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Inventory Model with Genralized Pareto Rate of Replinishment Having Time Dependent Demand

K. Srinivasaro[1], B. Punyavathi[2*]

[1,2]Department of Statistics, Andhra University, Visakhapatnam-530003, Andhra Pradesh, India

**Corresponding Author: punya.svu@gmail.com*

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Abstract - Generalized Pareto distribution gained lot of importance in income analysis and life testing experiments due to its long upper tail. The characteristics of the life time of a commodity in production processes dealing with deteriorated items match with the statistical characteristics of the Generalized Pareto distribution. Hence, in this paper we develop and analyze an economic production quantity model with generalized Pareto rate of production and deterioration. Here it is assumed that the production quantity is random and follows a Generalized Pareto distribution. It is further assumed that the life time of the commodity is random and follows a Generalized Pareto distribution. Considering that the demand rate is time dependent and follows a power pattern the instantaneous state of inventory at any given time in the cycle length under the assumptions of shortages are allowed and fully backlogged is derived. With plausible cost considerations the total production cost of the system is derived and minimized with respect to production up time, production down time and production quantity. The sensitivity analysis of the model reveals that the random production and random life time have significant influence on production. It is further observed that the deteriorating distribution and life time distribution parameters have tremendous influence on optimal operate policies of the system. This model also includes the model without shortages as a limiting case. This model is useful for analyzing the production systems dealing with deteriorating items.

Keywords- Generalized Pareto distribution, Random Production, Economic Production Quantity Model, Sensitivity Analysis, Production Scheduling, Deterioration.

I. INTRODUCTION

Recently the operation research scientists working in the area of inventory control and management have focused their attention towards Economic Production Quantity models with random production and life time of the commodities. Goyal and Giri [1], Ruxian Lie, et al. [2] and Pentico and Drake [3] have presented the literature review on inventory models dealing with both random as well as deterministic. In random life time inventory models it is customary to consider that the life of the commodity is random and follows a probability distribution. Various distributions such as exponential, gamma, weibul, pareto and Generalized Pareto Distributions are used for characterizing the life of the commodity Ghare and Schrader[4], Shah and Jaiswal [5], Cohen [6], Aggarwal [7], Dave and Shah[8],Pal[9], Kalpakam and Sapna [10], Griri and Chaudhuri [11], Tadikamalla [12], Covert and Philip [13], Philip [14], Goel and Aggarwal [15], Venkata Subbaiah, et al. [16], Nirupama Devi, et al.[17], Srinivas Rao,et al. [18], Xu and Li [19], Rong et al. [20], Srinivas Rao,et al.[21], Chakrabarthy and Chang[22] and Lin [23], Biswajit Sarkar [24]. In all these models the authors assumed that the replenishment is instantaneous.

But in Economic Production Quantity models the replenishment/production is finite. The finite rate of production in Economic Production Quantity models is studied by Mukherjee and Pal [25], Goswami [26] and Goyal and Giri [27]. The uniform rate of production in Economic Production Quantity models is studied by Panda and Chatarjee [28], Mandal [29], Sana, et al.[30]. The Economic Production Quantity models with two rates of production is studied by Perumal and Arivarignan [31]. Alternatives rates of production in Economic Production Quantity models is studied by Pal and Mandal [32] and Sen and Chakrabarthy [33], Venkata subbaiah, et al. [34]. Eassy and Srinivas Rao [35] have studied Economic Production Quantity models with stock dependent production.

Deviating from the finite or fixed rates of production, Sridevi et al. [36], Srinivasarao et al.[37], Lakshmanrao et al. [38], Srinivasarao et al. [39], Madhulatha et al. [40] have studied and analyzed Economic Production Quantity models with random production having variable rate of production. They assumed that economic production quantity is random and follows either exponential or weibul distribution. But in production processes the production quantity per a unit time follows a probability distribution which has long upper tail. That is the production rate increases with time up to certain

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time. There after the production stops due to various causes. This type of production process can be well characterized by a Generalized Pareto Distribution. Very little work has been reported in literature regarding Economic Production Quantity models with Generalized Pareto Rate of production and Generalized Pareto Rate of deterioration. Hence, in this paper we develop and analyze an Economic Production Quantity model with Generalized Pareto Rate of deterioration and production.

Using differential calculus the instantaneous state of inventory is derived under the assumptions that shortages are allowed and fully backlogged. Assuming that the demand is time dependent and the demand rate follows a power pattern demand and is of the form $\lambda(t) = \frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$ where n' is the indexed parameter, T' is the cycle length and 'd' total demand the model is analyzed . For different values of the index parameter the demand rate includes constant or increasing or decreasing rates of demand. With suitable cost considerations the total production cost is derived and minimized with respect to production down time and up time. The optimal production quantity is derived. This model is extended to the case of without shortages. In both the cases the sensitivity analysis is carried with respect to changes in input parameters and costs.

section 1, contains the introduction of Economic Production Quantity models with random production and life time of the commodities, section 2, contains the assumptions of the model, section 3, contains an inventory system in which the stock level is zero at time t=1, section 4, in this section we obtain the optimal policies of the inventory system under study, in section 5, we discuss the solution procedure of the model through a numerical illustration by obtaining the production uptime, production down time, optimum ordering quantity and the total cost of an inventory system, In section 6, the sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter at a time. In this section 7, the inventory model for deteriorating items without shortages is developed and analysed. Here, it is assumed that shortages are not allowed and the stock level is zero at time t = 0, in section 8, we obtain the optimal policies of the inventory system under study. Section 9, we discuss the numerical illustration of the model, under study and in section the discussed the conclusions.

II. ASSUMPTIONS OF THE MODEL

The following assumptions are made for developing the model:

The demand rate is a power function of time, which is $\lambda(t) = \frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}$, Where 'n' is the indexed parameter, T' is the cycle length and 'd' total demand.

(1)

The production quantity is random and follows a Generalized Pareto distribution having probability density function of the form.

$$f(t) = \frac{1}{\alpha}\left(1 - \frac{\beta t}{\alpha}\right)^{\frac{1}{\beta}-1} \quad ; \quad (\nu \neq 0); \;\; 0 < t < \frac{\alpha}{\beta}$$

Therefore, the instantaneous rate of production is

$$k(t) = \frac{f(t)}{1-F(t)} = \alpha\beta t^{\beta-1}; \alpha > 0, \beta > 0, t > 0 \qquad (2)$$

i.) Lead time is zero.
ii.) Cycle length, T is known and fixed.
iii.) Shortages are allowed and fully backlogged.
iv.) A deteriorated unit is lost.
v.) The life time of the item is random and follows a generalized Pareto distribution. Then the instantaneous rate of deterioration is

$$\lambda(t) = \frac{1}{\lambda_1 - \lambda_2 t}; 0 < t < \frac{\lambda_1}{\lambda_2} \qquad (3)$$

III. INVENTORY MODEL WITH SHORTAGES

Consider an inventory system in which the stock level is zero at time t=1. The Stock level increases during the period (0, t_1), due to excess of production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2, the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3, the production starts again and fulfils the backlog after satisfying the demand. During (t_3, T), the back orders are fulfilled and inventory level reaches zero at the end of the cycle T. The Schematic diagram representing the instantaneous state of inventory is given in Figure 1.

