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On Neutrosophic Semi-preopen Sets and Semi-preclosed Sets in a Neutrosophic Topological Space

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Abstract - In this paper we introduce the concept of Neutrosophic semi-preopen sets and Neutrosophic semi-preclosed sets in Neutrosophic topological spaces. After giving the fundamental definitions of neutrosophic set, operations on a neutrosophic set and neutrosophic topology, we introduce Neutrosophic semi-preopen sets and Neutrosophic semi-preclosed sets and some of their properties are derived.

Keywords: *Neutrosophic set, Neutrosophic topology, Neutrosophic semi-preopen sets, Neutrosophic semi-preclosed sets.*

I. INTRODUCTION

The concept of Fuzzy sets was introduced by Zadeh [10] in 1965 where each element had a degree of membership. Later in 1986, K. Atanassov [1,2,3] introduced the concept of Intuitionistic Fuzzy Set. It is a generalization of Fuzzy set where, besides the degree of membership, a degree of non-membership was also assigned to each element. The Neutrosophic set was introduced by Florentin Smarandache [4,5] as a generalization of Intuitionistic Fuzzy set. Later, A.A.Salama and S.A.Albowi [7,8] introduced Neutrosophic topological spaces.

This paper is organized as follows. The section 1 consists of some basic definitions and properties which are used in later sections. In section 2, we define Neutrosophic Semi-preopen sets and some of their properties are studied. The section 3 deals with the definition and properties of Neutrosophic semi- preclosed sets.

II. PRELIMINARIES

In this section the basic definitions for Neutrosophic sets and its operations are given.

Definition 2.1 [8] Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle ; x \in X \}$ where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set A.

Remark 2.2 [8] A Neutrosophic set $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle\}$ can be identified to an ordered triplet $\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ in $]^-0,1^+[$ on X.

Remark 2.3 [8] For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ for the Neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle ; x \in X \}$.

Remark 2.4 [8] Every intuitionistic fuzzy set (IFS) A of a non empty set X is a Neutrosophic set (NS) having the form $A = \{ \langle x, \mu_A(x), 1 - (\mu_A(x) + \gamma_A(x)), \gamma_A(x) \rangle ; x \in X \}$.

In neutrosophic set theory, the empty set 0_N and full set 1_N are defined as follows.

0_N may be defined as:

(0_1) $0_N = \{ \langle x,\ 0,0,1\rangle : x \in X \}$
(0_2) $0_N = \{ \langle x,\ 0,1,1\rangle : x \in X \}$
(0_3) $0_N = \{ \langle x,\ 0,1,0\rangle : x \in X \}$
(0_4) $0_N = \{ \langle x,\ 0,0,0\rangle : x \in X \}$

1_N may be defined as:

(1_1) $1_N = \{ \langle x,\ 1,0,0\rangle : x \in X \}$
(1_2) $1_N = \{ \langle x,\ 1,0,1\rangle : x \in X \}$
(1_3) $1_N = \{ \langle x,\ 1,1,0\rangle : x \in X \}$
(1_4) $1_N = \{ \langle x,\ 1,1,1\rangle : x \in X \}$

Definition 2.5 [8] The complement of a NS $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ on X denoted by $C(A)$ can be defined by any one of the following ways:

(C_1) $C(A) = \{ \langle x,\ 1-\mu_A(x), 1-\sigma_A(x), 1-\gamma_A(x) \rangle : x \in X \}$
(C_2) $C(A) = \{ \langle x,\ \gamma_A(x),\ \sigma_A(x),\ \mu_A(x)\rangle : x \in X \}$
(C_3) $C(A) = \{\langle x, \gamma_A(x), 1-\sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Definition2.6 [8] Let $A = \{\langle x,\ \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \epsilon X\}$ and $B = \{\langle x,\ \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \epsilon X\}$ be two neutrosophic sets of a non empty set X. Then we may consider two possible definitions for subsets ($A \subseteq B$)

A ⊆ B may be defined by any one of the following ways:

