International Journal of Scientific Research in Mathematical and Statistical Sciences
Research Paper
Vol.6, Issue.1, pp.131-137, February (2019)
E-ISSN: 2348-4519
DOI: https://doi.org/10.26438/ijsrmss/v6i1.131137

Comparison of Process Capability Indices under AR (2) process

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Available online at: www.isroset.org

Received: 08/Feb/2019, Accepted: 20/Feb/2019, Online: 28/Feb/2019

Abstract— Statistical Process Control (SPC) tools are applicable in all fields. Process Capability Analysis (PCA) is one of the essential SPC tools. In Process Capability, it measures the ability of an in-control process to produce the desired product. Process Capability Indices (PCIs) are defined to measure the capability of the process. Along with in control process PCIs also assumes that the process characteristic is Normally distributed and the observations on characteristic are independent. The assumption of independent observations has violated in many industrial processes. The present paper focuses on the effect of this violation of independence which is also known as autocorrelation effect. ARMA models are appropriate for autocorrelated processes.

Keywords— *Autocorrelated process, Estimation, Process Capability Indices (PCIs), Statistical Process Control.*

I. INTRODUCTION

Process Capability Analysis (PCA) is an essential Statistical Process Control (SPC) tool for assessing the ability of the process. PCA is conducted under the assumption that the process is under statistical control, observations are independent, and process characteristic follows Normal distribution. Many times the assumption of independence is violated due to some circumstances it may, due to an industrial process where data exhibits some degree of autocorrelation. There is voluminous literature available on process capability indices [1]. Shore [2] First attempted the concept of Process Capability Indices (PCIs) under autocorrelated data, he described some of the undesirable effects that autocorrelations may have on the sampling distribution of estimates of the mean and standard deviation, and thus on the PCIs calculated using it via Monte Carlo simulation. He suggested two approaches for studying the PCIs under autocorrelation. He also showed that as the degree of autocorrelation increases bias in the PCIs increases. The variance of the estimators of C_p and C_{pk} under autocorrelated processes has been found [3] and for also C_{pm} and C_{pmk} [4]. From the study of [3] and [4], it uses the autocorrelated process of order one AR (1), it has found that for the autocorrelated process the PCIs are biased and that bias decreases as n increases.

Similarly, the bias in the indices increases as $|\phi|$(degree of autocorrelation) increases. In many manufacturing industries, the processes are with the high inertia causing values of output characteristic to be interrelated; this is known as a characteristic variable is autocorrelated. Most often the ARMA models were used for analyzing these types of processes. AR (1) and AR (2) are common. The objective of this paper is to study the effect of autocorrelation on different indices and their estimate of variances. The autoregressive process of order two AR (2) is used to generate the data. Extensive simulation work has done for analyzing the effect of the AR(2) process on the estimators of PCIs and their variances.

In this regard the rest of the paper is organized as follows, Section II contains definitions of basic PCIs. Section III contains the basic terminology about stationary Gaussian processes. In section IV, expressions for variances of the estimators of the PCIs C_p, C_{pk}, $C_{pm,}$ and C_{pmk} are given. Section V presents a study of the effect of autocorrelation on PCIs C_p, C_{pk}, C_{pm} and variances of estimators of C_p, C_{pk}, C_{pm} and C_{pmk} and also on expected values. Section V also contains standard error of the sample mean and sample standard deviation. In section VI conclusions are given.

II. PROCESS CAPABILITY INDICES

The Process Capability Indices C_p, C_{pk}, C_{pm} and C_{pmk} are the most widely used among several PCIs for assessing the capability of the process defined in (i) –(iv).

$$(i) \quad C_p = \frac{USL - LSL}{6\sigma}$$

$$(ii) \quad C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = \frac{a - |\mu - b|}{3\sigma} = \frac{d - |2\mu - m|}{6\sigma}$$

$$(iii) \quad C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{C_p}{\sqrt{1+\xi^2}}$$

where T is the target value and $\xi = \frac{\mu - T}{\sigma}$

$$(iv) \quad C_{pmk} = \min\left\{\frac{USL-\mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\} = \frac{C_{pk}}{\sqrt{1+\xi^2}}. = \frac{a - |\mu - b|}{3\sqrt{\sigma^2 + (\mu - T)^2}}$$

where USL and LSL are upper and lower specification limits respectively. μ is the process mean, σ is the process standard deviation. Further, the quantities

$$a = \frac{USL - LSL}{2}, \; b = \frac{USL + LSL}{2},$$

$$d = USL - LSL, \quad m = USL + LSL$$

III. STATIONARY GAUSSIAN PROCESS

Let $\{X_t\}$ is a process such that $Var(X_t) < \infty$ for each $t \in W \subset R$, where R is the set of real numbers, then the autocovariance function $\gamma_x(.,.)$ of $\{X_t\}$ is given as