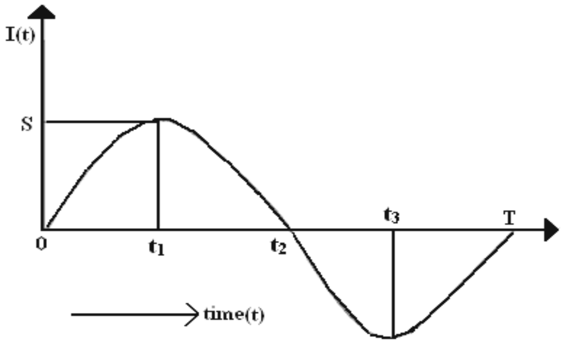

Fig 1: Schematic diagram representing the inventory level.

The differential equations governing the system in the cycle time [0, T] are

$$\frac{d}{dt}I(t)+\left(\frac{1}{\lambda_1-\lambda_2 t}\right)I(t)=\frac{1}{\alpha-\beta t}-\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\qquad 0\le t\le t_1 \tag{4}$$

$$\frac{d}{dt}I(t)+\frac{I(t)}{\lambda_1-\lambda_2 t}=-\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\quad t_1\le t\le t_2 \tag{5}$$

$$\frac{d}{dt}I(t)=-\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\qquad t_2\le t\le t_3 \tag{6}$$

$$\frac{d}{dt}I(t)=\frac{1}{\alpha-\beta t}-\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}};\quad t_3\le t\le T \tag{7}$$

Solving the equations (4) – (7) using the initial conditions, I (0) = 0, I(t_1) = S, I(t_2) = 0 and I(T) = 0, the on hand inventory at time 't' is obtained as

$$I(t)=S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\left(\frac{1}{\alpha-\beta u}-\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du;\quad 0\le t\le t_1 \tag{8}$$

$$I(t)=S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_{t_1}^{t}\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du;\quad t1\le t\le t_2 \tag{9}$$

$$I(t)=\frac{d}{T^{\frac{1}{n}}}\left(t_2^{\frac{1}{n}}-t^{\frac{1}{n}}\right);\qquad t_2\le t\le t_3 \tag{10}$$

$$I(t)=\frac{d}{T^{\frac{1}{n}}}\left(T^{\frac{1}{n}}-t^{\frac{1}{n}}\right)+\log\left(\frac{\alpha-\beta T}{\alpha-\beta t}\right)^{\frac{1}{\beta}};\qquad t_3\le t\le T \tag{11}$$

Stock loss due to deterioration in the interval (0, t) is

$$L(t)=\int_0^t k(t)dt-\int_0^t \lambda(t)dt-I(t)\,,0\le t\le t_2$$

This implies,

$$L(t)=\begin{cases}\log\left(\frac{\alpha}{\alpha-\beta t}\right)^{\frac{1}{\beta}}-d\left(\frac{t}{T}\right)^{\frac{1}{n}}-S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} \\ +(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\left(\frac{1}{\alpha-\beta u}-\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du; \quad 0\le t\le t_1 \\ \log\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}}-d\left(\frac{t}{T}\right)^{\frac{1}{n}}-S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} \\ +(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du; \; t_1\le t\le t_2\end{cases}$$

Stock loss due to deterioration in the cycle of length T is

$$L(T)=\log\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}}-\lambda(t)t_2$$

Ordering quantity Q in the cycle of length T is

$$Q=\log\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}}+\log\left(\frac{\alpha-\beta t_3}{\alpha-\beta T}\right)^{\frac{1}{\beta}} \tag{12}$$

From equation (8) and using the condition I (0) = 0, we obtain the value of ‘S’ as

$$S=(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_0^{t_1}\left(\frac{1}{\alpha-\beta u}-\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du \tag{13}$$

when t = t_3, then equations (10) and (11) become

$$I(t_3)=\frac{d}{T^{\frac{1}{n}}}\left(t_2^{\frac{1}{n}}-t_3^{\frac{1}{n}}\right)$$

$$and\ I(t_3)=\frac{d}{T^{\frac{1}{n}}}\left(T^{\frac{1}{n}}-t_3^{\frac{1}{n}}\right)+\log\left(\frac{\alpha}{\alpha-\beta t_3}\right)^{\frac{1}{\beta}}$$

Equating the equations and on simplification, one can get

$$t_2=T\left[1+\frac{1}{d}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)^{\frac{1}{\beta}}\right]^n \tag{14}$$

Let $K(t_1,t_{2,}t_3)$ be the total cost per unit time. The total cost is the sum of the set-up cost, cost of the units, the inventory holding cost and shortage cost. The total cost per unit time is

$$K(t_1,t_{2,}t_3)=\frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\left(\int_0^{t_1}I(t)dt+\int_{t_1}^{t_2}I(t)dt\right)+\frac{\pi}{T}\left(\int_{t_2}^{t_3}-I(t)dt+\int_{t_3}^{T}-I(t)dt\right) \tag{15}$$

Substituting the values of I(t) and Q in equation (12), one can obtain $K(t_1,t_{2,}t_3)$ as

$$\begin{aligned}K(t_1,t_{2,}t_3)&=\frac{A}{T}+\frac{C}{T}\log\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}}+\log\left(\frac{\alpha-\beta t_3}{\alpha-\beta T}\right)^{\frac{1}{\beta}}\\&+\frac{h}{T}\left\{\int_0^{t_1}\left[S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\right.\right.\\&\left.\int_t^{t_1}\left(\frac{1}{\alpha-\beta u}-\frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}du\right]dt\\&+\int_{t_1}^{t_2}\left[S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_{t1}^{t}\frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right.\\&\left.\left.(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}du\right]dt\right\}+\frac{\pi}{T}\left\{\int_{t_2}^{t_3}\left[\frac{d}{T^{\frac{1}{n}}}\left(t_2^{\frac{1}{n}}-t^{\frac{1}{n}}\right)+\log\left(\frac{\alpha-\beta T}{\alpha-\beta t}\right)^{\frac{1}{\beta}}\right]dt\right.\end{aligned} \tag{16}$$

Substituting the values of ‘S’ and ‘t_2’ from equations (13) and (14) in the total cost equation (16), we obtain

$$K(t_1,t_3)=\frac{A}{T}+\frac{C}{T}\frac{1}{\beta}\log\left[\frac{\alpha(\alpha-\beta t_3)}{(\alpha-\beta t_1)(\alpha-\beta T)}\right]$$

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$$+\frac{h}{T}\Bigg\{\int_0^{t_1}\left[\frac{1}{\lambda_2+1}\left[\lambda_1^{\lambda_2+1}-\left[\lambda_1-\lambda_2\,T\left(1+\frac{1}{d\beta}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)^n\right)^{\frac{1}{n_2}+1}\right]\right]\right]$$

$$\int_0^{t_1}\left[\left(\frac{1}{\alpha-\beta u}-\frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}du\right.$$

$$-\int_0^{t_1}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_t^{t_1}\left(\frac{1}{\alpha-\beta u}-\frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}du\right]\Bigg]dt$$

$$-\int_{t_1}^{t_2}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_{t_1}^{t}\frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}du\right]dt\Bigg\}$$

$$+\frac{\pi}{T}\left\{d\,\frac{T}{n+1}\left[\left[\frac{1}{d\beta}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)+1\right]^{n+1}-1\right]+\frac{1}{\beta}(t_3-T)-\frac{\alpha}{\beta^2}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right\} \tag{17}$$

IV. OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this section we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1 and t_3, we obtain the first order partial derivatives of $K(t_1,t_3)$ given in equation (17) with respect to t_1 and t_3 and equate them to zero. The condition for minimization of $K(t_1,t_3)$ is

$$D=\begin{vmatrix}\dfrac{\partial^2K(t_1,t_3)}{\partial t_1^2} & \dfrac{\partial^2K(t_1,t_3)}{\partial t_1\partial t_3}\\ \dfrac{\partial^2K(t_1,t_3)}{\partial t_1\partial t_3} & \dfrac{\partial^2K(t_1,t_3)}{\partial {t_3}^2}\end{vmatrix}>0$$