(1) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)\ \forall\ x \in X$
(2) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)\ \forall\ x \in X$

Remark 2.7 [8] For any neutrosophic set A, the following conditions holds:

(1) $0_N \subseteq A,\ 0_N \subseteq 0_N$
(2) $A \subseteq 1_N,\ 1_N \subseteq 1_N$

Definition 2.8 [8] Let X be any nonempty set, and $A = \{\langle x,\ \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \epsilon X\}$ and $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ be two NSs on X. Then,

(1) $A \cap B$ may be defined by any one of the following ways:

$(a) A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), and\ \gamma_A(x) \vee \gamma_B(x)\rangle$
$(b) A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), and\ \gamma_A(x) \vee \gamma_B(x)\rangle$

(2) $A \cup B$ may be defined by any one of the following ways:

$(a)\ A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), and\ \gamma_A(x) \wedge \gamma_B(x)\rangle$

$(b)\ A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), and\ \gamma_A(x) \wedge \gamma_B(x)\rangle$

The operations of intersection and union can be generalized to arbitrary family of NSs as follows:

Definition 2.9 [8]**:** Let $\{A_i : i \in I\}$ be an arbitrary family of NSs in X. Then,

(1) $\cap_{i\epsilon I} A_i$ may be defined by any one of the following ways

$(a)\ \cap_{i\epsilon I} A_i = \langle x, \wedge_{i\epsilon I} \mu_{A_i}(x), \wedge_{i\epsilon I} \sigma_{A_i}(x), \vee_{i\epsilon I} \gamma_{A_i}(x) \rangle$

$(b)\ \cap_{i\epsilon I} A_i = \langle x, \wedge_{i\epsilon I} \mu_{A_i}(x), \vee_{i\epsilon I} \sigma_{A_i}(x), \vee_{i\epsilon I} \gamma_{A_i}(x) \rangle$

(2) $\cup_{i\epsilon I} A_i$ may be defined by any of the following ways

$(a)\ \cup_{i\epsilon I} A_i = \langle x, \vee_{i\epsilon I} \mu_{A_i}(x), \vee_{i\epsilon I} \sigma_{A_i}(x), \wedge_{i\epsilon I} \gamma_{A_i}(x) \rangle$

$(b)\ \cup_{i\epsilon I} A_i = \langle x, \vee_{i\epsilon I} \mu_{A_i}(x), \wedge_{i\epsilon I} \sigma_{A_i}(x), \wedge_{i\epsilon I} \gamma_{A_i}(x) \rangle$

Remark 2.10 [8] The following conditions are satisfied by any two Neutrosophic sets A and B

(1) $C(A \cap B) = C(A) \cup C(B)$
(2) $C(A \cup B) = C(A) \cap C(B)$

Definition 2.11 [8] A neutrosophic topology [NT] on a nonempty set X is a family τ of neutrosophic subsets in X satisfying the following axioms.

(a) $0_N, 1_N \ \epsilon\ \tau$, (b) $G_1 \cap G_2 \ \epsilon\ \tau$ for any $G_1, G_2 \ \epsilon\ \tau$, (c) $\cup G_i \ \epsilon\ \tau$ for every $\{G_i : i \in J\} \subseteq \tau$

Then the pair (X, τ) is called a neutrosophic topological space [NTS].

Definition 2.12 [8]**:** Suppose τ is a neutrosophic topology on a non empty set X. Then elements of τ are called neutrosophic open sets [NOS] and the complement of a NOS is called a neutrosophic closed set [NCS].

Example 2.13: Let $X = \{x\}$ and $A = \{\langle x, 0.3, 0.7, 0.8\rangle : x \epsilon X\}$ $B = \{\langle x, 0.5. 0.6, 0.3\rangle : x \epsilon X\}$ $C = \{\langle x, 0.5. 0.7, 0.3\rangle : x \epsilon X\}$ $D = \{\langle x, 0.3. 0.6, 0.8\rangle :$

$x \epsilon X\}$ be the neutrosophic subsets of X.

