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Strong Regularity in Near-Rings

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Abstract— The aim of this paper is to prove some concepts related with strong regularity and strong reducedness in near-rings. We arrive a conclusion that all reduced and regular near-rings are strongly regular and strongly reduced near-rings. Also we discuss the idea of semicentral in near-rings

Keywords— Near-rings, Regular, Reduced, Strongly regular, Strongly reduced, Right semicentral

I. INTRODUCTION

Throughout this paper we work with right near-rings.

Mason[1] introduced the notion of strong regularity of near-rings and characterized left regular zero-symmetric unital near-rings. Reddy and Murty [2] extended the results in [1] to arbitrary near-rings and proved that the concepts of left regularity , left strong regularity and right regularity in near-rings are equivalent and these imply right strong regularity. Yong Uk Cho and Yasuyuki Hirano[3] showed that the strong regularity in near-ring is equivalent property (*) in[2]. Narmada and Anil Kumar[4] characterize the strong regularity of near-rings.

We will use the following notations:

Given a near-ring N , $N_0 = \{ n \; in \; N \colon n0 = 0\}$ which is called the zero-symmetric part of $N, N_c = \{n \; in \; N \colon n0 = n\} = \{ n \; in \; N : nn' = n \; for \; every \; n' \; in \; N\}$ which is called constant part of N .Clearly , N_0 and N_c are subnear-rings of N. A near-ring N is called zero-symmetric if $N = N_0$ and N is called a constant near-ring if $N = N_c$.

For basic concepts and notations we shall refer to Pilz[5].

II. PRELIMINARIES

The concept of homomorphism on near-rings are like on rings. Z_n is an abelian group under addition modulo n. Let N, N' be near-rings, $h : N \to N'$ is called near-ring homomorphism if for every $m, n \; in \; N$ such that $h(m + n) = h(n) + h(m) \; and \; h(mn) = h(m)h(n)$. A near-ring N is called left strongly regular if for every $a \; in \; N$, there exist $x \; in \; N \; such \; that \; a = xa^2$ and left regular if for every $a \; in \; N$, there is an $x \; in \; N \; such \; that \; a = xa^2 \; and \; a = axa$. Right strong regularity and right regularity can be defined in symmetric way. N is strongly regular if it is both left and right strongly regular. We can say that N is reduced if N has no nonzero nilpotent elements, that is , for each $a \; in \; N \; a^n = 0,$ for some positive integer $n \; implies \; a = 0$. In ring theory Mc Coy proved that N is reduced if and only if for each $a \; in \; N \; a^2 = 0 \; implies \; a = 0$. A near-ring N is said to be strongly reduced if for $a \; in \; N, a^2 \; in \; N_c \; implies \; a \; in \; N_c$.

An idempotent $e \; in \; N$ is right semicentral if $en = ene$ for each $n \in N$. Similarly, left semicentral can be defined in a symmetric way. A near-ring in which every idempotent is right semicentral is called right semicentral near-ring. An idempotent $e \; in \; N$ is central if $en = ne$ for each $x \in N$.

III. CHARACTERIZATION OF HOMOMORPHISM ON Z_n ([6]):

Suppose $f \colon Z_m \to Z_n$ is a group homomorphism and assume $f(1) = k$, then for $m \; in \; N \; f(m) = mk \; and \; f(-m) = -mk$. Thus $x \; in \; Z, f(x) = xf(1) = xk$

$f(0) = 0f(1)$. Then for $x \; in \; Z_m, f\ (x) = xk \; for \; some \; k \; in \; Z_n$.

That is $f \colon Z_m \to Z_n$ is a group homomorphism and $f(1) = k$, then the homomorphism has the form $f(x) = xk \; (mod \; n)$.

Conversely, if k is a solution of $mx \equiv 0 (mod \; n) \; then \; f(x) = xk \; (mod \; n)$ is a homomorphism from $Z_m \to Z_n$.

Theorem 3.1 : The function $f \colon Z_m \to Z_n$ given by $f(x) = xk$ for some $k \; in \; Z_n$, fixed is homomorphism of groups if and only if $mk \equiv 0 (mod \; n)$.

Corollary 3.2 : The function $f: Z_m \to Z_n$ is a homomorphism and $f(x) = xk$ where k is the solution the system $mk \equiv 0(mod\ n)$ and $(m,n) = 1$, is an on to homomorphism.

Proposition 3.3 : Suppose $f: Z_m \to Z_n$ is a ring homomorphism and assume $f(1) = k$, since every ring homomorphism is a group homomorphism, $f(x) = xk\ (mod\ n)$ is a ring homomorphism if and only if k is the solution of the system $mk \equiv 0(mod\ n)$. Also $k = f(1) = f(1.1) = [f(1)]^2 = k^2(k\ is\ idempotent)$. That is k is also a solution of the system $x^2 = x(mod\ n)$.

IV. STRONGLY REGULAR NEAR-RINGS

G Pilz[5], gives the description of near-rings of low order and from the above argument of homomorphism on near-rings , [6] gives the idea to construct the near-rings of low order on $Z_n\ , n \leq 7$. We know that every strongly regular near-ring is strongly reduced (Proposition 1 of [3]).

Lemma 4.1 : ([3]) Let N be a strongly reduced near-ring. If for any $, b \in N\ with\ ab \in N_c$, then $ba \in N_c$, and for every $x \in N\ , axa\ , bxa \in N_c$. Furthermore, $b^n \in N_c\ implies\ ab \in N_c$, for each positive integer n .

Theorem 4.2 : If N is strongly regular , then it is strongly reduced and regular.

Proof. By Proposition 1 of [3] , N is strongly reduced. Let $a \in N$. Then, there exists $x \in N$ such that $a = xa^2$. Since $(a - axa)a = 0 \in N_c$, by lemma 4.1, $a(a - axa), axa(a - axa) \in N_c$ .Thus $(a - axa)^2 \in N_c$ and this implies $a - axa \in N_c$. Hence, $0 = (a - axa)a = a - axa$ showing that $= axa$. Thus, N is regular.

Theorem 4.3 : A near-ring N is strongly regular if and only if it is right semicentral and regular .

Proof. Assume that N is strongly regular . Then N is strongly reduced and regular, by Theorem 4.2. Let $e^2 = e$. Now, $(e - ene)e = 0 \in N_c$. Hence, $en(en - ene), ene(en - ene) \in N_c$ by Lemma 4.1. So, $(en - ene)^2 \in N_c$ and this implies $en - ene \in N_c$. Hence, $0 = (en - ene)e = en - ene$, showing that $en = ene$. Thus , N is right semicentral.

Conversely, assume that N is right semicentral and regular. Let $a \in N$. Since N is regular,$a = axa$, for some $x \in N$. Then xa is idempotent. Now, $a = (ax)a = \big(a(xa)x(xa)\big)a = (axa)x^2a^2 = ax^2a^2 = ya^2$, where $y = ax^2 \in N$.

Theorem 4.4 [4] : Let N be a near-ring. Then N is strongly regular if and only if it is strongly reduced and regular.

From $Clay's$ [7] table [6] classifies reduced and regular near-rings of order ≤ 7 , which are strongly regular and strongly reduced near-rings.

Table 1

$Groups$	Zero-symmetric, reduced and regular	Nonzero-symmetric, reduced and regular
Z_2		3
Z_3	3	4
Z_4	8	9
Z_5	7,8,10	9
Z_6	27,47	24,35,48,49,52,53
Z_7	18,20,21,22,23,24	19

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