Differentiating $K(t_1,t_3)$ given in equation (17) with respect to t_1 and equating to zero, one can obtain

$$\frac{C}{\alpha-\beta t_1}+\left[\frac{1}{\lambda_2+1}\left[\lambda_1^{\frac{1}{\lambda_2}+1}-\left[\lambda_1-\lambda_2T\left(1+\frac{1}{d\beta}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right)^n\right]^{\frac{1}{\lambda_2}+1}\right]\right.$$

$$\left[\left(\frac{1}{\alpha-\beta t_1}-\frac{dt_1^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 t_1)^{-\frac{1}{\lambda_2}}\right]$$

$$+\frac{1}{\alpha-\beta t_1}\frac{1}{\lambda_2+1}\left[(\lambda_1-\lambda_2 t_1)-(\lambda_1-\lambda_2 t_1)^{-\frac{1}{\lambda_2}}\lambda_1^{\frac{1}{\lambda_2}+1}\right]$$

$$\left.+\frac{dt_1^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\frac{(\lambda_1-\lambda_2t_1)^{-\frac{1}{\lambda_2}}}{\lambda_2+1}\left[\lambda_1^{\frac{1}{\lambda_2}+1}-\left[\lambda_1-\lambda_2T\left(1+\frac{1}{d\beta}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right)^n\right]^{\frac{1}{\lambda_2}+1}\right]\right]=0 \tag{18}$$

Differentiating $K(t_1,t_3)$ given in equation (17) with respect to t_3 and equating to zero, one can obtain

$$-\frac{C}{\alpha-\beta t_3}+\left[\frac{Tn}{(\alpha-\beta t_3)d}\left[\lambda_1-\lambda_2T\left(1+\frac{1}{d\beta}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right)^n\right]^{\frac{1}{\lambda_2}}\left[1+\frac{1}{d\beta}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right]^{n-1}\int_0^{t_1}\left(\frac{1}{\alpha-\beta u}-\right.\right.$$

$$\left.\left.\frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]+\pi\left[\frac{T}{\alpha-\beta t_3}\left(1+\frac{1}{d\beta}\log\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right)^n-\frac{t_3}{\alpha-\beta t_3}\right]=0 \tag{19}$$

Solving the equations (18) and (19) simultaneously, we obtain the optimal time at which production is to be stopped t_1^* of t_1 and the optimal time t_3^* of t_3 at which the should be restarted after accumulation of backorders is obtained. the optimum ordering quantity $Q^*$of Q in the cycle of length T is obtained by substituting the optimal values of t_1^*, t_3^*in equation (12).

V. NUMERICAL ILLUSTRATION

In this section, we discuss the solution procedure of the model through a numerical illustration by obtaining the production uptime, production down time, optimum ordering quantity and the total cost of an inventory system. Here, it is assumed that the commodity is of deteriorating nature and shortages are allowed and fully back logged. For demonstrating the solution precedence, the others deteriorating parameter '$\boldsymbol{\lambda_1}$' is considered to vary 100, 105, 110, and 115, the values of the other deteriorating parameter '$\boldsymbol{\lambda_2}$' is considered to vary 10, 11, 12, and 13, the values of other parameters and costs associated with model are:

A=700,750,800,850;C=10,10.5,11,11.5,h=1,1.1,1.2,1.3;π=2,2.5,3,3.5,α=10,11,12,13;
β=0.5,0.525,0.55,0.575,λ_1=100,105,110,115;λ_2=10,11,12,13,n=2,2.5,3,3.5; d=100,105,110,115, T=12 months

Substituting these values the optimal ordering quantity Q*,production uptime, production down time, total cost are costs are computed and presented in Table 1.

From Table 1 it is observed that the deterioration parameters and production parameters have a tremendous influence on the optimal production times, ordering quantity and total cost. If the ordering cost ‘A’ increases from 700 to 850, the optimal ordering quantity Q^* increases from 0.718 to 0.723, the optimal production down time t_1^* increases from 3.613 to 3.702, the optimal production uptime t_3^* decreases from 5.022 to 5.016, the total cost per unit time K^*, increases from 46.522 to 58.652. As the cost parameter ‘C’ increases from 10 to 11.5, the optimal ordering quantity Q^* increases from 0.718 to 0.718, the optimal production down time t_1^* decreases from 3.613 to 3.614, the optimal production uptime t_3^* decreases from 5.022 to 5.021, the total cost per unit time K^*, increases from 46.522 to 46.607.As the holding cost ‘h’ increases from 1 to 1.3, the optimal ordering quantity Q^* decreases from 0.718 to 0.71, the optimal production down time t_1^* decreases from 3.613 to 3.471, the optimal production uptime t_3^* decreases from 10.92 to 10.20, the total cost per unit time K^*, increases from 5.022 to 5.023. As the shortage cost ‘π’ increases from 2 to 3.5, the optimal ordering quantity Q^* decreases from 0.718 to 0.716, the optimal production down time t_1^* decreases from 3.613 to 3.605, the optimal production uptime t_3^* increases from 5.022 to 5.049. The total cost per unit time K^*, decreases from 46.522 to 45.372.

As the production rate parameter ‘α’ varies from 10 to 13, the optimal ordering quantity Q^* decreases from 0.718 to 0.54, the optimal production down time t_1^* decreases from 3.613 to 3.61, the optimal production uptime t_3^* increases from 5.022 to 5.023, the total cost per unit time K^*, decreases from 46.522 to 46.416. Another production rate parameter ‘β’ varies from 0.5 to 0.575, the optimal ordering quantity Q^* increases from 0.718 to 0.803, the optimal production down time t_1^* increases from 3.603 to 3.615, the optimal production uptime t_3^* decreases from 5.022 to 5.018, the total cost per unit time K^*, increases from 46.522 to 46.728.

As the indexing parameter ‘n’ varies from 2 to 3.5, the optimal ordering quantity Q^* decreases from 0.718 to 0.71, the optimal production down time t_1^* decreases from 3.613 to 3.467, the optimal production uptime t_3^* increases from 5.022 to 5.024. As the total cost per unit time K^*, increases from 46.522 to 41.839. Another demand parameter ‘d’ varies from 100 to 115, the optimal ordering quantity Q^* decreases from 0.718 to 0.714, the optimal production down time t_1^* increases from 3.613 to 3.543, the optimal production uptime t_3^* increases from 5.022 to 5.023, the total cost per unit time K^*, decreases from 46.522 to 45.222.

As the deterioration rate parameter ‘$\boldsymbol{\lambda_1}$’ varies from 100 to 115, the optimal ordering quantity Q^* increases from 0.718 to 0.718, the optimal production down time t_1^* decreases from 3.613 to 3.611, the optimal production uptime t_3^* decreases from 5.022 to 5.022, the total cost per unit time K^*, decreases from 46.522 to 46.504. Another deterioration rate parameter ‘$\boldsymbol{\lambda_2}$’ increases from 10 to 13, the optimal ordering quantity Q^* increases from 0.718 to 0.718, the optimal production down time t_1^* increases from 3.613 to 3.615, the optimal production uptime t_3^* increases from 5.022 to 5.022, the total cost per unit time K^*, increases from 46.522 to 46.527.

