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Optimal Replenishment Strategy under Combined Criteria

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***Abstract*:** This article focuses on determining an optimal replenishment policy for items with three-parameter Weibull distribution deterioration where it represents the time to deterioration of a product. It is also observed that the demand of a consumer product usually varies with its cost and hence, the demand rate should be taken as price dependent. Holding cost is a linear function of time. Here replenishment strategy is developed under combined criteria of demand, deterioration and holding cost. The applicability of the model lies in the management of inventories of deteriorating products and for the particular items for which demand falls due to increase of its cost. Numerical illustrations and sensitivity analysis are provided to illuminate the effect of change of model parameters.

Keywords: Weibull Distribution, Time-varying Holding Cost, Price Dependent Demand, Deterioration

AMS Subject Classification No: 90B05

I. Introduction

Inventory management for deteriorating items is very challenging task in all types of business and making proper inventory decision is a key factor of success. Deterioration makes product demand and value dull. So it is a trending research area for all the researchers of inventory area. Deterioration makes product demand and value dull.

The instantaneous rate of deterioration of inventory at time t is defined by $\theta(t) = \alpha\,\beta(t-\gamma)^{\beta-1}$, where γ is the location parameter, $t \geq \gamma$; α is the scale parameter and β is the shape parameter; $\alpha > 0, \beta > 0 \,\&\, \gamma > 0$ represents the three-parameter Weibull Distribution deterioration rate at any time $t > 0$. Three parameter Weibull distribution is applicable for items with any initial value of rate of deterioration and which start deterioration only after a definite period of time.

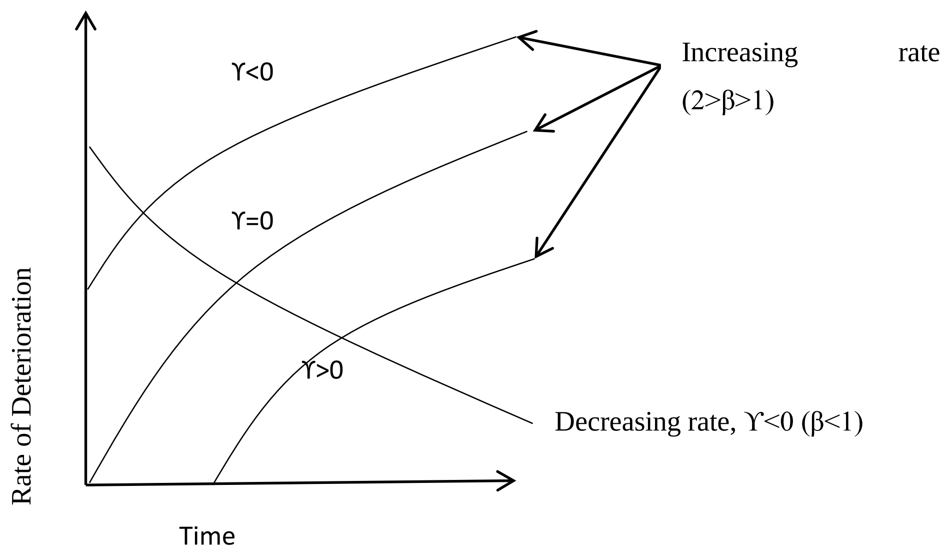

Figure 1 : Rate of deterioration and time relationship for three-parameter Weibull distribution

Ghare and Schrader [11] considered the inventory model for exponentially decaying inventory. Covert and Philip [13] obtained an inventory model for deteriorating items by taking the two-parameter Weibull distribution deterioration with constant demand and holding cost. Philip [3] developed a generalised inventory model of a three-parameter Weibull distribution deterioration and constant demand. Jalan et al. [1] and Chakrabarty et al. [20] made an extension of model of Covert and Philip [13] and Philip [3] by including a three-parameter Weibull distribution deterioration, time-dependent demand rate and shortages in the inventory. Wu et al. [4, 5], Deng [12] derived an inventory model for deteriorating items by following the Weibull distribution with time-varying and ramp type demand respectively. Banerjee and Agrawal [14], Ghosh and chaudhuri [15], Sanni and Chukwu [19] obtained a solution for an inventory model with Weibull deterioration and various types of deterministic demand (namely, trended and time quadratic). Ghosh et al. [16] developed a production model with Weibull demand. Goyal and Giri [17], Manna and Chaudhuri [18], Sahoo and Tripathy [6] did an attempt in his paper to obtain the optimal ordering quantity of deteriorating items for time-dependent deterioration rate and demand. Tripathy and Pradhan [9] introduced a model with Weibull distribution deterioration and power demand inventory. Tripathy and Pradhan [8] developed an integrated partial backlogging inventory model having Weibull demand and variable deterioration rate. Mohanty and Tripathy [2] formulated inventory model for deteriorating items with exponentially decreasing demand under imprecise environment. Tripathy and Sukla [10] has presented interactive fuzzy inventory model with ramp type demand and Weibull deterioration. Tripathy and Bag [7] explored inventory model with default risk under conditional delay.