$$\gamma_x(r,s) = Cov(X_r, X_s) = E[(X_r - EX_r)(X_s - EX_s)] \quad r,s \in W$$

The time series $\{X_t\ t \in W\}$ is said to be stationary if $E|X_t^2| < \infty,\ t \in Z,\ \ E(X_t) = m,\ t \in Z$ and $\gamma_x(r,s) = \gamma_x(r+t, s+t),\ \ r,s,t \in Z$; therefore $\mu_x(t)$ is independent of t, and $\gamma_x(t+h,t)$ is independent of t for each $h \in Z$. If $\{X_t\}$ is a stationary process, the autocovariance function (ACVF) of $\{X_t\}$ is given by

$$\gamma_x(h) = Cov(X_{t+h}, X_t)$$

and the autocorrelation function (ACF) is given by [5]

$$\rho_x(h) = \frac{\gamma_x(h)}{\gamma_x(0)} = Cor(X_{t+h}, X_t).$$

$\{X_t\}$ is a Gaussian process if all of its joint distributions are multivariate normal. A process is said to be stationary Gaussian if it is stationary and Gaussian simultaneously. Let $\{X_1, X_2, .. X_n\}$ be a random sample of size n from a stationary Gaussian process $\{X_t\}$. Let $\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n}$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ be the sample mean and sample variances respectively. The expected values and variances of $\overline{X}$, S^2 and S are given [3]:

$$E(\overline{X}) = \mu_x \ , \ V(\overline{X}) = \frac{\sigma_x^2}{n} g(n,\rho_i)$$

$$E(S^2) = \sigma_x^2 f(n,\rho_i)$$

$$V(S^2) = \frac{2\sigma_x^4}{(n-1)^2} F(n,\rho_i)$$

$$E(S) \approx \left[E(S^2)\right]^{1/2} = \sigma_x\left[f(n,\rho_i)\right]^{1/2}$$

and

$$Var(S) \approx \frac{Var(S^2)}{4E(S^2)} = \left[\frac{\frac{2\sigma_x^4}{(n-1)^2}F(n,\rho_i)}{4\sigma_x^4 f(n,\rho_i)}\right] = \sigma_x^4 \frac{F(n,\rho_i)}{2(n-1)^2 f(n,\rho_i)}$$

where $\rho_i = \rho_X(i)$ for i= 1,2,...n is the autocorrelation of X at lag i ,

$$f(n,\rho_i) = 1 - \frac{2}{n(n-1)}\sum_{i=1}^{n-1}(n-i)\rho_i$$

$$F(n,\rho_i) = n + 2\sum_{i=1}^{n-1}(n-i)\rho_i^2 + \frac{1}{n}\left[n + 2\sum_{i=1}^{n-1}(n-i)\rho_i\right]^2 - \frac{2}{n}\sum_{i=0}^{n-1}\sum_{j=0}^{n-i}(n-i-j)\rho_i\rho_j$$

$$g(n,\rho_i) = 1 + \frac{2}{n}\sum_{i=1}^{n-1}(n-i)\rho_i .$$

IV. VARIANCE OF ESTIMATORS OF PROCESS CAPABILITY INDICES

Let $\{X_1, X_2, X_3, \ldots X_n\}$ be a sample of size n from a stationary Gaussian process $\{X_t\}$. The usual estimators of C_p, C_{pk}, C_{pm}, and C_{pmk} are:

$$\hat{C}_p = \frac{USL - LSL}{6S}$$

$$\hat{C}_{pk} = \min\left\{\frac{USL - \overline{X}}{3S}, \frac{\overline{X} - LSL}{3S}\right\}$$

$$= \frac{a - |\overline{X} - b|}{3S} = \frac{d - |2\overline{X} - m|}{6S}$$

$$\hat{C}_{pm} = \frac{USL - LSL}{6\sqrt{S^2 + (\overline{X} - T)^2}}$$

$$= \frac{C_p}{\sqrt{1 + \hat{\xi}^2}} \quad where \quad \hat{\xi} = \frac{\overline{X} - T}{S},$$

and

$$\hat{C}_{pmk} = \min\left\{\frac{USL - \overline{X}}{3\sqrt{S^2 + (\overline{X} - T)^2}}, \frac{\overline{X} - LSL}{3\sqrt{S^2 + (\overline{X} - T)^2}}\right\}$$

$$= \frac{C_{pk}}{\sqrt{1 + \hat{\xi}^2}} = \frac{a - |\overline{X} - b|}{3\sqrt{S^2 + (\overline{X} - T)^2}}$$

the approximations for the variances of $\hat{C}_p$ and $\hat{C}_{pk}$ has been found [3]:

$$Var(\hat{C}_p) \approx C_p^2 \frac{F(n, \rho_i)}{2(n-1)^2 f^3(n, \rho_i)}$$

and

$$Var(\hat{C}_{pk}) \approx \frac{C_{pk}^2}{f(n, \rho_i)}\left[\frac{g(n, \rho_i)}{9nC_{pk}^2} + \frac{F(n, \rho_i)}{2(n-1)^2 f^2(n, \rho_i)}\right]$$

and also for the variances of $\hat{C}_{pm}$ and $\hat{C}_{pmk}$ [4]:

$$Var(\hat{C}_{pm}) \approx C_p^2 \left[\frac{\frac{2F(n, \rho_i)}{(n-1)^2} + \frac{4g(n, \rho_i)\xi^2}{n}}{4\left[f(n, \rho_i) + \xi^2\right]^3}\right]$$

and

$$Var(\hat{C}_{pmk}) \approx C_{pk}^2 \left[\frac{1}{f(n, \rho_i) + \xi^2}\right] \times \left\{\frac{F(n, \rho_i)}{2(n-1)^2\left[f(n, \rho_i) + \xi^2\right]^2} + \frac{g(n, \rho_i)}{9n}\left[\frac{1}{C_{pk}} + \frac{6\xi}{2\left[f(n, \rho_i) + \xi^2\right]}\right]^2\right\}$$

where, $\xi = \frac{\mu - T}{\sigma}$. If $\mu = T$ then,

$$Var(\hat{C}_{pm}) = C_p^2\left[\frac{F(n, \rho_i)}{2(n-1)^2 f^3(n, \rho_i)}\right]$$ which equals the variance of $\hat{C}_p$, and

$$Var(\hat{C}_{pmk}) \approx \frac{C_{pk}^2}{f(n, \rho_i)}\left[\frac{F(n, \rho_i)}{2(n-1)^2 f^2(n, \rho_i)} + \frac{4\sigma^2 g(n, \rho_i)}{n}\left[\frac{1}{a - |2\mu - b|}\right]^2\right]$$

$$= \frac{C_{pk}^2}{f(n, \rho_i)}\left[\frac{F(n, \rho_i)}{2(n-1)^2 f^2(n, \rho_i)} + \frac{g(n, \rho_i)}{9nC_{pk}^2}\right]$$

which equals the variance of $\hat{C}_{pk}$.

V. EFFECTS OF AUTOCORRELATION ON PROCESS CAPABILITY INDICES PCIs

EFFECTS OF AUTOCORRELATION ON SAMPLE MEAN AND SAMPLE STANDARD DEVIATION

Consider the example [2]; where a quality characteristic is normally distributed with mean 40 and standard deviation 7. The specification level are USL = 61 and LSL = 19. Different target values are considered: T = 40, 41, 42, 43, 44, 45. We then compare the two processes. A process with independent observations and a process with observations following an AR (2) model, $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$ where $\{\varepsilon_t\}$ is a series of uncorrelated errors, $\varepsilon_t \sim$ N (0, σ^2) and σ = 7. In Table 1 for each process, the mean, the standard deviation and the capability indices C_p, C_{pk}, C_{pm}, and C_{pmk} are calculated. The values of C_{pk} and C_{pmk} are not shown here because if the process is targeted at its mean then, $C_p = C_{pk}$ and $C_{pm} = C_{pmk}$. The second order autoregressive process is stationary if the parameters ϕ_1 and ϕ_2 are such that $\phi_1 + \phi_2 < 1$, $\phi_1 - \phi_2 < 1$, $|\phi_2| < 1$, so we considered ϕ_1 = 0.01, 0.26, 0.51 and ϕ_2 = 0.05, 0.1, 0.15 which satisfy the above constraints.

we have for an AR (2) process $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$

$$Var(X_t) = \frac{(1 - \phi_2)\,\sigma^2}{(1 + \phi_2)(1 - \phi_1 - \phi_2)(1 + \phi_1 - \phi_2)},$$

$$\rho(0) = 1, \; \rho(1) = \frac{\phi_1}{1 - \phi_2},$$

$$\rho(s) = \phi_1 \rho(s-1) + \phi_2 \rho(s-2) \qquad s \geq 3$$

where, σ^2 is white noise variance and ρ_i is autocorrelation function.