Table 1.*Optimal values of t_1^*, t_3^*, Q^* and K^* for different values of parameters*

A	C	h	π	α	β	n	λ_1	λ_2	d	t_1	t_3	Q	K
700	10	1	2	10	0.5	2	100	10	100	3.613	5.022	0.718	46.522
750										3.643	5.02	0.719	50.566
800										3.672	5.018	0.721	54.61
850										3.702	5.016	0.723	58.652
	10.5									3.614	5.021	0.718	46.55
	11.0									3.614	5.021	0.718	46.579
	11.5									3.614	5.021	0.718	46.607
		1.1								3.567	5.022	0.715	45.65
		1.2								3.519	5.023	0.713	44.817
		1.3								3.471	5.023	0.71	44.024
			2.5							3.61	5.031	0.717	46.138
			3							3.608	5.04	0.716	45.754
			3.5							3.605	5.049	0.716	45.372
				11						3.612	5.022	0.616	46.476

				12						3.611	5.023	0.54	46.442
				13						3.61	5.023	0.481	46.416
					0.525					3.614	5.021	0.743	46.595
					0.55					3.614	5.019	0.772	46.663
					0.575					3.615	5.018	0.803	46.728
						2.5				3.552	5.023	0.715	44.568
						3				3.505	5.024	0.712	43.048
						3.5				3.467	5.024	0.71	41.839
							105			3.612	5.022	0.718	46.516
							110			3.611	5.022	0.718	46.51
							115			3.611	5.022	0.718	46.504
								11		3.614	5.022	0.718	46.524
								12		3.614	5.022	0.718	46.526
								13		3.615	5.022	0.718	46.527
									105	3.59	5.022	0.717	46.079
									110	3.566	5.022	0.715	45.645
									115	3.543	5.023	0.714	45.222

VI. SENSITIVITY ANALYSIS OF THE MODEL

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (–15%, –10%, –5%, 0%,5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2. The relationship between the parameters and the optimal values of the production schedule is shown in Figure 2.

It is observed that the costs are having a significant influence on the optimal ordering quantity and production schedules. As the ordering cost A decreases, the optimal production down time t_1^*, total cost per unit time K^* and optimal ordering quantity $Q^*$are decreasing, the optimal production up time t_3^*increases.. As the cost per unit C decreases, the optimal ordering quantity Q^*, the optimal production down time $t_1^*$and total cost per unit time K^* are decreasing and optimal production uptime t_3^*increases.

As the holding cost 'h' decreases, the optimal production down time t_1^*, total cost per unit time K^* and optimal ordering quantity $Q^*$are increasing and the optimal production up time t_3^*decreases.. As shortage cost 'π' decreases, the optimal production uptime t_3^*decreases, the optimal production down time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* are increasing.

As production rate parameter 'α' decreases, the optimal production uptime t_3^*decreases, the optimal production down time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* are increasing. As production rate parameter 'β 'decreases, the optimal production down time $t_3^*$are increasing, the optimal production down time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* are decreasing.

As the deterioration rate parameter '$\boldsymbol{\lambda_1}$' decreases, the optimal production uptime t_3^*decreases, the optimal production down time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* are increasing. As the deterioration rate parameter '$\boldsymbol{\lambda_2}$' decrease, the optimal production down time $t_3^*$is increasing, the optimal production down time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* are decreasing.

As the indexing parameter 'n' increases, the optimal production down time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* are increasing. As the demand parameter 'd' decreases, the optimal production down time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* are increasing.

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Table 2.Sensitivity analysis of the model-with shortages

Parameters/Costs	Optimal policies	Change in parameters						
		-15%	-10%	-5%	0%	5%	10%	15%
A	t_1^*	3.551	3.572	3.593	3.613	3.634	3.655	3.675
	t_3^*	5.026	5.025	5.023	5.022	5.02	5.019	5.017
	Q^*	0.714	0.715	0.717	0.718	0.719	0.72	0.721
	K^*	38.026	40.859	43.691	46.522	49.353	52.184	55.014
C	t_1^*	3.612	3.612	3.613	3.613	3.614	3.614	3.614
	t_3^*	5.023	5.023	5.022	5.022	5.021	5.021	5.021
	Q^*	0.718	0.718	0.718	0.718	0.718	0.718	0.718
	K^*	46.438	46.466	46.494	46.522	46.55	46.579	46.607
h	t_1^*	3.681	3.659	3.636	3.613	3.59	3.567	3.543
	t_3^*	5.021	5.021	5.022	5.022	5.022	5.022	5.023
	Q^*	0.721	0.72	0.719	0.718	0.717	0.715	0.714
	K^*	47.906	47.435	46.973	46.522	46.081	45.65	45.228
π	t_1^*	3.615	3.614	3.614	3.613	3.613	3.613	3.612
	t_3^*	5.017	5.018	5.02	5.022	5.024	5.025	5.027
	Q^*	0.718	0.718	0.718	0.718	0.718	0.718	0.717
	K^*	46.753	46.676	46.599	46.522	46.445	46.368	46.291
α	t_1^*	3.616	3.615	3.614	3.613	3.612	3.612	3.611
	t_3^*	5.021	5.021	5.022	5.022	5.022	5.022	5.022
	Q^*	0.967	0.865	0.784	0.718	0.663	0.616	0.575
	K^*	46.639	46.59	46.553	46.522	46.497	46.476	46.458
β	t_1^*	3.611	3.612	3.613	3.613	3.614	3.614	3.615
	t_3^*	5.027	5.025	5.023	5.022	5.021	5.019	5.018
	Q^*	0.654	0.673	0.695	0.718	0.743	0.772	0.803
	K^*	46.27	46.361	46.445	46.522	46.595	46.663	46.728
n	t_1^*	3.659	3.643	3.628	3.613	3.6	3.587	3.575
	t_3^*	5.021	5.021	5.022	5.022	5.022	5.022	5.022
	Q^*	0.72	0.719	0.719	0.718	0.717	0.716	0.716
	K^*	47.974	47.463	46.98	46.522	46.089	45.679	45.289
λ_1	t_1^*	3.617	3.616	3.614	3.613	3.612	3.611	3.611
	t_3^*	5.022	5.022	5.022	5.022	5.022	5.022	5.022
	Q^*	0.718	0.718	0.718	0.718	0.718	0.718	0.718
	K^*	46.548	46.538	46.53	46.522	46.516	46.51	46.504
λ_2	t_1^*	3.612	3.613	3.613	3.613	3.613	3.614	3.614
	t_3^*	5.022	5.022	5.022	5.022	5.022	5.022	5.022
	Q^*	0.718	0.718	0.718	0.718	0.718	0.718	0.718
	K^*	46.52	46.521	46.521	46.522	46.523	46.524	46.525

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d	t_1^*	3.681	3.659	3.636	3.613	3.59	3.566	3.543
	t_3^*	5.021	5.021	5.022	5.022	5.022	5.022	5.023
	Q^*	0.721	0.72	0.719	0.718	0.717	0.715	0.714
	K^*	47.914	47.44	46.976	46.522	46.079	45.645	45.222