The work of researchers who used Weibull distribution deterioration as deterioration pattern and various form of demand with holding cost, for developing inventory models are summarized below

Table 1. Comparison of demand pattern, deterioration and holding cost

Reference	Deterioration Function	Demand Pattern	Holding Cost
Banerjee and Agrawal [14]	Three-parameter Weibull distribution	Trended	Constant
Chakrabarty et al. [20]	Three-parameter Weibull distribution	Trended	Constant
Covert and Philip [13]	Two-parameter Weibull distribution	Constant	Constant
P. S. Deng [12]	Two-parameter Weibull distribution	Ramp type	Constant
Ghosh and Chaudhury [15]	Two-parameter Weibull distribution	Time-Quadratic	Constant
Jalan et al. [1]	Three-parameter Weibull distribution	Trended	Constant
G. C. Philip [3]	Two-parameter Weibull distribution	Constant	Constant
Sahoo and Tripathy [6]	Three-parameter Weibull distribution	Trended	Time-Quadratic
Sanni and Chukwu [19]	Three-parameter Weibull distribution	Time-Quadratic	Constant
K. S. Wu [5]	Two-parameter Weibull distribution	Time-Varying	Constant
Kun –Shan Wu [4]	Two-parameter Weibull distribution	Ramp type	Constant
Present Study	Three-parameter Weibull distribution	Price dependent	Time-Varying

The current study focuses on a certain kind of demand pattern where demands of the products are dependent upon its selling price. This inventory model is applicable for deteriorating items and assumes that deterioration follows a three-parameter Weibull distribution. The unique advantage of the functional form of Weibull distribution accommodates with both increasing and decreasing deterioration. Holding cost is also time-varying. An analytical solution of the model is betalked and illustrated with the help of several numerical illustrations and sensitivity of the optimal solution is also examined.

II. Assumptions and Notations

The model is developed with the following assumptions and notations

i. $D(p) = ap^{-b}$ represents the price-dependent demand rate, where p is the selling price per unit item, $a > 0$ is scale parameter and $b > 0$ is shape parameter.

ii. $\theta(t) = \alpha\,\beta(t-\gamma)^{\beta-1}$, where $\alpha > 0, \beta > 0\ \&\ \gamma > 0$ represents the three-parameter Weibull distribution deterioration rate at any time $t > 0$.

iii. c is the constant purchase cost per unit item.

iv. A is the ordering cost per cycle.

v. $h(t) = h + \lambda t$, where λ >0 represents the inventory holding cost per unit of item per unit time.

vi. T is the total cycle length.

vii. Shortages are not allowed.

viii. π is the profit rate.

ix. $I_1(T) = 0$, where $I_1(t)$ is the inventory level at any time t.

III. Mathematical Model

Let $I_1(t)$ be the inventory level at any time t is governed by the following differential equation.

$$\frac{dI_1(t)}{dt} = -\theta(t)\, I_1(t) - D(p),\ \ 0 \le t \le T \tag{1}$$

With $I_1(0) = I_0$ *and* $I_1(T) = 0$

I_0 is the initial order quantity.

The inventory gradually depletes over time due to two key factors of demand and deterioration. The first term of the right hand side of equation (1) indicates the depletion of inventory per unit time due to deterioration only, while the second term represents that due to demand only. The negative sign indicates that the inventory level decreases over time due to these two factors.