Table 1: Mean, Standard deviation (STD), C_p and C_{pm} of a process not autocorrelated vs. a process following an AR (2) model. *d= μ -T

		Mean	STD	C_p
No Autocorrelation		40	7	1
ϕ_1 =0.01	ϕ_2=0.05	40	7.182	0.975
	ϕ_2=0.1	40	7.379	0.949
	ϕ_2=0.15	40	7.593	0.922
ϕ_1 =0.26	ϕ_2=0.05	40	7.467	0.937
	ϕ_2=0.1	40	7.707	0.908
	ϕ_2=0.15	40	7.975	0.878
ϕ_1 =0.26	ϕ_2=0.05	40	8.512	0.822
	ϕ_2=0.1	40	8.955	0.782
	ϕ_2=0.15	40	9.491	0.738

		C_{pm}					
*d= μ -T →		0	1	2	3	4	5
No Autocorrelation		1	0.99	0.962	0.919	0.868	0.814
ϕ_1 =0.01	ϕ_2=0.05	0.975	0.965	0.939	0.899	0.851	0.8
	ϕ_2=0.1	0.949	0.94	0.916	0.879	0.834	0.785
	ϕ_2=0.15	0.922	0.914	0.891	0.857	0.816	0.77
ϕ_1 =0.26	ϕ_2=0.05	0.937	0.929	0.906	0.87	0.826	0.779
	ϕ_2=0.1	0.908	0.901	0.879	0.846	0.806	0.762
	ϕ_2=0.15	0.878	0.871	0.851	0.822	0.785	0.744
ϕ_1 =0.26	ϕ_2=0.05	0.822	0.817	0.801	0.776	0.744	0.709
	ϕ_2=0.1	0.782	0.777	0.763	0.741	0.714	0.682
	ϕ_2=0.15	0.738	0.734	0.722	0.703	0.68	0.653

We observe in Table 1 that higher the autocorrelation levels lower the capability index value. Through simulation study the effect of autocorrelation on the expected value of the sample mean and standard error studied. We generated 10000 samples from a no autocorrelated process and 10000 samples from AR (2) process for each of the following cases: n = 15, 50, 100, 200; ϕ_1 = 0.01, 0.26, 0.51 and ϕ_2 = 0.05, 0.1, 0.15. Table 2 shows results for no autocorrelated process and Table 3 shows results for the autocorrelated process.

Table 3 shows that the autocorrelation does not affect the expected value of the sample mean; while a different situation occurs with the expected value of the standard deviation. For example, for n = 15 and (ϕ_1, ϕ_2) = (0.01, 0.1) the estimated expected value of the standard deviation is 6.77 in autocorrelated process, for (ϕ_1, ϕ_2) = (0.26, 0.1) is 6.92 and (ϕ_1, ϕ_2)=(0.51,0.1) is 7.46. For independent observations, the value is 6.84. As n increases, the estimated expected value of the standard deviation increases slightly, for autocorrelated data. For example, for (ϕ_1, ϕ_2) = (0.51, 0.15) the estimated expected values for n = 15, 50, 100 are 7.42, 8.4, 8.49 respectively.

Table 2: Expected values and standard error of the sample mean and sample standard deviation for no autocorrelated process.

n	Mean		Std Deviation	
	Average	Std Error	Average	Std Error
15	40.01	1.81	6.84	1.31
50	39.95	1.02	6.95	0.69
100	40	0.7	6.97	0.49
200	40	0.5	7	0.35

Table 3: Expected values and standard error of the sample mean and sample standard deviation for autocorrelated process following AR (2).

			Mean			Std		
	n	ϕ_1 ↓ ϕ_2 →	0.05	0.1	0.15	0.05	0.1	0.15
Ave Rage	15	0.01	39.97	39.88	39.98	6.82	6.77	6.9
		0.26	40.1	40.15	40.03	6.94	6.92	7
		0.51	39.79	39.94	40.2	7.47	7.46	7.42
	50	0.01	40.03	40.01	40	6.95	6.97	7.03
		0.26	40.02	39.98	40.02	7.17	7.22	7.27
		0.51	40.13	40.06	40.01	8.03	8.2	8.4
	100	0.01	40.01	39.94	40	6.97	7.03	7.04
		0.26	40.02	40.05	40.03	7.22	7.29	7.36
		0.51	40	39.95	40.04	8.15	8.38	8.59
Std Error	15	0.01	1.9	2.02	2.12	1.33	1.25	1.31
		0.26	2.67	2.69	2.9	1.41	1.38	1.45
		0.51	4	4.28	4.84	1.81	1.77	1.87
	50	0.01	1.03	1.11	1.17	0.7	0.71	0.7
		0.26	1.38	1.59	1.68	0.78	0.82	0.83
		0.51	2.11	2.44	2.82	1.11	1.22	1.3
	100	0.01	0.73	0.81	0.84	0.51	0.51	0.49
		0.26	1.01	1.09	1.16	0.57	0.58	0.6
		0.51	1.55	1.79	2.01	0.79	0.88	0.92

