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Special Type Double Sampling Plans with Minimum Variance

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Abstract— Acceptance sampling plans are usually defined by the Lot size, sample sizes at stage one and stage two along with acceptance numbers at stage one and two. The design of DSP are based on the AOQL, LTPD by without assuming the Outgoing Quality (OQ) and Total Inspection (TI) as random variables. In this paper, OQ and TI are treated random variables and distributional properties are obtained. Based on these properties, a Special Type double sampling plan with minimum variance, treating the outgoing quality and total inspection as random variables under total rectification, is introduced and are illustrated with the practicability of the plan.

Keywords— *Special Type Double Sampling plans; Outgoing Quality; Total Inspection*

I. INTRODUCTION

Acceptance sampling is the method which deals with procedures to take a decision whether a lot is to accept or reject based on the *n* random sample of the product drawn from the lot. It protects the consumer and producer from receiving and distributing a product with low quality. The main objective of the acceptance sampling is to choose the best sampling plan which will have high discriminating power, the OC curve of the plan is very close to the ideal OC cure. The OC curve is in turn fixed by suitably chosen parameters. Usually the two points on the OC curve namely (AQL,α) and (LQL,β) are prescribed, where AQL is acceptable quality level, α is producer risk, LQL is limiting quality level and β is consumer risk.

Hall (1979) has given a new approach to rectifying inspection plans by introducing Outgoing Quality (OQ) and Total Inspection (TI) as random variables. By considering the standard deviations of these random variables, the extremes in lot to lot quality can be revealed.

In this paper, a Special Type Double Sampling Plan with Minimum Variance (STDSPMV) is introduced, following Hall's procedure, under total rectification. The following are the organisation of the paper. In Section 2, Special Type Double sampling plan is defined. In Section 3, distributional properties of OQ and TI are obtained. Special Type Double Sampling Plan with Minimum Variance is derived in Section 4. In Section 5, the procedure for determination of STDSPMV is explained and is illustrated with an example in Section 6.

II. SPECIAL TYPE DOUBLE SAMPLING PLANS

A Special Type Double Sampling Plan (STDSP) can be found in the literature and it can be more useful if one wants to fix the OC curve with an acceptance number either as Zero or One, Hahn (1974) and Kaviayarasu and Devika (2017). Though this plan is not applicable frequently, it can be used for costly or destructive testing as the acceptance numbers are very low. The STDSP is defined by

(i) A random sample of n_1 items are selected from the lot and the number of defectives in the sample, d_1, is observed.
(ii) If $d_1 \geq 1$, the lot is rejected.
(iii) If $d_1 = 0$, a second random sample of size n_2, items are drawn from the lot and number of defectives in the second sample, d_2, is observed.
(iv) If the combined number of observed defectives $d_1 + d_2 \leq 1$ the lot is accepted, otherwise the lot is rejected.

III. DISTRIBUTIONAL PROPERTIES OF OQ AND TI

Consider a production process under statistical control has 100p percent defectives in the long run. Lots of sizes N are made up of this product have varying number of defectives characterized by the density function.

$$p(X;N)=\binom{N}{X}p^{X}\,(1-p)^{N-X}\ ;\quad X = 0, 1,\ldots, N$$

The output of this process is said to be of quality p. Assume lots of size N of this product to be submitted to a double sampling inspection plan defined by (N, n_1, n_2, c_1, c_2). The OC curve of this process is characterized by Type B, situation of Dodge and Romig (1959).

From each lot of size N, form two portions, one consisting of sampled portion and the other unsampled portion. Unsampled portion of a lot is defined by (N-n_1) when a decision is reached at first stage or (N-n_1– n_2) when a decision is reached at the end of second stage.

We define the following

N : Lot size

n_i : Sample size at stage i

c_i : Acceptance number at stage i

r_i : Rejection number at stage i

x : Number of defective in the sample

p_{ai} : Probability of acceptance of a lot at stage i

p_{ri} : Probability of rejection of a lot at stage i

M : A constant, whose value is strictly less than one

S_{N-n_1} : Number of defectives in the unsampled portion of a lot before rectification and after a decision is reached at the end of first stage

$S_{N-n_1-n_2}$: Number of defectives in the unsampled portion of a lot before rectification and after a decision is reached at the end of second stage

b (x; n, p) : The probability distribution of x based on sample size n from a product under control with process average p.

p_{a1} and p_{a2} are defined by

$$p_{a_1}=\sum_{x=0}^{c_1}\frac{e^{-z}z^{x}}{x!}\ ;\qquad p_{a_2}=\sum_{k=c_1+1}^{c_2}\left[\frac{e^{-z}z^{k}}{k!}\sum_{x=0}^{c_2-k}\frac{e^{-z}z^{x}}{x!}\right]$$