Using $\theta(t) = \alpha\,\beta(t-\gamma)^{\beta-1}$ and $D(p) = ap^{-b}$, the solution of the equation (1) will be

$$I_1(t) = I_0.e^{\alpha(-\gamma)^{\beta}}.e^{-\alpha(t-\gamma)^{\beta}} - ap^{-b}.e^{-\alpha(t-\gamma)^{\beta}}\int_0^t e^{\alpha(t-\gamma)^{\beta}}dt \tag{2}$$

Using $I_1(0)=I_0$ *and* $I_1(T)=0$, then equation (2) becomes

$$I_1(0)=I_0=ap^{-b}.e^{-\alpha(-\gamma)^\beta}\int_0^T e^{\alpha(t-\gamma)^\beta}dt \tag{3}$$

Combining equation (2) & (3), we get

$$I_1(t)=ap^{-b}.e^{-\alpha(t-\gamma)^\beta}\int_t^T e^{\alpha(t-\gamma)^\beta}dt \tag{4}$$

If there is no decay, the differential equation of the inventory level $I_2(t)$ at any time t can be written as

$$\frac{dI_2(t)}{dt}=-D(p),\ 0\le t\le T \tag{5}$$

with the condition $I_2(T)=0$

The solution of equation (5) is

$$I_2(t)=(T-t)ap^{-b},\ 0\le t\le T \tag{6}$$

$I_1(0)$ and $I_1(t)$ be the initial stock levels at time 0 and *t* respectively, the amount of stock depleted in time *t* due to both demand and deterioration is $I_1(0)-I_1(t)$.

Similarly when there is no decay, the amount of stock depleted in time *t* due to demand only is $I_2(0)-I_2(t)$.

Let $Z(t)$ be the stock loss due to deterioration in time interval $[0,\ t]$, which is written as

$$Z(t)=[I_1(0)-I_1(t)]-[I_2(0)-I_2(t)]$$

$$=ap^{-b}.e^{-\alpha(-\gamma)^\beta}\int_0^T e^{\alpha(t-\gamma)^\beta}dt-ap^{-b}.e^{-\alpha(t-\gamma)^\beta}\int_t^T e^{\alpha(t-\gamma)^\beta}dt-ap^{-b}t \tag{7}$$

As of now loss to the system is due to either decay or demand, the order quantity in each cycle is as follows.

$$Q_T=Z(T)+D(p)T$$

$$=ap^{-b}.e^{-\alpha(-\gamma)^\beta}\int_0^T e^{\alpha(t-\gamma)^\beta}dt \tag{8}$$

The total cost per cycle is

$$C(T,p)=Ordering\ Cost+Purchase\ Cost+Holding\ Cost$$

$$= A + cQ_T + \int_0^T h(t) I_1(t) dt$$

$$= A + cap^{-b}(1-\alpha(-\gamma)^{\beta})\left(T + \frac{\alpha(T-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1}\right)$$
$$+ hap^{-b}\left(\frac{T^2}{2} - \frac{2\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T(-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha T(T-\gamma)^{\beta+1}}{\beta+1}\right)$$
$$+ \lambda ap^{-b}\left(\frac{T^3}{6} + \frac{\alpha T^2(T-\gamma)^{\beta+1}}{2(\beta+1)} - \frac{3\alpha T(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3\alpha(T-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}\right) \quad (9)$$

Hence the average system cost is

$$C^*(T, p) = \frac{C(T, p)}{T}$$

$$= \frac{1}{T}\left[\begin{array}{l} A + cap^{-b}(1-\alpha(-\gamma)^{\beta})\left(T + \frac{\alpha(T-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1}\right) \\ + hap^{-b}\left(\frac{T^2}{2} - \frac{2\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T(-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha T(T-\gamma)^{\beta+1}}{\beta+1}\right) \\ + \lambda ap^{-b}\left(\frac{T^3}{6} + \frac{\alpha T^2(T-\gamma)^{\beta+1}}{2(\beta+1)} - \frac{3\alpha T(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3\alpha(T-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}\right) \end{array}\right] \quad (10)$$

Now profit can be written as a function of the cycle length and price as given below

$$\pi(T, p) = pD(p) - C^*(T, p)$$

$$= ap^{1-b} - \frac{1}{T}\left[\begin{array}{l} A + cap^{-b}(1-\alpha(-\gamma)^{\beta})\left(T + \frac{\alpha(T-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1}\right) \\ + hap^{-b}\left(\frac{T^2}{2} - \frac{2\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T(-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha T(T-\gamma)^{\beta+1}}{\beta+1}\right) \\ + \lambda ap^{-b}\left(\frac{T^3}{6} + \frac{\alpha T^2(T-\gamma)^{\beta+1}}{2(\beta+1)} - \frac{3\alpha T(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3\alpha(T-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}\right) \end{array}\right] \quad (11)$$