Then, the family $\tau = \{0_N, A, B, C, D, 1_N\}$ of Neutrosophic sets in X is a neutrosophic topology on X.

Definition 2.14 [8]: Suppose (X, τ) is a NTS and $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ is a NS in X. Then the neutrosophic closure and neutrosophic interior of A is defined as follows

$NCl(A) = \cap \{K : K \text{ is a } NCS \text{ in } X \text{ and } A \subseteq K\}$.
$NInt(A) = \cup \{G : G \text{ is } NOS \text{ in } X \text{ and } G \subseteq A\}$.
$NCl(A)$ is a NCS and $NInt(A)$ is a NOS in X.

Remark 2.15 [8] For any neutrosophic set A in a NTS(X, τ), we have

$(a)\ NCl(C(A)) = C(NInt(A))$,
$(b)\ NInt(C(A)) = C(NCl(A))$.

Proposition 2.16 [8] Let (X, τ) be a NTS and A, B be two NSs in X. Then the following conditions hold:

$(a)\ NInt(A) \subseteq A$, (b) $A \subseteq NCl(A)$, (c) $A \subseteq B \Rightarrow NInt(A) \subseteq NInt(B)$, (d) $A \subseteq B \Rightarrow NCl(A) \subseteq NCl(B)$, (e) $NInt(NInt(A)) = NInt(A)$, (f) $NCl(NCl(A)) = NCl(A)$, (g) $NInt(A \cap B) = NInt(A) \cap NInt(B)$, (h) $NCl(A \cup B) = NCl(A) \cup NCl(B)$, (i) $NInt(0_N) = 0_N$, (j) $NInt(1_N) = 1_N$, (k) $NCl(0_N) = 0_N$, (l) $NCl(1_N) = 1_N$, (m) $A \subseteq B \Rightarrow C(B) \subseteq C(A)$, (n) $NCl(A \cap B) \subseteq NCl(A) \cap NCl(B)$, (o) $NInt(A \cup B) \supseteq NInt(A) \cup NInt(B)$.

Definition 2.17: [6,9] Let X be a NTS and A be a NS of X. Then,

(a) A is said to be a neutrosophic semi open [NSO] set if and only if $A \subseteq NCl(NInt(A))$

(b) A is said to be neutrosophic semi closed [NSC] set if and only if $NInt(NCl(A)) \subseteq A$

(c) A is said to be a neutrosophic preopen [NPO] set if and only if $A \subseteq NInt(NCl(A))$

(d) A is said to be a neutrosophic preclosed [NPC] set if and only if $NCl(NInt(A)) \subseteq A$

III. NEUTROSOPHIC SEMI-PREOPEN SET IN A NEUTROSOPHIC TOPOLOGICAL SPACE.

In this section, the neutrosophic semi-preopen set is introduced and their properties are studied.

Definition 3.1: Let X be a NTS and A be a NS in X. Then, A is said to be a neutrosophic semi-preopen [NSPO] set of X if and only if $A \subseteq NCl\left(NInt(NCl(A))\right)$.

Theorem 3.2 Let (X, τ) be a NTS. Then the union of any two NSPO sets in X is also a NSPO set in X.

Proof: Let A and B be any two NSPO sets of X.
Then, $A \subseteq NCl\left(NInt(NCl(A))\right)$ and $B \subseteq NCl\left(NInt(NCl(B))\right)$ $\Rightarrow A \cup B \subseteq NCl\left(NInt(NCl(A))\right) \cup NCl\left(NInt(NCl(B))\right)$

$= NCl(NInt(NCl(A)) \cup NInt(NCl(B)))$ $[by\ proposition\ 1.16\ (h)]$
$\subseteq NCl(NInt(NCl(A) \cup NCl(B)))$ $[by\ proposition\ 1.16\ (o)]$
$= NCl(NInt(NCl(A \cup B)))$ $[by\ proposition\ 1.16\ (h)]$
$\therefore A \cup B \subseteq NCl(NInt(NCl(A \cup B)))$.
Therefore $A \cup B$ is a NSPO set in X.