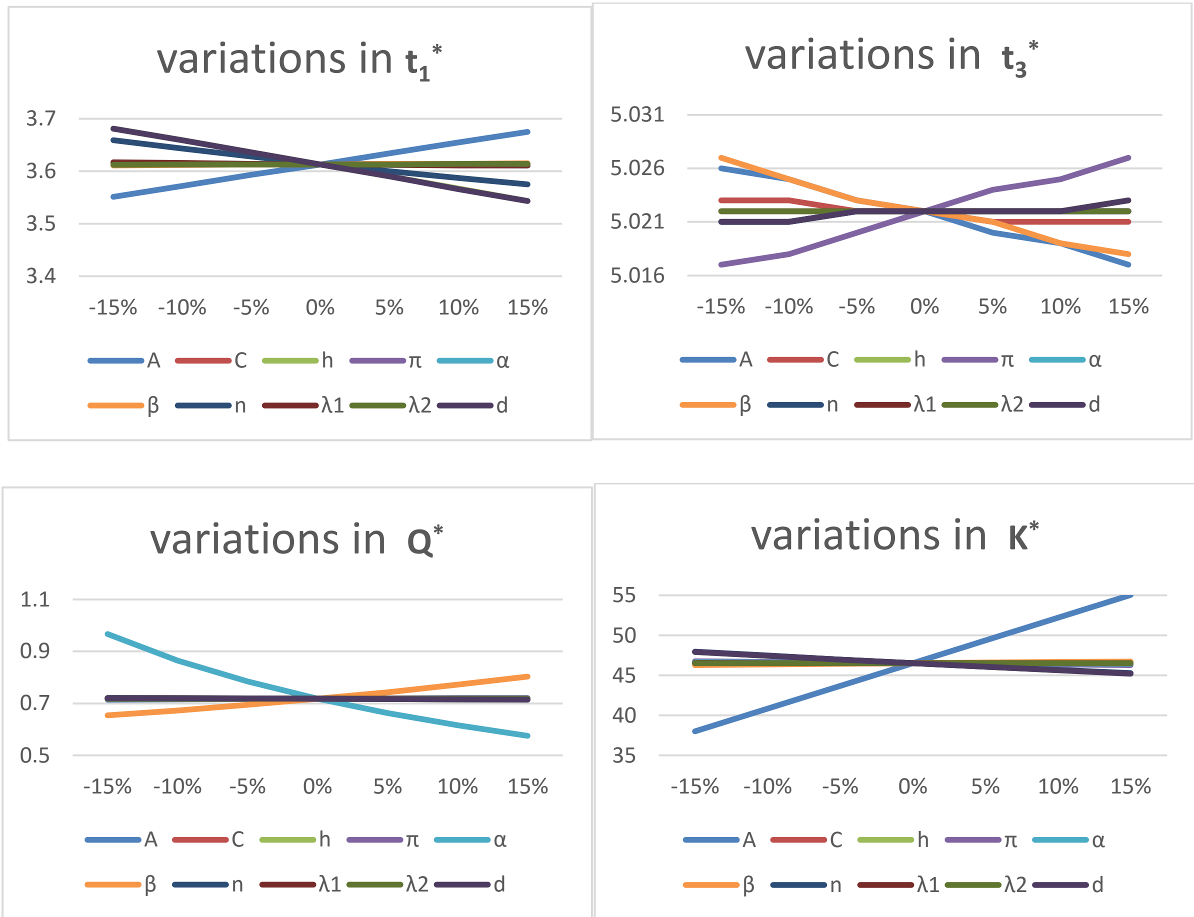

Fig 3: Relationship between optimal values and parameters with shortages

VII. INVENTORY MODEL WITHOUT SHORTAGES:

In this section, the inventory model for deteriorating items without shortages is developed and analysed. Here, it is assumed that shortages are not allowed and the stock level is zero at time t = 0. The stock level increases during the period (0, t_1), due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, T). At time T, the inventory reaches zero. The Schematic diagram representing the instantaneous state of inventory is given in Figure 3.

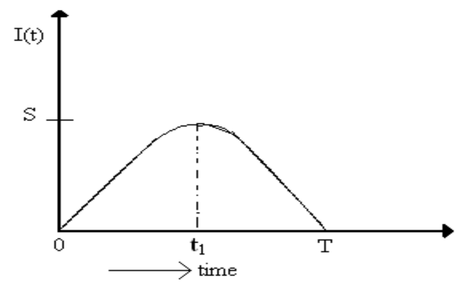

Fig:2 Schematic diagram representing the inventory level.

The differential equations governing the system in the cycle time [0, T] are:

$$\frac{d}{dt}I(t) + h(t)I(t) = \frac{1}{\alpha-\beta t} - \frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}; \qquad 0 \le t \le t_1 \tag{20}$$

$$\frac{d}{dt}I(t) + \frac{I(t)}{h(t)} = -\frac{dt^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}; \qquad t_1 \le t \le T \tag{21}$$

where, h(t) is as given in equation (3), with the initial conditions, I (0) = 0, I(t_1) = S, and I(T)=0.
Substituting h (t) given in equation (3) in equation (20) and (21) and solving the differential equations, the on-hand inventory at time 't' is obtained as

$$I(t) = S\left(\frac{\lambda_1 - \lambda_2 t}{\lambda_1 - \lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1 - \lambda_2 t)^{\frac{1}{\lambda_2}} \int_t^{t_1} \left(\frac{1}{\alpha - \beta u} - \frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du; \quad 0 \le t \le t_1 \tag{22}$$

$$I(t) = S\left(\frac{\lambda_1 - \lambda_2 t}{\lambda_1 - \lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1 - \lambda_2 t)^{\frac{1}{\lambda_2}} \int_{t_1}^{t} \frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du; \quad t_1 \le t \le T \tag{23}$$

Stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t k(t)dt - \int_0^t \lambda(t)dt - I(t), \quad 0 \le t \le T$$

This implies

$$L(t) = \begin{cases} \log\left(\frac{\alpha}{\alpha - \beta t}\right)^{\frac{1}{\beta}} - d\left(\frac{t}{T}\right)^{\frac{1}{n}} - S\left(\frac{\lambda_1 - \lambda_2 t}{\lambda_1 - \lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1 - \lambda_2 t)^{\frac{1}{\lambda_2}} \int_t^{t_1} \left(\frac{1}{\alpha - \beta u} - \frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du; & 0 \le t \le t_1 \\ \log\left(\frac{\alpha}{\alpha - \beta t_1}\right)^{\frac{1}{\beta}} - d\left(\frac{t}{T}\right)^{\frac{1}{n}} - S\left(\frac{\lambda_1 - \lambda_2 t}{\lambda_1 - \lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1 - \lambda_2 t)^{\frac{1}{\lambda_2}} \int_t^{t_1} \left(\frac{1}{\alpha - \beta u} - \frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du; & t_1 \le t \le T \end{cases}$$

Ordering quantity Q in the cycle of length T is

$$Q = \int_0^{t_1} k(t)dt = \log\left(\frac{1}{\alpha - \beta t_1}\right)^{\frac{1}{\beta}} \tag{24}$$

From equation (22) and using the condition I (0) = 0, we obtain the value of 'S' as

$$S = (\lambda_1 - \lambda_2 t)^{\frac{1}{\lambda_2}} \int_0^{t_1} \left(\frac{1}{\alpha - \beta u} - \lambda\right)(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du \tag{25}$$

Let $K(t_1)$ be the total cost per unit time. Since the total cost is the sum of the set up cost, cost of the units, the inventory holding cost. Therefore, the total cost is

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right) \tag{26}$$

Substituting the value of I (t), Q and S given in equation's (22), (23), (24) and (25) in equation (26) and on simplification, we obtain $K(t_1)$ as

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\log\left(\frac{1}{\alpha - \beta t_1}\right)^{\frac{1}{\beta}}$$
$$+\frac{h}{T}\left\{\left[\int_0^{t_1} (\lambda_1 - \lambda_2 t)^{\frac{1}{\lambda_2}} \int_0^{t_1} \left(\frac{1}{\alpha - \beta u} - \lambda\right)(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du\right.\right.$$

$$\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}} \int_t^{t_1} \left[\left(\frac{1}{a-\beta u} - \frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du\right] dt\Bigg]$$

$$+\left[\int_{t_1}^{T}\left[(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}} \int_0^{t_1}\left(\frac{1}{\alpha-\beta u}-\lambda\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du \left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}\right.\right.$$

$$\left.\left.(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}} \int_{t_1}^{t} \frac{du^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} (\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du\right] dt\right\} \tag{27}$$

