1}{L_0}$ in a way represents the ratio of the likelihood of P_1 to that of P_0 with a sample drawn from P_0. Therefore $\frac{P_1}{P_0}$ is the ratio of a smallest probability to a larger probability and hence is expected to be small. The null hypothesis "H0: The sample is drawn from P_0 " can be tested using the percentiles of the likelihood ratio $\frac{L_1}{L_0}$.

Therefore statistics $T_1 = \frac{L_1}{L_0}$ can be taken as test statistics to test the above null hypothesis.

Since the distribution of $\frac{L_1}{L_0}$ is not analytical, we have computed the percentiles of empirical sampling distributions of $T_1 = \frac{L_1}{L_0}$ with the help of 10,000 simulation runs of sample size n=2,..,10 are given in table 1.

Table1: Percentiles of Sampling Distribution Of $T_1 = \frac{L_1}{L_0}$

n	**0.99865**	**0.99**	**0.975**	**0.95**	**0.05**	**0.025**	**0.01**	***0.0050***	***0.00135***
2	1.4956	1.4785	1.4538	1.3936	0.8368	0.8267	0.8204	0.8185	0.8171
3	1.6160	1.4503	1.3655	1.2965	0.7835	0.7645	0.7522	0.7465	0.7411
4	1.7305	1.5369	1.1317	1.3438	0.7479	0.7202	0.7002	0.6887	0.6790
5	1.8640	1.6267	1.5076	1.3992	0.7179	0.6866	0.6602	0.6453	0.6259
6	1.9104	1.6754	1.5384	1.4644	0.6974	0.6595	0.6239	0.6067	0.5914
7	2.1733	1.7277	1.6005	1.4276	0.6734	0.6342	0.5956	0.5748	0.5453
8	2.3034	1.8447	1.6608	1.5067	0.6490	0.6072	0.5682	0.5427	0.5157
9	2.2789	1.8422	1.6568	1.5151	0.6340	0.5892	0.5473	0.5224	0.4849
10	2.3018	1.9082	1.7136	1.5569	0.6150	0.5699	0.5216	0.4970	0.4584

The power of the test statistic T_1 is also tabulate for 3 different levels of significance(1%, 2.5% &5%) at the sample sizes n=2,3 … 10.by simulating sampling from P_1 and using the values of T_1. The count of T_1 values that fall beyond the table values of table – 1. These are given in the Table –2.

Table 2: Power of T_1

n	**0.9900**	**0.9750**	**0.9500**
2	9997	9987	9919
3	9846	9650	9328
4	9831	9627	9300
5	9885	9638	9282
6	9822	9752	9240
7	9789	9528	9142
8	9826	9610	9245
9	9788	9488	9107
10	9771	9528	9100

These tables indicate that even with the help of a small sample of size as small as 2 the power remains to be at more than 99%. It is therefore concluded that the T1 statistic proposed in this section cannot discriminate between the null and alternative population with a high power values.

III. TEST STATISTICS BASED ON QUANTILES

Let $x_1, x_2, \ldots x_n$ denote sample from DD. The correlation type goodness of fit test as given by [6&7] in this case using percentiles can be formed as

$$T_2 = \frac{\sum_{i=1}^{n} X_i v_i}{\sqrt{\left(\sum_{i=1}^{n} X_i^2 \sum_{i=1}^{n} v_i^2\right)}} \tag{3.1}$$

This statistic represent the correlation between x_i & v_i i=1,2…n where v_i is the i^{th} quantile of the null population. The statistic T_2 is simulated through Monte Carlo method based on 10,000 simulations. .Table-3 represents the percentiles of T_2 for sample sizes n=2,3…10 and various levels of significance

Table3: Percentiles Of T2 Based On Quantile

n	**0.99865**	**0.9900**	**0.9750**	**0.9500**	**0.0500**	**0.0250**	**0.01**	**0.005**	**0.00135**
2	1	0.9999	0.9999	0.9999	0.0244	0.0244	0.0244	0.0244	0.0244
3	0.9999	0.9998	0.9992	0.9984	0.0032	0.0027	0.0026	0.0026	0.0026
4	0.9998	0.9991	0.9972	0.9952	0.0017	0.0007	0.0005	0.0005	0.0005
5	0.9996	0.9969	0.9936	0.9923	0.0020	0.0005	0.0002	0.0001	0.0002
6	0.9989	0.9937	0.9901	0.9896	0.0012	0.0007	0.0001	0.0001	0.0001
7	0.9987	0.9931	0.9883	0.9874	0.0025	0.0012	0.0005	0.0003	0.0001
8	0.9968	0.9874	0.9856	0.9839	0.0010	0.0003	0.0001	0.0001	0.0001
9	0.9961	0.9855	0.9838	0.9808	0.0007	0.0002	0.0001	0.0001	0.00001
10	0.9951	0.9831	0.9823	0.9713	0.0007	0.0002	0.0001	0.0001	0.00001