Corollary 3.3 Let (X, τ) be a NTS. If $\{A_\alpha\}_{\alpha \epsilon \Delta}$ is a collection of NSPO sets in X. Then, $\cup_{\alpha \epsilon \Delta} A_\alpha$ is a NSPO set in X.

Proof: Let A_α be a NSPO set for each $\alpha \epsilon \Delta$. Then for each $\alpha \epsilon \Delta$, $A_\alpha \subseteq NCl\left(NInt(NCl(A_\alpha))\right) \Rightarrow \cup_{\alpha \epsilon \Delta} A_\alpha \subseteq \cup_{\alpha \epsilon \Delta} NCl\left(NInt(NCl(A_\alpha))\right) = NCl\left(\cup_{\alpha \epsilon \Delta} NInt(NCl(A_\alpha))\right)$ $[by\ proposition\ 1.16\ (h)]$

$\subseteq NCl\left(NInt(\cup_{\alpha \epsilon \Delta \Delta}\ NCl(A_\alpha))\right)$ $[by\ proposition\ 1.16\ (o)]$

$= NCl\left(NInt(Ncl(\cup_{\alpha \epsilon \Delta} A_\alpha))\right)$

$[by\ proposition\ 1.16\ (h)]$

$\therefore \cup_{\alpha\epsilon\Delta} A_\alpha \subseteq NCl\left(NInt\left(Ncl(\cup_{\alpha\epsilon\Delta} A_\alpha)\right)\right)$.

Therefore, $\cup_{\alpha\epsilon\Delta} A_\alpha$ is a NSPO set in X.

Remark 3.4: The intersection of any two NSPO sets of a neutrosophic topological space need not be a NSPO set as shown in the following example.

Example 3.5: Let $X = \{a, b\}$ and
$A = \{\langle 0.3, 0.2, 0.7 \rangle \langle 0.4, 0.1, 0.5 \rangle\}$
$B = \{\langle 0.2, 0.3, 0.4 \rangle \langle 0.5, 0.4, 0.5 \rangle\}$
$C = \{\langle 0.3, 0.3, 0.4 \rangle \langle 0.5, 0.4, 0.5 \rangle\}$
$D = \{\langle 0.2, 0.2, 0.7 \rangle \langle 0.4, 0.1, 0.5 \rangle\}$
Then $\tau = \{0_N, A, B, C, D, 1_N\}$ is a NTS on X. Let
$A_1 = \{\langle 0.8, 0.1, 0.5 \rangle \langle 0.4, 0.2, 0.7 \rangle\}$ and
$A_2 = \{\langle 0.5, 0.2, 0.3 \rangle \langle 0.6, 0.5, 0.3 \rangle\}$.
Here, $NCl\left(NInt\left(NInt(A_1)\right)\right) = 1_N$
and $NCl\left(NInt\left(NCl(A_2)\right)\right) = 1_N$.
Therefore, A_1 and A_2 are NSPO sets of X.
But $A_1 \cap A_2 = \{\langle 0.5, 0.1, 0.5\rangle\langle 0.4,0.2,0.7\rangle\}$ is not a NSPO set in X.

Theorem 3.6: Let A be a NSPO set in a NTS X and suppose $A \subseteq B \subseteq NCl\ (A)$. Then B is a NSPO set in X.

Proof: A is a NSPO set.

$\therefore A \subseteq NCl\left(NInt\left(NCl(A)\right)\right)$

$\Rightarrow NCl(A) \subseteq NCl\left(NCl\left(NInt\left(NCl(A)\right)\right)\right)$

$= NCl\left(NInt\left(NCl(A)\right)\right)$

$[by\ proposition\ 1.16\ (f)]$

$\therefore NCl(A) \subseteq NCl\left(NInt\left(NCl(A)\right)\right)$

Given, $A \subseteq B \subseteq NCl\ (A)$. Hence it follows that

$B \subseteq NCl\left(NInt\left(NCl(A)\right)\right)$.