VIII. OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL WITHOUT SHORTAGES:

In this section, we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1, we equate the first order partial derivatives of $K(t_1)$ with respect to t_1 equate them to zero. The condition for minimum of $K(t_1)$ is $\frac{d^2K(t_1)}{dt_1^2} > 0$

Differentiating $K(t_1)$ with respect to t_1 and equating to zero we get

$$\frac{C}{\alpha-\beta t1} + h\left[\frac{\lambda_1-\lambda_2 t}{(\alpha-\beta t1)(\lambda_2+1)}\left[(\lambda_1-\lambda_2 t1)^{\frac{1}{\lambda_2}} + 1 - [\lambda_1 - \lambda_2 T[1+\frac{1}{d\beta}log\left(\frac{\alpha-\beta T}{\alpha-\beta t3}\right)\right]\right] \tag{28}$$

Solving the equation (28), we obtain the optimal time at which the production is to be stopped at t_1^* of t_1.

The optimal ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal value of t_1 in equation (24).

IX. NUMERICAL ILLUSTRATION OF MODEL WITHOUTSHORTAGES

In this section, we discuss the numerical illustration of the model. For demonstrating the solution procedure of the model, the deteriorating rate parameter '$\boldsymbol{\lambda_1}$' is considered to vary 100,105,110,115 and the other deteriorating rate parameter '$\boldsymbol{\lambda_2}$' is considered to vary 5,5.5,6,6.5 the values of other parameters and costs associated with model are:

A=2000,2100,2200,2300; C=10,10.5,11,11.5,

h=0.85,0.9,0.95,1;

α=10,10.5,11,11.5; β=0.85,0.875,0.9,0.925,

λ_1=100,105,110,115;λ_2=5,5.5,6,6.5,

n=0.5,0.525,0.55,0.575; d=5,5.5,6,6.5, T=12 moths.

Substituting these values, the optimal ordering quantity *Q**, production down time and total cost are computed and presented in Table 3.

Table 3. *Optimal values of t_1^*, t_3^*, Q^* and K^* for different values of parameters*

A	C	h	α	β	n	λ_1	λ_2	d	t_1	Q	K
2000	10	0.85	10	0.85	0.5	100	5	5	3.333	0.274	170.178
2100									3.256	0.265	178.195
2200									3.179	0.257	186.226
2300									3.102	0.248	194.27
	10.5								3.333	0.274	170.187
	11.0								3.332	0.274	170.195
	11.5								3.331	0.274	170.204
		0.9							3.188	0.258	170.19
		0.95							3.144	0.253	170.157
		1							3.1	0.248	170.11
			10.5						3.333	0.256	170.166
			11						3.334	0.24	170.155
			11.5						3.334	0.226	170.146

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				0.875					3.233	0.284	170.222
				0.9					3.232	0.305	170.238
				0.925					3.232	0.326	170.254
					0.525				3.413	0.283	170.125
					0.55				3.478	0.291	170.031
					0.575				3.53	0.297	169.915
						105			3.334	0.274	170.174
						110			3.335	0.274	170.171
						115			3.335	0.274	170.169
							5.5		3.333	0.274	170.182
							6		3.233	0.263	170.215
							6.5		3.232	0.263	170.222
								5.5	3.266	0.266	170.207
								6	3.2	0.259	170.196
								6.5	3.134	0.252	170.149

From Table 3 it is observed that the deterioration parameters and production rate parameters have a tremendous influence on the optimal production times, ordering quantity and total cost.

X .SENSITIVITY ANALYSIS OF THE MODEL WITHOUT SHORTAGES:

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (–15%, –10%, –5%, 0%,5%, 10%, 15%) at a time for the model under study. The results are presented in Table 4. The relationship between the parameters and the optimal values of the production schedule is shown in Figure 4. It is observed that the costs are having significant influence on the optimal ordering quantity and production schedules.

As the ordering cost A decreases, the optimal production time t_1^*, the optimal ordering quantity Q^* increases and total cost per unit time K^* is decreasing. As the cost per unit C decreases, the optimal production timet_1^*, is increasing and total cost per unit time K^* and optimal ordering quantity Q^* is decreasing. As the holding cost 'h' decreases, the optimal production down time $t_1^*$and optimal ordering quantity Q^* are increasing and total cost per unit time K^* is decreasing.

As production parameter rate 'α' decreases, the optimal production time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K are increasing. As another production rate parameter 'β' decreases, the optimal production time t_1^* is increasing and the optimal ordering quantity Q* and the total cost per unit time K^* are decreasing.

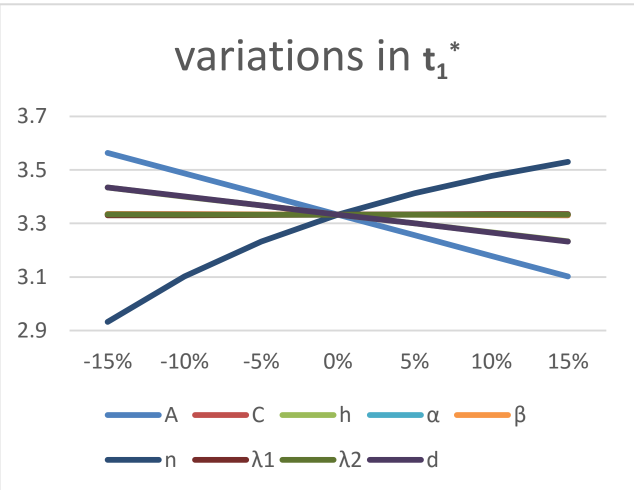
As the deterioration rate parameter 'λ_1' decreases, the optimal production time t_1^*, the optimal ordering quantity Q^* are the total cost per unit time K^*increases. As the deterioration rate parameter 'λ_2' decrease, the optimal production time$t_1^*$and the optimal ordering quantity Q^* are increasing and the total cost per unit time K^* is decreasing.

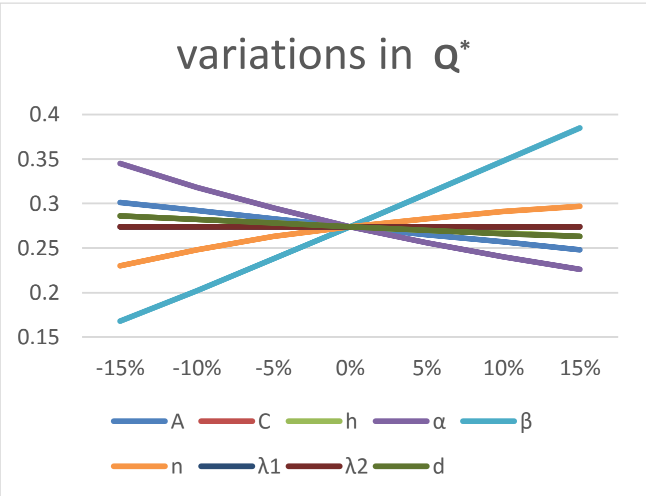
As demand indexing parameter 'n' decreases, the optimal production time t_1^*, the optimal ordering quantity Q^* and the total cost per unit time K^* decreases.. As demand parameter 'd' decreases, the optimal production time t_1^*, the optimal ordering quantity Q^* are increasing, the total cost per unit time K^* decreases.