EFFECTS OF AUTOCORRELATION ON PCIS C_P, C_{PK} AND C_{PM}

Through a simulation study, we analyze the effect of the autocorrelated process of order two AR (2) on capability indices estimators. Comparing the estimated expected values of the capability indices estimators shown in table 4, 5 and 6 with the theoretical values in table 1, it has observed that for autocorrelated processes the estimators are biased, bias that decreases as n increases. Also bias in the index increases as the degree of autocorrelation increases. For example, for (ϕ_1, ϕ_2) = (0.51, 0.15) and n = 15, 50 and 100 the expected value of $\hat{C}_p$ are 1, 0.86, 0.82 while the true value is 0.738. For (ϕ_1, ϕ_2) = (0.26, 0.1) and n = 15, 50 and 100 the expected value of $\hat{C}_{pk}$ are 0.94, 0.93, 0.93 while the true value is 0.91. For n = 50 and (ϕ_1, ϕ_2) = (0.26, 0.15) the expected values of $\hat{C}_{pm}$ are 0.95, 0.94, 0.88, 0.84, 0.82 and 0.8 when μ –T = 0, 1, 2, 3, 4 and 5 respectively while the true values are 0.87, 0.87, 0.85, 0.82, 0.78, 0.74.

Table 4: Expected values and standard error of the capability index C_p, C_{pk} and C_{pm} for no autocorrelated processes.

				C_{pm}					
	n	C_p	C_{pk}	*d= μ -T					
				0	1	2	3	4	5
Average	15	1.2	1.06	1.02	1.02	0.99	0.94	0.89	0.83
	50	1.21	1.02	1.01	1	0.97	0.93	0.87	0.82
	100	1.20	1.01	1	0.99	0.97	0.92	0.87	0.82
Std Error	15	0.28	0.25	0.21	0.2	0.19	0.19	0.17	0.15
	50	0.12	0.12	0.1	0.1	0.1	0.1	0.08	0.08
	100	0.08	0.08	0.07	0.07	0.07	0.06	0.06	0.05

Table 5: Expected values and standard error of the capability index C_p, C_{pk} for autocorrelated processes.

			C_p			C_{pk}		
	n	ϕ_1↓ ϕ_2→	0.05	0.1	0.15	0.05	0.1	0.15
Average	15	0.01	1.06	1.07	1.07	0.98	0.99	0.99
		0.26	1.05	1.05	1.05	0.95	0.94	0.93
		0.51	1	1.01	1	0.85	0.85	0.83
	50	0.01	1.01	1.02	1.01	0.97	0.97	0.97
		0.26	0.98	0.98	0.97	0.93	0.93	0.91
		0.51	0.88	0.88	0.86	0.8	0.8	0.77
	100	0.01	1.01	1	1	0.98	0.97	0.97
		0.26	0.97	0.97	0.96	0.93	0.93	0.91
		0.51	0.86	0.84	0.82	0.81	0.79	0.76
Std Error	15	0.01	0.21	0.23	0.23	0.21	0.22	0.22
		0.26	0.23	0.22	0.23	0.22	0.21	0.22
		0.51	0.25	0.27	0.27	0.24	0.25	0.28
	50	0.01	0.11	0.11	0.11	0.11	0.11	0.11
		0.26	0.1	0.11	0.12	0.11	0.12	0.12
		0.51	0.12	0.12	0.13	0.13	0.13	0.14
	100	0.01	0.07	0.07	0.07	0.07	0.07	0.07
		0.26	0.07	0.08	0.08	0.08	0.08	0.08
		0.51	0.09	0.08	0.09	0.09	0.09	0.1

Table 6a: Expected values and standard error of the capability index C_{pm} for autocorrelated process.

			Mean					
n	ϕ_1	ϕ_2	*d= μ -T					
			0	1	2	3	4	5
15	0.01	0.05	1.02	1.01	1	0.94	0.89	0.84
		0.1	1.02	1.01	0.98	0.95	0.89	0.83
		0.15	1.01	1	0.97	0.95	0.88	0.84
	0.26	0.05	1	0.98	0.96	0.93	0.87	0.83
		0.1	0.98	0.96	0.97	0.91	0.87	0.83
		0.15	0.97	0.97	0.95	0.92	0.86	0.82
	0.51	0.05	0.89	0.89	0.87	0.85	0.81	0.78
		0.1	0.89	0.87	0.86	0.84	0.81	0.78
		0.15	0.85	0.86	0.83	0.83	0.81	0.77
50	0.01	0.05	1	1	0.96	0.92	0.87	0.82
		0.1	1	0.99	0.96	0.92	0.87	0.82
		0.15	0.99	0.98	0.96	0.92	0.87	0.81
	0.26	0.05	0.97	0.96	0.93	0.9	0.85	0.8
		0.1	0.96	0.95	0.93	0.89	0.85	0.8
		0.15	0.95	0.94	0.92	0.88	0.84	0.8
	0.51	0.05	0.86	0.85	0.83	0.81	0.78	0.73
		0.1	0.83	0.83	0.82	0.8	0.77	0.72
		0.15	0.82	0.81	0.79	0.77	0.75	0.72
100	0.01	0.05	1	0.99	0.96	0.92	0.87	0.82
		0.1	1	0.99	0.96	0.92	0.87	0.81
		0.15	0.99	0.99	0.96	0.92	0.86	0.81
	0.26	0.05	0.97	0.96	0.93	0.89	0.85	0.8
		0.1	0.96	0.95	0.93	0.88	0.84	0.79
		0.15	0.94	0.94	0.91	0.88	0.84	0.79
	0.51	0.05	0.85	0.84	0.83	0.8	0.78	0.73
		0.1	0.83	0.82	0.81	0.79	0.75	0.72
		0.15	0.8	0.8	0.78	0.76	0.73	0.7

Table 6b: Standard error of the capability index C_{pm} for autocorrelated process.