We have $A \subseteq B$

$\therefore NCl\left(NInt\left(NCl(A)\right)\right) \subseteq NCl\left(NInt\left(NCl(B)\right)\right)$

$[by\ proposition\ 1.16\ (c) and 1.16\ (d)]$

$\therefore B \subseteq NCl\left(NInt\left(NCl(B)\right)\right)$. Hence B is a NSPO set.

Theorem 3.7: Every NPO set in a NTS X is a NSPO set.

Proof: Let A be a NPO set. Then $A \subseteq NInt\left(NCl(A)\right)$ $\therefore$ $NCl(A) \subseteq NCl\left(NInt\left(NCl(A)\right)\right)$

$[by\ proposition\ 1.16\ (d)]$ We have $A \subseteq NCl(A)$ Therefore, $A \subseteq NCl\left(NInt\left(NCl(A)\right)\right)$ Hence A is a NSPO set in X.

Remark 3.8: Converse of the above theorem need not be true as shown in the example below.

Example 3.9: Let $X = \{a, b\}$ and
$A = \{\langle 0.3, 0.2, 0.7 \rangle \langle 0.4, 0.1, 0.5 \rangle\}$
$B = \{\langle 0.2, 0.3, 0.4 \rangle \langle 0.5, 0.4, 0.5 \rangle\}$
$C = \{\langle 0.3, 0.3, 0.4 \rangle \langle 0.5, 0.4, 0.5 \rangle\}$
$D = \{\langle 0.2, 0.2, 0.7 \rangle \langle 0.4, 0.1, 0.5 \rangle\}$
Then $\tau = \{0_N, A, B, C, D, 1_N\}$ is a NTS on X. Let
$P = \{\langle 0.3,0.5,0.4\rangle \langle 0.3,0.4,0.6\rangle\}$
Then, P is a NSPO set in X but not a NPO set.

Theorem 3.10: Every NSO set in a NTS X is a NSPO set in X.

Proof: Suppose A is a NSO set.
Then, $A \subseteq NCl\left(NInt(A)\right)$
we have $A \subseteq NCl(A)$

$\therefore NCl\left(NInt(A)\right) \subseteq NCl\left(NInt\left(NCl(A)\right)\right)$

$[by\ proposition\ 1.16\ (c) and 1.16 (d)]$

Hence it follows that $A \subseteq NCl\left(NInt\left(NCl(A)\right)\right)$

$\therefore A$ is a NSPO set.

Remark 3.11: Converse of the above theorem need not be true as shown in the following example:

Example 3.12: Let $X = \{a, b\}$ with
$A = \{\langle 0.3,0.5,0.4\rangle \langle 0.6,0.2,0.5\rangle\}$
$B = \{\langle 0.2,0.6,0.7\rangle \langle 0.5,0.3,0.1\rangle\}$
$C = \{\langle 0.3,0.6,0.4\rangle \langle 0.6,0.3,0.1\rangle\}$
$D = \{\langle 0.2,0.5,0.7 \rangle\langle 0.5,0.2,0.5\rangle\}$ $\tau = \{0_N, A, B, C, D, 1_N\}$ is a neutrosophic topology on X.

Then, $P = \{\langle 0.4,0.6,0.4\rangle \langle 0.5,0.3,0.4\rangle\}$ is a NSPO set but not a NSO set.

Theorem 3.13: Every NOS in a NTS X is a NSPO set in X.

Proof: Suppose A is a NOS in X. As every NOS is a NSO set, from theorem 3.10 it easily follows that A is a NSPO set.