Table 4. Sensitivity analysis of the model without shortages

Parameters/ Costs	**Optimal values**	**Change in parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	3.564	3.487	3.41	3.333	3.256	3.179	3.102
	Q^*	0.301	0.292	0.283	0.274	0.265	0.257	0.248
	K^*	146.214	154.187	162.175	170.178	178.195	186.226	194.27
C	t_1^*	3.335	3.335	3.334	3.333	3.333	3.332	3.331

	Q^*	0.274	0.274	0.274	0.274	0.274	0.274	0.274
	K^*	170.153	170.161	170.17	170.178	170.187	170.195	170.204
H	**t_1^***	3.434	3.4	3.367	3.333	3.3	3.267	3.234
	Q^*	0.286	0.282	0.278	0.274	0.27	0.267	0.263
	K^*	170.057	170.108	170.149	170.178	170.197	170.205	170.204
α	**t_1^***	3.333	3.333	3.333	3.333	3.333	3.334	3.334
	Q^*	0.345	0.318	0.295	0.274	0.256	0.24	0.226
	K^*	170.226	170.208	170.192	170.178	170.166	170.155	170.146
β	**t_1^***	3.335	3.334	3.334	3.333	3.333	3.332	3.332
	Q^*	0.168	0.202	0.238	0.274	0.311	0.348	0.385
	K^*	170.097	170.123	170.15	170.178	170.206	170.235	170.264
N	**t_1^***	2.933	3.102	3.232	3.333	3.413	3.478	3.53
	Q^*	0.23	0.248	0.263	0.274	0.283	0.291	0.297
	K^*	169.678	170.013	170.157	170.178	170.125	170.031	169.915
λ_1	**t_1^***	3.33	3.331	3.332	3.333	3.334	3.335	3.335
	Q^*	0.274	0.274	0.274	0.274	0.274	0.274	0.274
	K^*	170.194	170.188	170.183	170.178	170.174	170.171	170.169
λ_2	**t_1^***	3.334	3.333	3.333	3.333	3.333	3.333	3.333
	Q^*	0.274	0.274	0.274	0.274	0.274	0.274	0.274
	K^*	170.174	170.175	170.177	170.178	170.18	170.182	170.184
d	**t_1^***	3.434	3.401	3.367	3.333	3.3	3.266	3.233
	Q^*	0.286	0.282	0.278	0.274	0.27	0.266	0.263
	K^*	170.053	170.106	170.148	170.178	170.198	170.207	170.206





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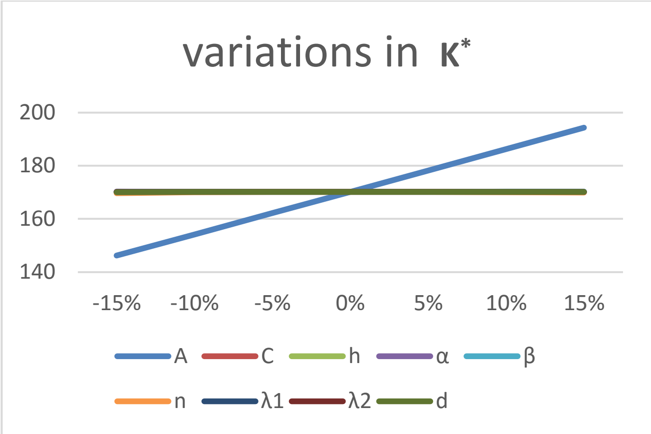

Fig 4: Relationship between optimal values and parameters without shortages

XI. CONCLUSION

This paper introduces a new and novel Economic Production Quantity model with Random production for deteriorating items having Random life time with time dependent demand. Here the life time of the commodity is characterize with the Generalized Pareto Distribution, Since the statistical characteristics of the deteriorating items match with the distribution Characteristics. The production quantity per a unit time is considered to be random and follows a Generalized Pareto Distribution. In many production processes the production quantity per a unit time is random due to various Random factors such as availability of raw materials, tool wear of the materials, skill level of the employees, power supply. The Generalized Pareto Distribution characterizes the increasing rate of production and deterioration. It is also having a finite range. Assuming that the demand is time dependent and follows a power pattern demand the instantaneous sate of inventory is derived. The optimal production schedules (production uptime and production down time), production quantity are derived. The sensitivity analysis of the model reveals that the life time distribution, as well as production quantity per a unit time distribution having significant influence on optimal, production policies. It is also observed that the various costs associated with the model have significant influence on the production schedules. Allowing shortages in the production processes will yield more benefits. This model is much use full for batch production systems dealing with deteriorated items. It is possible to extended this model with permissible delay in payments under inflation. It is also possible to develop Economic Production Quantity models for multi commodity items having Generalized Pareto Rate of production and deterioration which will be taken up elsewhere.

REFERENCES

[1]. Goyal, S. K and Giri, B. C,"Recent trends in modeling of deteriorating inventory", *European Journal of Operational Research*, Vol. 134, No.1, pp. 1-16, 2001.
[2]. Ruxian, LI., Lan, H. and Mawhinney, R. J, "A review on deteriorating inventory study", *Journal of Service Science Management*, Vol. 3, No. 1, pp. 117-129, 2010.
[3]. Pentico, D. W. and Drake, M. J, "A survey of deterministic models for the EOQ and EPQ with partial backordering", *European Journal of Operational Research*, Vol. 214, Issue. 2, pp. 179-198, 2011.
[4]. Ghare, P. M and Schrader, G. F, "A model for exponentially decaying inventories', *Journal of Industrial engineering",* Vol. 14, pp. 238-2430, 1963.
[5]. Shah, Y. and Jaiswal, M. C, "An order level inventory model for a system with a constant rate of deterioration", *OPSEARCH*, Vol. 14, pp. 174-184, 1977.
[6]. Cohen, M. A,"Joint pricing and ordering for policy exponentially decaying inventories with known demand", *Naval Research Logistics. Q*, Vol. 24, pp. 257-268, 1977.
[7]. Aggarwal, S. P, "A note on an order level inventory model for system with constant rate of deterioration", *OPSEARCH*, Vol. 15, No. 4, pp.184–187, 1978 .
[8]. Dave, U and Shah, Y.K, "A probabilistic inventory model for deteriorating items with time proportional demand", *Journal of Operational Research Society*, Vol. 32, pp. 137-142, 1982.
[9]. Pal, M, 'An inventory model for deteriorating items when demand is random", *Calcutta Statistical Association Bulletin*, Vol. 39, pp. 201-207, 1990.
[10]. Kalpakam, S and Sapna, K. P, "A lost sales (S-1, S) perishable inventory system with renewal demand", *Naval Research Logistics*, Vol. 43, pp. 129-142, 1996.
[11]. Giri, B. C and Chaudhuri, K. S,"An economic production lot-size model with shortages and time dependent demand", *IMA Journal of Management Mathematics*, Vol. 10, No.3, pp. 203-211, 1999.
[12]. Tadikamalla, P.R, "An EOQ inventory model for items with gamma distributed deteriorating", *AIIE TRANS*, Vol. 10, pp. 100-103, 1978.
[13]. Covert, R.P. and Philip, G.C, "An EOQ model for items with Weibull distribution deterioration", *AIIE. TRANS*, Vol.5, pp. 323-326, 1973.
[14]. Philip, G. C, "A generalized EOQ model for items with Weibull distribution", *AIIE TRANS*, Vol. 16, pp. 159-162, 1974.
[15]. Goel, V. P and Aggarwal, S. P, "Pricing and ordering policy with general Weibull rate deteriorating inventory", *Indian journal Pure and Applied Mathematics*, Vol. 11(5), pp. 618 -627, 1980.