And $p_a = p_{a_1}+p_{a_2};\ p_r = p_{r_1}+p_{r_2};\ p_a + p_r = 1$

Following Venkatesan (2006), define a random variable X_1 as

X_1 = 0 if lot is rejected at stage 1

= 1 if go to second sample

When X_1 = 1, define another random variable X_2 as

$$X_2=\begin{cases}0:\text{lot is rejected at stage 2}\\ 1:\text{lot is accepted at stage 2}\end{cases}$$

The following probability models are used

$$b(X;\ n,\ p)=\binom{n}{x}p^{x}\left(1-p\right)^{n-x}\ ;\quad x = 0, 1, 2, \ldots, n \tag{1}$$

$$P\left(S_{N-n}=j\right)=p^{j}\left(1-p\right)^{N-n-j}\ ;\quad n = n_1;\ j = 0, 1, 2, \ldots, (N-n_1)$$

$$= n_1 + n_2\ ;\ j = 0, 1, \ldots, (N- n_1- n_2)$$

Then,

$$P\left(X_1=j\right)=\begin{cases}p_{r_1}:j=0\\ p_k:j=1\end{cases}$$

Where,

$$p_k = b(k; n_1, p); \quad k = 1,2,\ldots,c_2$$

and $$P(X_2 = i) = \begin{cases} p_{r_2} : i = 0 \\ p_{a_2} : i = 1 \end{cases}$$

The above models follow the assumption that the lot of size N contains x defective items where the conditional distribution of the number of defective items in the samples is given by

$$P(x \mid X) = \frac{\binom{X}{x}\binom{N-X}{n-x}}{\binom{N}{n}}; \qquad X = 0, 1, 2, \min(n, x)$$

$$P(X; N) = \binom{N}{X} p^X (1-p)^{N-X}; \; X = 0, 1, \ldots, N \tag{2}$$

Generally, in a double sampling plan, one assumes that $r_1 = r_2 = c_2 + 1$ with $c_1 < c_2$ and $n_2 = kn_1$ $(k = 1,2)$; Duncan (1986).

Define the random variable Outgoing Quality (OQ), as the ratio of the number of defectives in the unsampled portion of the lot after inspection and rectification to the total number of items in the lot. Assuming screening to be conducted without error, symbolically one gets,

$$OQ = \frac{1}{N}\left[\left(1 - \frac{X_1(X_1-1)}{2}\right) S_{N-n_1} X_1 + \frac{X_1(X_1-1)}{2} S_{N-n_1-n_2} X_2\right]; \qquad X_1 = 0,1\; ;\; X_2 = 0,1 \tag{3}$$

Directly determining the probability distribution of OQ from equation (3) is very difficult as it involves the sums, differences and products of the following independent and dependent random variables X_1, $X_2(X_1-1)/2$, S_{N-n_1} and $S_{N-n_1-n_2}$.

Hence, to determine the probability distribution of OQ, all possible probability values of OQ were enumerated and combined. This leads to compact form, after rearrangement, which is given below

$$P(OQ = j \mid N) = \begin{cases} p_{a_1} b(j; N-n_1, p) + p_{a_2} b(j; N-n_1-n_2, p) + p_r : j = 0 \\ p_{a_1} b(j; N-n_1, p) + p_{a_2} b(j; N-n_1-n_2, p) : j = 1,2,\ldots,(N-n_1-n_2) \\ p_{a_1} b(j; N-n_1, p) : j = (N-n_1-n_2)+1,\ldots,(N-n_1) \end{cases}$$

(4)

Thus

$$E(OQ) = AOQ = (N-n_1) p_{a_1} + (N-n_1-n_2) p_{a_2} \left(p/N\right) \tag{5}$$

and $$Var\left(OQ\right)=\frac{1}{N^2}\left[\begin{array}{l} p_{a_1}\left\{\left(N-n_1\right)p\left(1-p\right)+\left(N-n_1\right)^2 p^2\right\} \\ +p_{a_2}\left\{\left(N-n_1-n_2\right)p\left(1-p\right)+\left(N-n_1-n_2\right)^2 p^2\right\}\end{array}\right]-\left(AOQ\right)^2 \tag{6}$$

The random variable total inspection (TI), is defined as

$$TI=\left[1-\frac{X_1\left(X_1-1\right)}{2}\right]\left[n_1X_1+N\left(1-X_1\right)\right]+\left[\frac{X_1\left(X_1-1\right)}{2}\right]\left[\left(n_1+n_2\right)X_2+N\left(1-X_2\right)\right];\ X_1=0,1;\ \ X_2=0,1 \tag{7}$$