Remark 3.14: Converse of the above theorem need not be true as shown in the example below:

Example 3.15: Let $X = \{a, b, c\}$ with
$A = \{ \langle 0.4,0.5,0.2\rangle \langle 0.3,0.2,0.1\rangle \langle 0.9,0.6,0.8\rangle \}$
$B = \{ \langle 0.2,0.4,0.5\rangle \langle 0.1,0.1,0.2\rangle \langle 0.6,0.5,0.8\rangle \}$ $\tau = \{0_N, A, B, 1_N \}$ is a neutrosophic topology on X. Then,
$C = \{ \langle 0.5,0.6,0.1\rangle \langle 0.4,0.3,0.1\rangle \langle 0.9,0.8,0.5\rangle \}$ is a NSPO set but not a NSO set.

IV. NEUTROSOPHIC SEMI-PRECLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES.

In this section we introduce the concepts of the neutrosophic semi-preclosed sets and some of their properties are studied.

Definition 4.1: A neutrosophic set A of a NTS X is aid to be a neutrosophic semi-preclosed set in X if and only if $NInt\left(NCl\left(NInt(A)\right)\right) \subseteq A$.

Theorem 4.2: Let (X, τ) be a NTS and A be a NS of X. Then, A is a NSPC set if and only if $C(A)$ is a NSPO set.

Proof: Suppose A is a NSPC set in X. Then,
$NInt\left(NCl\left(NInt(A)\right)\right) \subseteq A$ taking compliments on both sides, we get $C(A) \subseteq C\left(NInt\left(NCl\left(NInt(A)\right)\right)\right) = NCl\left(NInt\left(NCl\left(C(A)\right)\right)\right)$
[*by remark* 1.15]
$\therefore C(A) \subseteq NCl\left(NInt\left(NCl\left(C(A)\right)\right)\right)$
hence $C(A)$ is NSPO set in X.
Conversely suppose $C(A)$ is a NSPO set in X. Then,
$C(A) \subseteq NCl\left(NInt\left(NCl\left(C(A)\right)\right)\right)$
taking compliments on both sides, we get
$C\left(NCl\left(NInt\left(NCl\left(C(A)\right)\right)\right)\right) \subseteq A$
$\Longrightarrow \quad NInt\left(NCl\left(NInt(A)\right)\right) \subseteq A$
[*by remark* 1.15]
hence it follows that A is a NSPC set in X.

Theorem 4.3: Let (X, τ) be a NTS. Then, the intersection of any two NSPC sets in X is also a NSPC set.

Proof: Let A and B be two NSPC sets in X. Then,
$NInt\left(NCl\left(NInt(A)\right)\right) \subseteq A$ and
$NInt\left(NCl\left(NInt(B)\right)\right) \subseteq B$
Therefore,
$NInt\left(NCl\left(NInt(A)\right)\right) \cap NInt\left(NCl\left(NInt(B)\right)\right) \subseteq A \cap B$

now by using proposition 1.16 (g) and 1.16 (n), we get
$NInt\left(NCl\left(NInt(A \cap B)\right)\right) \subseteq A \cap B$.
Hence, $A \cap B$ is a NSPC set.

Remark 4.4: The union of two NSPC sets in a NTS X need not be a NSPC as shown in the following example.

Example 4.5: Let $X = \{ a \}$ with
$A = \{ \langle 0.2,0.5,0.3\rangle \}$ $B = \{ \langle 0.1,0.5,0.7\rangle \}$.
Then, $\tau = \{0_N, A, B, 1_N \}$ is a neutrosophic topology on X.
Let $A_1 = \{ \langle 0, 0.5, 0.8\rangle \}$ and $A_2 = \{ \langle 0.1,0.2,0.3\rangle \}$.
$NInt(A_1) = 0_N$ and $NInt(A_2) = 0_N$.
Hence, A_1 and A_2 are NSPC sets in X but $A_1 \cup A_2$ is not a NSPC set.

Theorem 4.6: Every NPC set in a NTS X is a NSPC set.