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[16]. Venkata Subbaiah, K., Srinivasa Rao, K. and Satyanarayana, B.V.S,"An inventory model for perishable items having demand rate dependent on stock level", *OPSEARCH,* Vol. 41(4), pp. 222-235, 2004.
[17]. Nirupama Devi, K., Srinivasa Rao, K. and Lakshminarayana, J, "Perishable inventory model with mixtures of Weibull distribution having demand as a power function of time", *Assam Statistical Review,*Vol.15,No.2,pp.70-80,2001.
[18]. Srinivasa Rao, K., Vivekananda Murthy, M and Eswara Rao, S, "An optimal ordering and pricing policies of inventory models for deteriorating items with generalized Pareto life time", *A Journal on Stochastic Process and Its Applications,* Vol. 8(1), pp. 59-72, 2005.
[19]. Xu, X. H. and Li, R. J, "A two-warehouse inventory model for deteriorating items with time-dependent demand", *Logistics Technology,* No. 1, pp. 37-40, 2006.
[20]. Rong, M., Mahapatra, N.K. and Maiti, M,"A two-warehouse inventory model for a deteriorating item with partially/fully backlogged shortage and fuzzy lead time", *European Journal Of Operational Research,* Vol. 189, Issue. 1, pp. 59-75, 2008.
[21]. Srinivasa Rao, K., Srinivas. Y., Narayana, B. V. S. and Gopinath, Y, 'Pricing and ordering policies of an inventory model for deteriorating items having additive exponential lifetime", *Indian Journal of Mathematics and Mathematical Sciences,* Vol. 5(1), pp. 9-16, 2009.
[22]. Chakrabarti, T and Chaudhuri, K. S. (1997). 'An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles', *International Journal of production economics,* Vol.49, No.3, pp. 205-213.
[23]. Chang, H. J and Lin, W. F. (2010). 'A partial backlogging inventory model for non-instantaneous deteriorating items with stock dependent consumption rate under inflation', *Yugoslav Journal of Operations Research,* Vol.20 (1), pp. 35-54.
[24]. Biswajit Sarkar, "An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production", *Mathematics and Computation,* Vol. 218(17), pp. 8295-8308, 2012.
[25]. Mukherjee, S. P and Pal, M, "An order level production inventory policy for items subject to general rate of deterioration", *IAPQR Transactions,* Vol. 11, pp. 75-85, 1986.
[26]. Gowsami, A., Mahata, G. C. and Prakash, O. M., "Optimal retailer replenishment decisions in the EPQ model for deteriorating items with two level of trade credit financing", *International Journal of Mathematics in Operational Research,* Vol.2, No.1, pp. 17-39, 2010.
[27]. Goyal, S. K and Giri, B. C, "The production- inventory problem of a product with varying demand, production and deterioration rates", *European Journal of Operational Research,* Vol. 147, No. 3, pp. 549-557, 2003.
[28]. Panda, R. M. and Chaterjee, E,"On a deterministic single items model with static demand for deteriorating item subject to discrete time variabl", *IAPQR Trans,* Vol. 12, pp. 41-49, 1987.
[29]. Mandal, B. N. and Phaujdar, S,"An inventory model for deteriorating items and stock dependent consumption rate", *Journal of Operational Research Society,* Vol. 40, No. 5, pp. 483- 488, 1989.
[30]. Sana, S. S., Goyal, S. K. and Chaudhuri, K. S, "A production inventory model for a deteriorating item with trended demand and shortages", *European Journal of Operational Research,* Vol. 157, No. 2, pp. 357-371,2004.
[31]. Perumal, V. and Arivarignan, G,"A production model with two rates of productions and back orders", *International Journal of Management system,* Vol. 18, pp. 109-119, 2002.
[32]. Pal, M. and Mandal, B,"An EOQ model for deteriorating inventory with alternating demand rates", *Journal Of Applied Mathematics and Computing,* Vol. 4, No.2, pp. 392-397, 1997.
[33]. Sen, S. and Chakrabarthy, T, "An order- level inventory model with variable rate of deterioration and alternating replenishing rates considering shortages", *OPSEARCH,* Vol. 44(1), pp. 17-26, 2007.
[34]. Venkata Subbaiah, K., Uma Maheswara Rao, S.V. and Srinivasa Rao, K., "An inventory model for perishable items with alternating rate of production", *International Journal of Advanced Operations Management,* Vol. 3, No. 1, pp. 66-87, 2011.
[35]. Essey, K. M and Srinivasa Rao, K, "EPQ models for deteriorating items with stock dependent demand having three parameter Weibull decay", *International Journal of Operations Research,* Vol.14, No.3, pp. 271-300, 2012.
[36]. Sridevi, "Inventory model for deteriorating items with Weibull rate of replenishment and selling price dependent demand", *International Journal of Operational Research,* Vol. 9(3), pp. 329-349, 2010.
[37]. Srinivasa Rao, K., Nirupama Devi, K. and Sridevi, G, "Inventory model for deteriorating items with Weibull rate of production and demand as function of both selling price and time", *Assam Statistical Review,* Vol. 24, No.1, pp.57-78, 2010.
[38]. Lakshmana Rao, A. and Srinivasa Rao, K, "Studies on inventory model for deteriorating items with Weibull replenishment and generalized Pareto decay having demand as function of on hand inventory", *International Journal of Supply and Operations Management*, Vol. 1, Issue. 4, pp. 407-426, 2015.
[39]. Srinivasa Rao et al, "Inventory model for deteriorating items with Weibull rate of replenishment and selling price dependent demand", *International Journal of Operational Research,* Vol. 9(3), pp. 329-349, 2017.
[40]. Madhulatha,D."Econamic production quantity model with generalized Pareto rate of production and weibull decay having selling price dependent demand", Journal of Ultra scientist of physical sciences,Vol.29,No.11,pp.485-500,2017.
[41]. Ashwani Chandel , Vikram Jeet Singh, "Relative Investigation of Ant Colony Optimization and Genetic Algorithm based Solution to Travelling Salesperson Problem", International Journal of Computer Sciences and Engineering **Open Access**, Vol. 3, Issue.3, 2014.
[42]. Neetu Anand ,Tapas Kumar, "Prediction of User Interest and Behavior using Markov Model", International Journal of Scientific Research in, Vol.5, Issue.3, pp.119-123, June 2017.

AUTHORS PROFLE

[1]**.** Dr. K.Srinivasa Rao received Ph.D degree in statistics in 1988 from Andhra University, V**i**sakhapatnam, Andhra Pradesh**,** India. He taught the subjects operations research, Statistical inference, Time series analysis, Forecasting methods, He has guided 53 students for Ph.D. He also dynamics completed 11 Research Projects. He is expertise in Computer Networking, Classification, Data Mining, Knowledge Discovery, Information and Communication Technology, Image Processing, Supervised Learning, Unsupervised Learning, Signal Processing, Networking, Pattern Classification. He also published 8 books and also he given several invited talks in National/International workshops/ Conferences.

[2]**.** B.Punyavathi recieved Bachelor of Science from Sri Krishnadevaraya University (2009-2011), Anantapuram and Master of Science from Sri Venkateswara University(2011-2013), Tirupathi and presently pursuing Ph.D in the Department of Statistics, Andhra University, Visakhapatnam-530003, Andhra Pradesh, India. Attended National/International workshops/ Conferences and she presented three papers one National and International conferences.