Our aim is to determine the values of T and p that minimize the $C^*(T, p)$ and maximize the $\pi(T, p)$

Now $\frac{\partial C^*(T,p)}{\partial T} = 0$

$$\frac{\partial C^*(T,p)}{\partial T} = \frac{1}{T}\left[\begin{array}{l} cap^{-b}(1-\alpha(-\gamma)^{\beta})\left(1 + \alpha(T-\gamma)^{\beta}\right) \\ + hap^{-b}\left(T - \frac{2\alpha(T-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha(T-\gamma)^{\beta+1}}{\beta+1} + \alpha T(T-\gamma)^{\beta}\right) \\ + \lambda ap^{-b}\left(\frac{T^2}{2} + \frac{\alpha T(T-\gamma)^{\beta+1}}{(\beta+1)} + \frac{\alpha T^2(T-\gamma)^{\beta}}{2} - \frac{3\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{3\alpha T(T-\gamma)^{\beta+1}}{\beta+1} + \frac{3\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right) - C^*(T, p) \end{array}\right] \quad (12)$$

&

$$\frac{\partial \pi(T, p)}{\partial p} = 0$$

$$\frac{\partial \pi(T,\ p)}{\partial p} = a(1-b)\,p^{-b} + \frac{1}{T}\left[\begin{array}{l} cabp^{-b-1}(1-\alpha(-\gamma)^{\beta})\left(T + \frac{\alpha(T-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1}\right) \\ + habp^{-b-1}\left(\frac{T^2}{2} - \frac{2\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T(-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha T(T-\gamma)^{\beta+1}}{\beta+1}\right) \\ + \lambda abp^{-b-1}\left(\frac{T^3}{6} + \frac{\alpha T^2(T-\gamma)^{\beta+1}}{2(\beta+1)} - \frac{3\alpha T(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3\alpha(T-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}\right) \end{array}\right]$$

(13)

Also

$$\frac{\partial C^*(T,\ p)}{\partial p} = 0$$

$$\frac{\partial C^*(T,\ p)}{\partial p} = -\frac{1}{T}\left[\begin{array}{l} cabp^{-b-1}(1-\alpha(-\gamma)^{\beta})\left(T + \frac{\alpha(T-\gamma)^{\beta+1}}{\beta+1} - \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1}\right) \\ + habp^{-b-1}\left(\frac{T^2}{2} - \frac{2\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T(-\gamma)^{\beta+1}}{\beta+1} + \frac{\alpha T(T-\gamma)^{\beta+1}}{\beta+1}\right) \\ + \lambda abp^{-b-1}\left(\frac{T^3}{6} + \frac{\alpha T^2(T-\gamma)^{\beta+1}}{2(\beta+1)} - \frac{3\alpha T(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3\alpha(T-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3\alpha(-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)}\right) \end{array}\right]$$

(14)

Solution of equations (12) and (13) are obtained by using Mathmatica 10.0 software to find the optimal cycle length T* and the optimal selling price p* when the values of parameters a, b, c, h, $\lambda,\ \alpha,\ \beta,\ \gamma$, A are known.

For convexity of $C^*(T,\ p)$, we must have

$$\begin{vmatrix} \frac{\partial^2 C^*(T,\ p)}{\partial T^2} & \frac{\partial^2 C^*(T,\ p)}{\partial T \partial p} \\ \frac{\partial^2 C^*(T,\ p)}{\partial p \partial T} & \frac{\partial^2 C^*(T,\ p)}{\partial p^2} \end{vmatrix} > 0$$

(15)

IV. Numerical Illustrations & Sensitivity Analysis

This section presents and solves three numerical illustrations to explain how the solution procedure works in support of derived model.