Proof: Suppose A is a NPC set.
Then, $NCl\left(NInt(A)\right) \subseteq A$.
Now, by using proposition 1.16 (c), we get
$NInt\left(NCl\left(NInt(A)\right)\right) \subseteq NInt(A)$
we have $NInt(A) \subseteq A$.
Hence it follows that $NInt\left(NCl\left(NInt(A)\right)\right) \subseteq A$
Thus we get that A is a NSPC set.

Remark 4.7: The converse of the above theorem need not be true as shown by the following example.

Example 4.8: Let $X = \{ a \}$ with
$A = \{ \langle 0.4,0.5,0.3\rangle \}, B = \{ \langle 0.1,0.5,0.5\rangle \}$.
Then, $\tau = \{0_N, A, B, 1_N \}$ is a neutrosophic topology on X. Let $C = \{ \langle 0.3,0.6,0.5\rangle \}$. Then, C is a NSPC set but not a NPC set.

Theorem 4.9: Every NSC set in a NTS X is a NSPC set.

Proof: Suppose A is a NSC set in X.
Then, $NInt\left(NCl(A)\right) \subseteq A$.
We have $NInt(A) \subseteq A$.
Therefore by using proposition 1.16(c) and 1.16(d), it

follows that

$NInt\left(NCl\left(NInt(A)\right)\right) \subseteq NInt\left(NCl(A)\right)$ Therefore,

$NInt\left(NCl\left(NInt(A)\right)\right) \subseteq A$. Thus we get that A is a NSPC set.

Remark 4.10: The converse of the above theorem need not be true as shown in the example below.

Example 4.11: Let $X = \{ a \}$ with
$A = \{ \langle 0.2,0.5,0.3\rangle \}, B = \{ \langle 0.1,0.5,0.7\rangle \}$.
Then, $\tau = \{0_N, A, B, 1_N \}$ is a neutrosophic topology on X. Let $A_1 = \{ \langle 0, 0.5, 0.8\rangle \}$. A_1 is a NSPC set but not a NSC set.

Theorem 4.12: Every NCS in X is a NSPC set.

Proof: Suppose A is a NCS set in A. As every NCS set is a NSC set, it follows from the above theorem 4.9 that A is a NSPC set

Remark 4.13: The converse of the above theorem need not be true as shown in the following example.

Example 4.14: Let $X = \{a, b, c\}$ with
$A = \{ \langle 0.5,0.6,0.3\rangle \langle 0.1,0.7,0.9\rangle \langle 1,0.6,0.4\rangle \}$
$B = \{ \langle 0,0.4,0.7\rangle \langle 0.1,0.6,0.9\rangle \langle 0.5,0.5,0.8\rangle \}$ $\tau = \{0_N, A, B, 1_N \}$ is a neutrosophic topology on X. Then,
$C = \{ \langle 0.2,0.4,0.9\rangle \langle 0,0.2,0.9\rangle \langle 0.3,0.2,1\rangle \}$ is a NSPC set but not a NCS.

Theorem 4.15: If A is NSPC set in a NTS X and suppose $NInt(A) \subseteq B \subseteq A$, then B is also a NSPC set in X.

Proof: A is a NSPC set. $\therefore$ $NInt\left(NCl\left(NInt(A)\right)\right) \subseteq A$

Then by proposition 1.16 (c) and 1.16 (e) we get

$NInt\left(NCl\left(NInt(A)\right)\right) \subseteq NInt\ (A)$.

Given $NInt\ (A) \subseteq B$.

Hence it follows that $NInt\left(NCl\left(NInt(A)\right)\right) \subseteq B$.

Also, $B \subseteq A$. $\therefore$ by using proposition 1.16 (c) and 1.16 (d)

$NInt\left(NCl\left(NInt(B)\right)\right) \subseteq NInt\left(NCl\left(NInt(A)\right)\right)$

Thus, $NInt\left(NCl\left(NInt(B)\right)\right) \subseteq B$

Hence B is a NSPC set.

V. CONCLUSION

The basic aim of this paper is the introduction of two new sets – the semi-preopen set and the semi-preclosed set in a neutrosophic topological space. Then certain properties of these sets have been studied in detail.

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