As we can see from table -3, the percentile points of T_2 increase as the smple size increases as well as the significance level increases. The power of the test statistic T_1 is also tabulate for 3 different levels of significance (1%,2.5%,5%) at the sample sizes n=2,3 … 10.by simulating sampling from P_1 and using the values of T_2. The count of T_2 values that fall beyond the table values of table – 3.These are given in Table-4.

Table-4 **Power of T_2**

n	**0.99**	**0.975**	**0.95**
2	9901	9754	9500
3	9978	9877	9699
4	9980	9892	9774
5	9985	9897	9781
6	9972	9869	9723
7	9990	9926	9799
8	9987	9897	9742
9	9985	9902	9754
10	9991	9958	9938

IV. COMPARATIVE STUDY&CONCLUSION

The powers given in table 2 & 4 for the test statistics T_1 and T_2 respectively indicate that the two statistics do not discriminate between Dagum & Burr-III distributions significantly. i.e there is much similarity between these two distributions for larger values of n. If the sample size increases the Dagum distribution and Burr-3 distribution cannot be distinguished with respect to proposed test statistics. We may say that for a data from Dagum distribution and the application of Dagum distribution is quite complicated, the methodology of Burr-III distribution can be used in large samples.

ACKNOWLEDGMENT

The authors thank the editor and the reviewer for their helpful suggestions, comments and encouragement, which helped in improving the final version of the paper.

REFERENCES

[1] Arabin.K.D. and Kundu.D., "Discriminating between the log normal and log logistic distributions",Communications in Statistics Theory and Methods,Vol 39, pp.280-290, 2010.

[2] Chambers.E.A. and Cox.D.R "Discriminating Between alternative binary response models", Biometrika, Vol.54, pp 573-578, 1967.

[3] Dumonceaux.R. and Antle.C. E., "Discriminating between the log-normal and weibull distributions", Technometrics, Vol.15, pp 923-926, 1973.

[4] Gupta.D.R., Kundu.D., "Discriminating between gamma and generalized exponential distributions" , Journal of Statistical computation and Simulation, Vol.74,No.2,pp 107–121,2004.

[5] Gupta.D.R., Kundu.D.,Manglic.A., Discriminating between log normal and generalized exponential distributions , Journal of Statistical planning and Inference, Vol.127: pp 213–227, 2005.

[6] Rao.B.S. and Kantam.R.R.L., "Discriminating between generalized exponential distribution and some life test models based on population quantiles", Journal of Modern Applied StatisticalMethods, Vol.12 No.2, pp.336-343, 2013.

[7] Rao.B.S. and Kantam.R.R.L., "Discriminating between log-logistic and Rayleigh distributions", Pakistan Journal of Statistics and Operations Research, Vol. 10,No.1, pp. 1-7, 2014.

AUTHORS PROFILE

B.Srinivasa Rao did his M.Sc in Statistics from Acharya Nagarjuna University in 1996. He completed his doctoral research in the area of Software Reliability and Quality control methods from Acharya Nagarjuna University in 2010. Currently he is working as an Associate Professor of Statistics at R.V.R & J.C College of Engineering, Guntur ,putting an experience of 22 years in teaching. He published more than 35 research articles in reputed national and international Journals. His areas of specialization are Reliability, Quality control methods and Statistical inference. His teaching interests include Bio-Statistics, Quantitative techniques, Probability, Inferential Statistics and Stochastic process.

P.Sricharani is currently working as Assistant Professor in department of Basic science from Shri Vishnu Engineering college for women, Bhimavaram. She guided for 4 P.G Projects. she has 13 years of teaching experience. Her research interest is in the area of Statistical Quality Control and Statistical Inference. She has published many research papers in National and International journals.