			Standard Error					
n	ϕ_1	ϕ_2	*d= μ -T					
			0	1	2	3	4	5
15	0.01	0.05	0.2	0.21	0.21	0.19	0.18	0.16
		0.1	0.2	0.2	0.2	0.2	0.18	0.16
		0.15	0.2	0.2	0.19	0.19	0.18	0.17
	0.26	0.05	0.21	0.21	0.21	0.21	0.19	0.18
		0.1	0.21	0.2	0.22	0.2	0.19	0.19
		0.15	0.21	0.2	0.2	0.21	0.2	0.2
	0.51	0.05	0.22	0.21	0.22	0.22	0.21	0.21
		0.1	0.24	0.21	0.21	0.23	0.22	0.22
		0.15	0.23	0.23	0.23	0.24	0.24	0.24
50	0.01	0.05	0.1	0.11	0.1	0.09	0.09	0.08
		0.1	0.11	0.1	0.1	0.1	0.09	0.08
		0.15	0.1	0.1	0.1	0.1	0.09	0.09
	0.26	0.05	0.11	0.1	0.1	0.1	0.1	0.09
		0.1	0.11	0.11	0.11	0.11	0.1	0.1
		0.15	0.11	0.11	0.11	0.11	0.11	0.11
	0.51	0.05	0.12	0.12	0.11	0.12	0.11	0.11
		0.1	0.12	0.12	0.12	0.12	0.13	0.12
		0.15	0.12	0.13	0.13	0.12	0.13	0.13
100	0.01	0.05	0.07	0.07	0.07	0.07	0.06	0.06
		0.1	0.07	0.07	0.07	0.07	0.06	0.06
		0.15	0.07	0.07	0.07	0.07	0.06	0.06
	0.26	0.05	0.08	0.08	0.07	0.07	0.07	0.07
		0.1	0.07	0.07	0.07	0.07	0.07	0.07
		0.15	0.08	0.08	0.08	0.08	0.08	0.08
	0.51	0.05	0.09	0.08	0.08	0.08	0.08	0.08
		0.1	0.09	0.09	0.08	0.09	0.08	0.08
		0.15	0.09	0.09	0.09	0.09	0.09	0.09

EFFECT OF AUTOCORRELATION ON VARIANCE OF ESTIMATORS OF C_P, C_{PK}, C_{PM}, C_{PMK} UNDER AUTOREGRESSIVE PROCESS OF ORDER TWO AR (2)

To compare the variances of estimators of C_p, C_{pk}, C_{pm}, C_{pmk} a simulation study was carried out for a second-order stationary autoregressive process with parameter ϕ_1 and ϕ_2 . For C_p=1.33, ϕ_1 = 0.01, 0.26, 0.51, ϕ_2 = 0.05, 0.1, 0.15, and *n* = 10, 20 ... 200. Figure 1 shows fixing C_p, ϕ_2 and *n*, as ϕ_1 increases $\hat{\sigma}_{Cp}$. For fixed values of ϕ_2 the variance is larger for n <100. Partial results have shown in Figure 1. Similar results are also obtained for $\hat{\sigma}_{Cpk}$, $\hat{\sigma}_{Cpm}$, $\hat{\sigma}_{Cpmk}$.

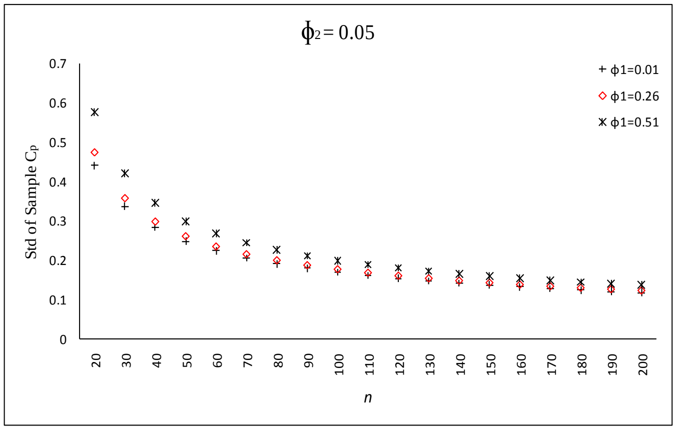


Figure 1. $\hat{\sigma}_{Cp}$ as function of the sample size with C_p= 1.33, ϕ_1 = 0.01, 0.26, 0.51, and ϕ_2 = 0.05.