The probability distribution of TI is defined as

$$P\left(TI=i\right)=\begin{cases} p_{a_1}:i=n_1 \\ p_{a_2}:i=n_1+n_2 \\ 1-p_a:i=N \end{cases} \tag{8}$$

Thus, $$E\left(TI\right) = ATI = n_1p_{a_1} + \left(n_1+n_2\right)p_{a_2} + N\left(1-p_a\right) \tag{9}$$

$$Var\left(TI\right)=\left(N-n_1\right)^2 p_{a_1}+\left(N-n_1-n_2\right)^2 p_{a_2}-\left(N\left(AOQ\right)/p\right)^2 \tag{10}$$

IV. SPECIAL TYPE DOUBLE SAMPLING PLANS WITH MINIMUM VARIANCE (STDSPMV)

In this section special type double sampling plans employing total rectification that minimizes the variance of the outgoing quality at some level of the process average are developed.
One seeks a double sampling plan which gives

$$\underset{\left(N,n_1,n_2,c_1,c_2\right)}{Min} Var_0\left(OQ\right)p \tag{11}$$

Subject to $$\left[\left(N-n_1\right)p_{a_{10}}+\left(N-n_1-n_2\right)p_{a_{20}}\right]=AOQ_0 \tag{12}$$

and $$p_{a_0}\leq M,\ M<1 \tag{13}$$

Where AOQ_0 is the designated value of the AOQ at the value of p_0 Var_0 (OQ) and p_{a0} are the values of Var (OQ) and p_a, each calculated for the value p_0.

The plan based on M=1 is quite an ideal plan as it would allow optimum plans to be plans for which p_{a0} = 1, unless p_0 = 0. Under such plan, sampling would be meaningless. For this reason the constraint M<1 is necessary. Since n_1, n_2, c_1, and c_2 are discrete quantities no exact solution to equation (12) may exist. For this moment, assume that equality in (12) is always attainable. Substituting the constraint equation (12) in (6) and simplifying, gives

$$Var(OQ)=\left[{}^{(1-p_0)}\!\big/_{N-AOQ_0}\right]+\left({}^{p_0}\!/_{N}\right)^2\left[\left(N-n_1\right)^2 p_{a_{10}}+\left(N-n_1-n_2\right)^2 p_{a_{20}}\right] \tag{14}$$

This equation indicates that subsequent to the selection of N, p_0 and AOQ_0, the Var (OQ) depends on the sample size n_1 and n_2.

There are usually many sampling plans which satisfy the constraints (12) and (13), of those, clearly the plan with the largest n_1 and n_2 is the plan which minimizes Var_0 (OQ). The plans which satisfy the constraints (12) and (13) are called the admissible plans. The admissible plan which has the largest n_1 and n_2 is called the MVDSP, denoted as (N, n_1' n_2', c_1', c_2'). It follows that

$$Min\,Var_0\left(OQ\right)=\left[\tfrac{1-p_0}{N}-AOQ_0\right]+\left(\tfrac{p_0}{N}\right)\left[\left(N-n_1^{'}-n_2^{'}\right)^2 p_{a_{20}}\right] \tag{15}$$

Solving (12) for n_1 on the assumption that $n_2 = kn_1$, one arrives at the relation

$$n_1=\frac{N\left[1-\frac{AOQ_0}{p_0 p_{a_0}}\right]}{\left[1+\frac{k\times p_{a_{20}}}{p_{a_0}}\right]} \tag{16}$$

Obviously, the largest value of n_1 from (16) occurs when $p_{a0} = M$, although this value of n_1 may not be attainable, it serves as upper bound for $n_1^{'}$, hence

$$n_1^{'}\le\frac{N\left[1-\frac{AOQ_0}{P_0 M}\right]}{\left(1+kM\right)} \tag{17}$$

Substituting the right hand side of (17) for $n_1^{'}$ and $n_2^{'}$ in (15) gives the attainable lower bound for the Min Var_0 (OQ) and if M = 1 in (17) and then substituting in (15) gives the unattainable lower bound for the Min Var_0 (OQ).