The items of the product like food stuffs, photographic films and electronic devices for which demand falls with increase of its cost, that is $D(p) = ap^{-b}$ where p is the selling price per unit item and a and b are arbitrary constants satisfying $a > 0$ and $b > 0$. Consider the inventory model with the following values of system parameters:

Illustration 1: $2 > \beta$ and $\gamma > 0$

Let a=100000, b=2.05, c=20, h=4, λ=0.2, A=80, α=0.4, β=8, γ =0.2. The optimum solution for T and p of equation (12) are T=0.853778 and p=42.4797. Putting the solution for T and p of equation (12) in equation (10) and (11), $C^*(T,\ p)$=1093.34 and $\pi(T,\ p)$ = 858.34

Illustration 2: $\beta < 1$ and $\gamma < 0$

Let a=100000, b=2.05, c=20, h=4, λ=0.2, A=80, α=0.4, β=0.7, γ = -4. The optimum solution for T and p of equation (12) are T=12.1150 and p=21.1016. Putting the solution for T and p of equation (12) in equation (10) and (11), $C^*(T,\ p)$=2090.63 and $\pi(T,\ p)$ = 1978.19

Illustration 3: $2 > \beta > 1$ and $\gamma < 0$

Let a=100000, b=2.05, c=20, h=4, λ=0.2, A=80, α=0.4, β=1.8, γ =-0.1. The optimum solution for T and p of equation (12) are T=0.550518 and p=44.1007. Putting the solution for T and p of equation (12) in equation (10) and (11), $C^*(T,\ p)$=1106.41 and $\pi(T,\ p)$ = 770.02

Illustration 4: $2 > \beta > 1$ and $\gamma = 0$

Let a=100000, b=2.05, c=20, h=4, λ=0.2, A=80, α=0.4, β=1.8, γ =0. The optimum solution for T and p of equation (12) are T=0.57078 and p=43.4397. Putting the solution for T and p of equation (12) in equation (10) and (11), $C^*(T,\ p)$=1116.62 and $\pi(T,\ p)$ = 789.80

Illustration 5: $2 > \beta > 1$ and $\gamma > 0$

Let a=100000, b=2.05, c=20, h=4, λ=0.2, A=80, α=0.4, β=1, γ =0.1. The optimum solution for T and p of equation (12) are T=27.4500 and p=412.4938. Putting the solution for T and p of equation (12) in equation (10) and (11), $C^*(T,\ p)$=94.7999 and $\pi(T,\ p)$ = 84.5956

Next, the effects of changes of model parameters are studied on the minimum average total cost per unit time and optimum profit. The sensitivity analysis is performed by changing each of parameters by +50%, +25%, -25% and -50% taking one parameter at a time while considering remaining parameters unchanged. The results are used in numerical illustration 1 and listed in table 2.

Table 2. Sensitivity analysis by changing model parameters

Parameters	% change in parameter	T	p	$C^*(T,\ p)$	Relative Change of Error (%) For $C^*(T,\ p)$	$\pi(T,\ p)$	Relative Change of Error (%) For $\pi(T,\ p)$
a	+50	0.730337	41.943	1629.16	49.0076	1337. 72	55.9114
	+25	0.788908	42.1899	1359.97	24.3867	1097.23	27.8316
	-25	0.924328	42.8318	829.812	-24.1030	621.319	-27.6139
	-50	1.0052	43.3262	569.159	-47.9431	386.674	-54.9510
b	+50	1.43817	41.7934	84.8256	-92.2416	-41.5541	-104.8412
	+25	1.13657	37.4471	282.561	-74.1562	65.4031	-92.3803
	-25	0.461206	59.8681	4048.9	270.3240	7036.66	719.7987
	-50	0.633744	872.55	2185.42	99.8848	8224.14	858.1448
c	+50	0.990718	62.8571	743.215	-32.0234	550.17	-35.9030
	+25	0.940561	52.7141	881.972	-19.3323	673.911	-21.4867
	-25	0.685135	31.9948	1462.94	33.8047	1165.31	35.7632
	-50	0.456837	21.3212	2236.62	104.5677	1788.21	108.3335
h	+50	0.760958	43.5573	1078.82	-1.3280	822.194	-4.2112
	+25	0.806533	43.0546	1102.87	0.8716	821.464	-4.2962
	-25	0.899443	41.8206	1105.14	1.0793	878.858	2.3904
	-50	0.94142	41.0783	1120.46	2.4805	901.196	4.9929
λ	+50	0.850832	42.4896	1093.42	0.0073	857.783	-0.0649
	+25	0.852304	42.4847	1093.38	0.0037	858.061	-0.0325
	-25	0.855213	42.4746	1093.31	-0.0027	858.62	0.0326
	-50	0.856731	42.4696	1093.27	-0.0064	858.902	0.0655
A	+50	0.949628	42.9731	1113.96	1.8860	819.201	-4.5598
	+25	0.909666	42.7541	1102.84	0.8689	835.695	-2.6382
	-25	0.768605	42.1031	1087.1	-0.5707	882.92	2.8637
	-50	0.635758	41.5581	1085.85	-0.6851	911.305	6.1706
α	+50	0.838117	42.4283	1096.37	0.2771	857.801	-0.0628
	+25	0.845328	42.4519	1094.97	0.1491	858.057	-0.0330
	-25	0.863995	42.5133	1091.41	-0.1765	858.658	0.0370
	-50	0.876948	42.5559	1088.99	-0.3979	859.025	0.0798
β	+50	0.895386	42.6099	1085.78	-0.6915	859.639	0.1513
	+25	0.878695	42.5552	1088.83	-0.4125	859.222	0.1028
	-25	0.81591	42.3802	1100.16	0.6238	856.339	-0.2331
	-50	0.755085	42.2522	1111.24	1.6372	851.475	-0.7998
γ	+50	0.891276	42.5984	1086.48	-0.6274	859.496	0.1347
	+25	0.874346	42.5442	1089.55	-0.3466	859.028	0.0802
	-25	0.830107	42.4081	1097.79	0.4070	857.356	-0.1146
	-50	0.803891	42.3325	1102.81	0.8662	856.0	-0.2726