Similar results are obtained for ϕ_2 = 0.1, 0.15.

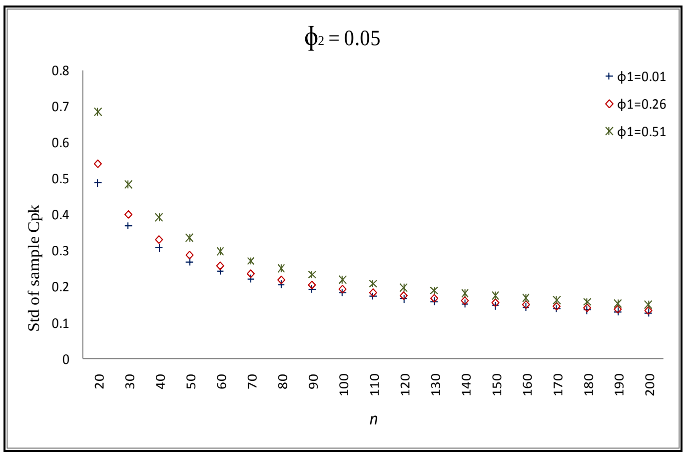
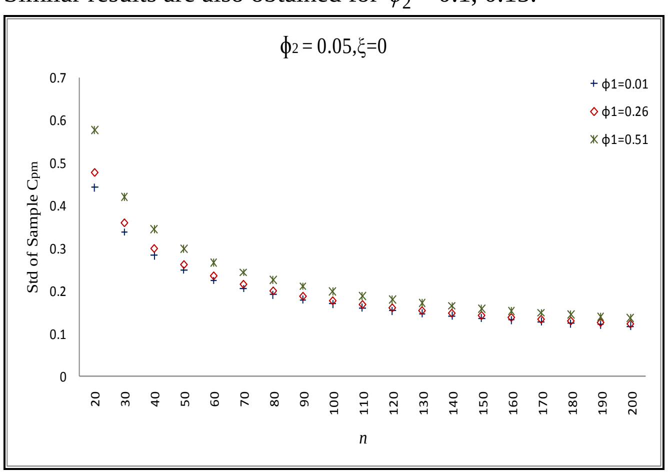
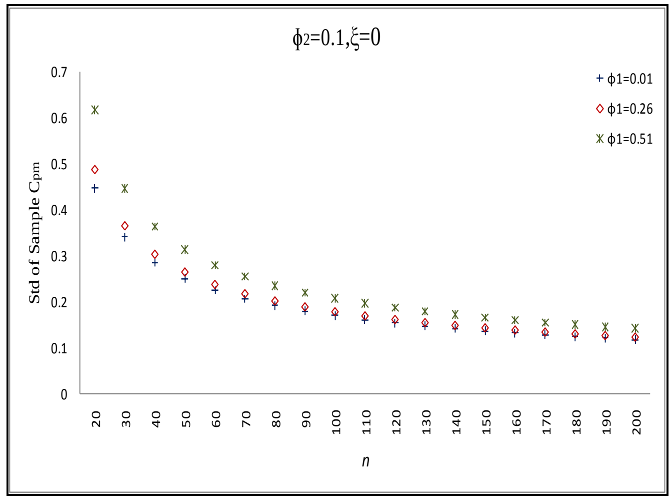


Figure 2. $\hat{\sigma}_{Cpk}$ as function of the sample size with C_p= 1.33, ϕ_1 = 0.01, 0.26, 0.51, and ϕ_2 = 0.05.

Similar results are also obtained for ϕ_2 = 0.1, 0.15.