Let us now consider the random variable TI. Average total inspection calculated under the constrains (12) and (13) denoted as ATI_0, is given by

$$ATI_0=N\left(1-{}^{AOQ_0}\!\big/_{p_0}\right)$$

Since ATI_0 is free of n_1 and n_2, all admissible plans are equally economical at the value of P_0. This is a critical result as it implies total freedom from economic consideration while searching the set of admissible plans for optimality.
For an admissible sampling plan, (10) calculated at p_0 becomes

$$Var_0\left(TI\right)=\left(N-n_1\right)^2 p_{a_{10}}+\left(N-n_1-n_2\right)^2 p_{a_{20}}-\left({}^{N\times AOQ}\!\big/_{p_0}\right)^2 \tag{18}$$

It is apparent that $n_1^{'}$ and $n_2^{'}$ will also minimize Var_0 (TI), hence

$$Min\,Var_0\left(TI\right)=\left(N-n_1^{'}\right)^2 p_{a_{10}}+\left(N-n_1^{'}-n_2^{'}\right)^2 p_{a_{20}}-\left({}^{N\times AOQ_0}\!\big/_{p_0}\right)^2 \tag{19}$$

Hence, the values of n_1 and n_2 which minimize Var_0 (OQ) also minimize Var_0 (TI). Substituting the right hand side of (17) for $n_1^{'}$ and $n_2^{'}$ in (19) gives the lower bound to the minimum variance total inspection.

V. DETERMINING THE STDSPMV

Obviously, only integer values of n_1, n_2, c_1 and c_2 can produce meaningful double sampling plans. Therefore, the constraint (12) may not be satisfied exactly. Integer values for n_1, n_2, c_1 and c_2 which minimize the variance and approximately satisfying the constraints (12) and (13) are obtained. Due to the excellence of the Poisson model in approximating the Binomial distribution especially in the range of p values of interest, $p \le 0.10$, substituting $n_1p_0 = z_0$ the constraints (12) and (13) can be written as

$$N\left(AOQ_0\right)=\left(Np_0-z_0\right)\sum_{x=0}^{c_1}\frac{e^{-z_0}z_0^x}{x!}+\left[Np_0-z_0\left(1+k\right)\right]\sum_{k=c_1+1}^{c_2}\left[\frac{e^{-z_0}z_0^x}{k!}\left(\sum_{x=0}^{c_2-k}\frac{e^{-z_0}z_0^x}{x!}\right)\right] \tag{20}$$

And $$\sum_{x=0}^{c_1}\frac{e^{-z_0}z_0^x}{x!}+\sum_{k=c_1+1}^{c_2}\left[\frac{e^{-z_0}z_0^k}{k!}\left(\sum_{x=0}^{c_1}\frac{e^{-z_0}z_0^x}{x!}\right)\right]\le M \tag{21}$$

The value of z_0 which satisfies (20) for a plan (N, n_1, n_2, c_1, c_2) is denoted as z*. All triputs (z*, c_1, c_2) satisfying (20) and (21) define the set of admissible double sampling plans (N, n_1, n_2, c_1, c_2) where $n_1=\langle z^*/p\rangle$, n_2= kn_1; k = 1, 2, and $\langle a\rangle$ is the largest integer greater than or equal to a.

The admissible sampling plan with largest n_1 and n_2 is the MVDSP. When solution of n_1 happens to be a fraction and if n_1 is taken as the next highest integer, the constraint (12) becomes an approximation. However, for all practical purposes the constraint (12) may be regarded as satisfied exactly.

To determine a MVDSP for a designated value of N, p_0, AOQ_0 and M, solve (20) for n_1 for a fixed value of c_1 with successive values of c_2 with $c_1 < c_2$. For each solution (N,n_1,n_2,c_1,c_2) calculate p_{a0}. All the plans for which $p_{a0} \le M$ are admissible plans. Then select the plan which has the largest value of n_1 for which $p_{ao} \le M$. The plan so selected will produce the minimum standard deviation of OQ and minimum standard deviation of TI at the designated value of the process average.

VI. EXAMPLE

The following example illustrates the practicability of determination of MVDSP. Consider $N = 1000$, $p_0 = 0.02$, $AOQ = 0.015$ and $n_2 = n_1$ one gets several plans, for different values $c_1 = 0$ and c_2 with $c_1 < c_2$ and M. The values of p_{a1} and p_{a2} are obtained by referring to cumulative Poisson table (1962). The plans, thus obtained are furnished below.

Min Var plan with N=1000	
(M,c_1, c_2, n_1)	SD (OQ)
(0.8,0,1,84)	0.00659
(0.85,0,1,102)	0.00645
(0.90,0,1,121)	0.00634
(0.95,0,1,134)	0.00620
(0.99,0,1,153)	0.00622

From the above table, it is evident that the least SD (OQ) is 0.00620 and thus the required MVSTDSP is (1000, 134, 134, 0, 1) with SD (OQ) is 0.00620 for M = 0.95. By adopting the above procedure, one can design a number of sampling plans with minimum variance for different values of M for k = 1 and for k = 2.

VII. CONCLUSION

Special Type double sampling plan with minimum variance by treating the outgoing quality and total inspection as random variables under total rectification, has been developed and has been illustrated with the simulated data with special reference to the upper bound and k values.

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