On the basis of the results shown in above table, the following observations can be made.

- $C^*(T, p)$ and $\pi(T, p)$ are moderately sensitive to changes in a.
- $C^*(T, p)$ and $\pi(T, p)$ are highly sensitive to changes in b.
- $C^*(T, p)$ and $\pi(T, p)$ are moderately sensitive to changes in c.
- $C^*(T, p)$ and $\pi(T, p)$ are almost sensitive to changes in h.
- $C^*(T, p)$ and $\pi(T, p)$ are all insensitive to changes in the parameter λ.
- $C^*(T, p)$ and $\pi(T, p)$ are almost sensitive to changes in A.
- $C^*(T, p)$ has low sensitivity and $\pi(T, p)$ is insensitive to changes in α.
- $C^*(T, p)$ and $\pi(T, p)$ are all slightly sensitive to changes in the parameter β and γ

The observation from above example is that the total cost function is strictly convex. Thus, the optimal value of T can be obtained with the help of total cost function of the model where the total cost per unit time of the inventory system is minimum.

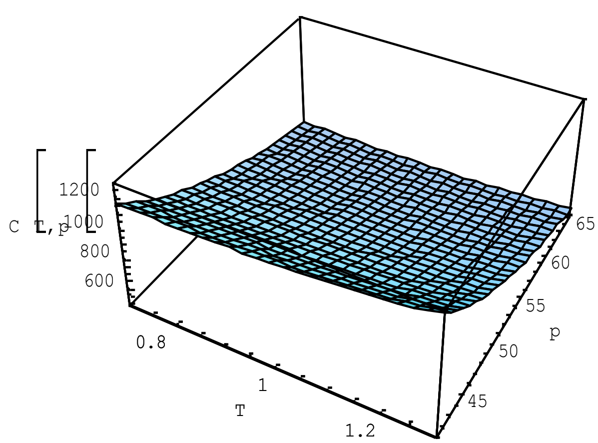

Figure 2: $C^*(T, p)$ vs. T and p

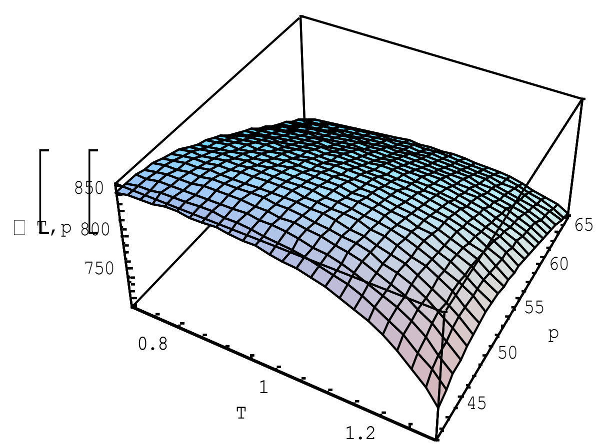

Figure 3: $\pi(T, p)$ vs. T and

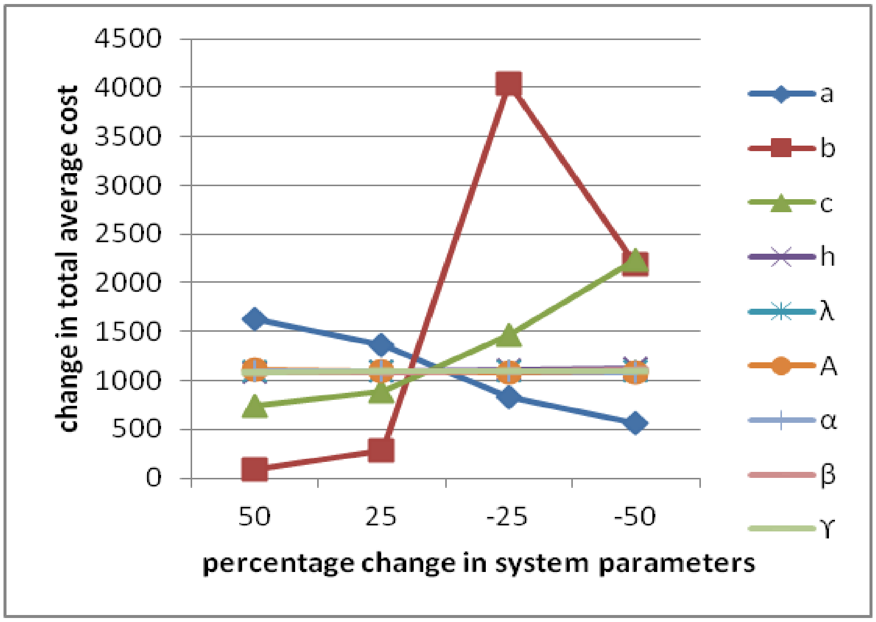

Figure 4: Change of average cost with the change of parameters

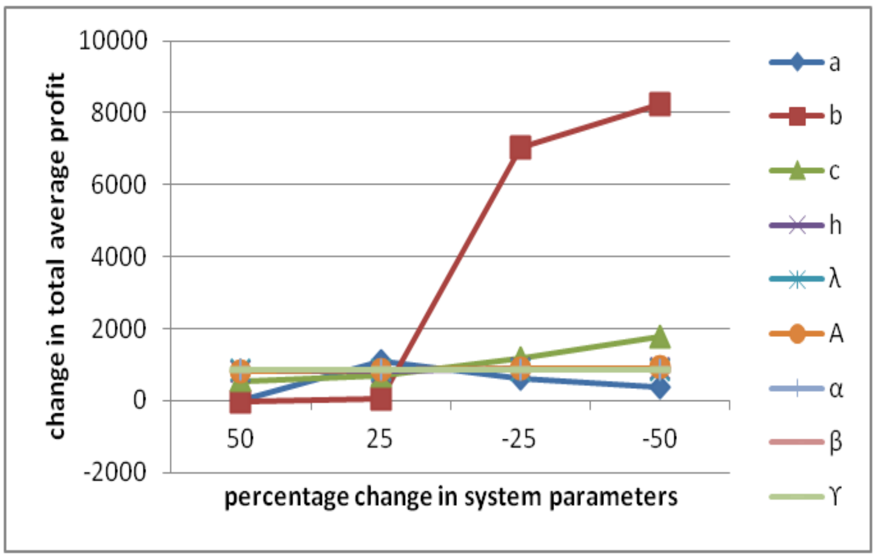

Figure 5: Change of average profit with the change of parameters

V. CONCLUSION

The classical economic order quantity model assumes a pre-determined constant demand rate and no effects on deterioration of items. In reality, not only demand varies with time, but also variable costs are affected by demand of product. In the proposed model, it represents a deterministic inventory model for deteriorating inventory model with three parameter weibull distribution deterioration rate and demand rate is function of selling price. The holding cost is time varying and linear function of time.

Furthermore, three numerical illustrations are provided in support of theory. Sensitivity analysis for parameters is discussed to assess the effects of optimum solutions with the change of parameters. The derived model is suitable for items with any initial value of the rate of deterioration and also which start deteriorating only after certain period of time. In real situation, almost demands of frequently used products are dependent on selling price for which this model is quite applicable.

A future research may be considered to extend the model under stochastic demand, random or non-instantaneous deterioration and credit policy.

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