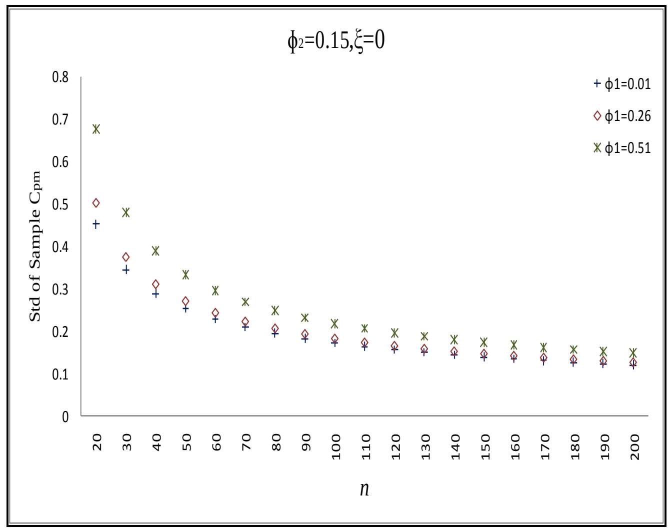
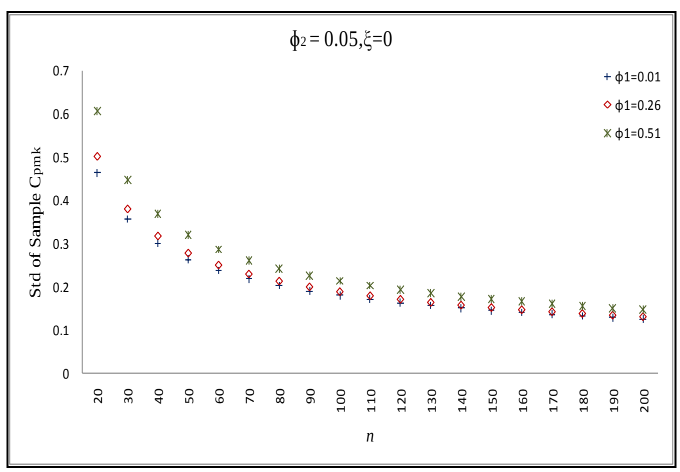
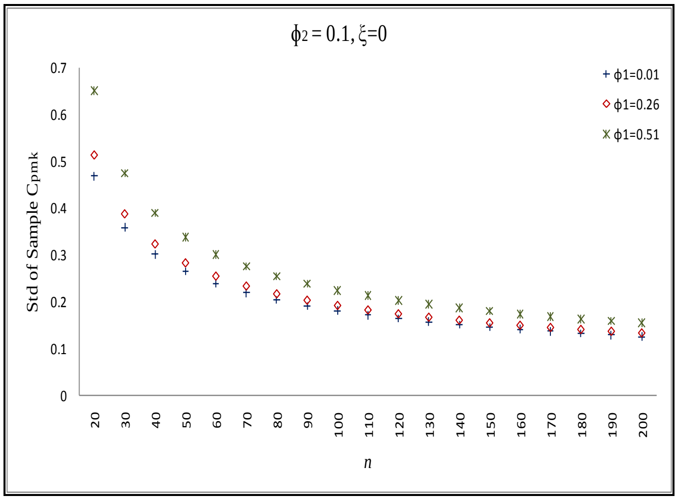


Figure 3. $\hat{\sigma}_{Cpm}$ as function of the sample size with C_p= 1.33, ϕ_1 = 0.01, 0.26, 0.51, and ϕ_2 = 0.05, 0.1, 0.15, ξ=0

Similar results are obtained for ξ= 5, 10.





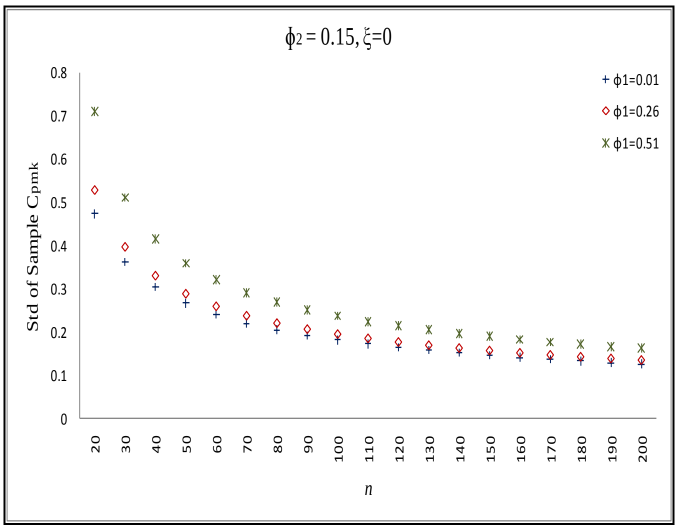


Figure 4. $\hat{\sigma}_{Cpmk}$ as function of the sample size with C_p= 1.33, ϕ_1 = 0.01, 0.26, 0.51, and ϕ_2 = 0.05, 0.1, 0.15, ξ=0

Similar results are obtained for ξ= 5, 10.

VI. Conclusions

We observed the effect of autocorrelation and found that it does not affect the expected value of the sample mean but affects the estimated expected value of the standard error that increases slightly for autocorrelated data.

Through a simulation study, we observed that the higher the autocorrelation level lower the capability index value. We also observed that for autocorrelated processes the estimators are biased, bias increases as the degree of autocorrelation increases and also it decreases as *n* increases. The variances of the estimators of C_{pm} and C_{pmk} have studied through a simulation study. It has found that for sample size *n*<100 the variances of the estimators decreases rapidly for all parameter combinations of AR (2) process. This situation happens in all the cases of estimators.

In the future scope of this study, one can find the remedial measures to remove the effect due to autocorrelation on the estimators of PCIs.

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