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A Fixed Point theorem on S_b-metric spaces with a weak S_b-metric

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Abstract— In this paper we define weak S_b-metric on a S_b-metric space which is an extension of weak S-metric on a S-metric space. We establish a fixed point theorem on S_b-metric space with a weak S_b-metric. Examples are given at relevant places. We obtain the result of K.Sarada., V.Jhansi Rani., V.Siva Rama Prasad., as a corollary. We also obtain Banach Contraction principle as a corollary.

Key Words— S-metric space and weak S-metric, S_b-metric space, weak S_b-metric, fixed point.

I. INTRODUCTION

In 2012, S.Sedghi, N.Shobe, A.Aliouchi introduced the notion of S-metric space as a generalization of metric space and proved fixed point theorems [1]. Many researchers studied the properties of S-metric spaces and proved fixed point theorems [2,3,4,5,6,7].

In 2016, S.Sedghi, A.Gholidahneh, T.Dosenovic, J.Esfahani, S.Radenovic introduced the notion of S_b-metric space which is a generalization of S-metric space and proved fixed point theorems for self maps on such spaces [8]. Subsequently many authors worked in this direction and proved fixed point theorems in S_b-metric spaces [9,10,11].

In 2016, K.Sarada, V.Jhansi Rani, V.Siva Rama Prasad introduced the notion of a weak S-metric on a S-metric space (X, S) and proved a fixed point theorem on a S-metric space with a weak S-metric on X [12].

In this paper we introduce weak S_b-metric on a S_b-metric space. We prove a fixed point theorem on S_b-metric space with a weak S_b-metric. Two examples of weak S_b-metric are given provided. We obtain the result of K.Sarada, V.Jhansi Rani, V.Siva Rama Prasad as a corollary of our main result [12]. We also obtain Banach contraction principle as a corollary to our main result.

In this paper, the set of Natural numbers is denoted by $\mathbb{N}$ and the set of Real numbers is denoted by $\mathbb{R}$.

II. PRELIMINARIES

In this section we present the necessary definitions and results which are used either tacitly or explicitly in the next section.

In 2012, S.Sedghi, N.Shobe, A.Aliouchi introduced the notion of a S-metric space as a generalization of metric space [1]

2.1 Definition [1]: Let X be a non-empty set. An S-metric on X is a function $S: X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,
(2.1.1) $S(x, y, z) = 0$ if and only if $x = y = z$
(2.1.2) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$
The function S is called an S- metric on X and the pair (X, S) is called an S-metric space.

2.2 Examples [1]:
2.2.1 Let $X = \mathbb{R}^n$ and $\|.\|$ a norm on X, then
$S(x, y, z) = \|y + z - 2x\| + \|y - z\|$ is an S-metric on X.
2.2.2 Let $X = \mathbb{R}^n$ and $\|.\|$ be a norm on X and define
$S(x, y, z) = \|x - z\| + \|y - z\|$ for all $x, y, z \in X$, then
(X, S) is an S-metric space.
2.2.3 Let X be a non empty set, d be a metric on X then
$S(x, y, z) = d(x, z) + d(y, z)$ is a S-metric on X.

2.3 Observations [1]: Let (X, S) be an S-metric space. Then
2.3.1 $S(x, x, y) = S(y, y, x)$
2.3.2 $S(x, x, z) \leq 2S(x, x, y) + S(z, z, y)$ and
2.3.3 $S(x, y, y) \leq S(x, x, y)$ for all $x, y, z \in X$.

In 2016, S.Sedghi, A.Gholidahneh, T.Dosenovic, J.Esfahani, S.Radenovic introduced the notion of S_b-metric space as follows [8].

2.4 Definition [8]: Let X be a non-empty set and $b \geq 1$ be a real number. Suppose that a mapping $S_b : X^3 \to [0, \infty)$ satisfies the following properties:
(2.4.1) $0 < S_b(x, y, z)$ for all $x, y, z \in X$ with $x \neq y \neq z$

(2.4.2) $S_b(x,y,z)=0$ if and only if $x=y=z$
(2.4.3) $S_b(x,y,z)\le b(S_b(x,x,a)+S_b(y,y,a)+S_b(z,z,a))$ for all $x,y,z,a\in X$.
Then the function S_b is called an S_b-metric on X and the pair (X,S_b) is called an S_b-metric space.

2.5 Remark [8]: It should be noted that the class of S_b-metric spaces is effectively larger than that of S-metric spaces. Indeed each S-metric space is an S_b-metric space with $b=1$.

2.6 Examples [8]:
(2.6.1) Let (X,S) be an S-metric space and $S_*(x,y,z)=S(x,y,z)^p$ where $p>1$ is a real number.
Note that S_* is an S_b-metric with $b=2^{2(p-1)}$.
Also (X,S_*) is not necessarily an S-metric space.
(2.6.2) Let $X=[0,1]$. Define $S: X\times X\times X\to\mathbb{R}^+$ by $S_b(x,y,z)=(|y+z-2x|+|y-z|)^2$, then (X,S_b) is a S_b-metric space for $b\ge 3$.

2.7 Lemma [8]: In an S_b-metric space (X,S_b), we have $S_b(x,x,y)\le bS_b(y,y,x)$ and
$S_b(y,y,x)\le bS_b(x,x,y)$.

2.8 Lemma [8]: In an S_b-metric space (X,S_b), we have
$S_b(x,x,z)\le 2bS_b(x,x,y)+b^2S_b(y,y,z)$.

2.9 Definition [8]: If (X,S_b) is an S_b-metric space, a sequence $\{x_n\}$ in X is said to be S_b- Cauchy sequence, if for each $\epsilon>0$ there exists $n_0\in\mathbb{N}$ such that $S_b(x_n,x_n,x_m)<\epsilon$ for each $m,n\ge n_0$.

2.10 Definition [8]: Let (X,S_b) be an S_b-metric space. A sequence $\{x_n\}$ in X is said to be S_b-convergent to a point $x\in X$ if for each $\epsilon>0$, there exists a positive integer n_0 such that $S_b(x_n,x_n,x)<\epsilon$ or $S_b(x,x,x_n)<\epsilon$ for all $n\ge n_0$ and denote this by $\lim_{n\to\infty}x_n=x$.

2.11 Definition [8]: An S_b-metric space (X,S_b) is called complete if every S_b- Cauchy sequence is S_b-convergent in X.

In 2016, K.Sarada, V.Jhansi Rani, V.Siva Rama Prasad introduced the notion of weak S-metric on S-metric space as follows [12].

2.12 Definition [12]: Suppose (X,S) is a S-metric space. A weak S-metric on X is a function $p: X^3\to[0,\infty)$ satisfying the conditions given below:
(2.12.1) $p(x,y,z)\le p(a,a,x)+p(a,a,y)+p(a,a,z)$ for $x,y,z,a\in X$
(2.12.2) for each $x\in X$, $p(x,x,.): X\to[0,\infty)$ is lower continuous
(2.12.3) to each $\epsilon>0$ there is a $\delta>0$ such that $p(a,a,x)<\delta$, $p(a,a,y)<\delta$ and $p(a,a,z)<\delta$ for some $a\in X$ imply that $S(x,y,z)<\epsilon$.

2.14 Remark [12]: For a weak S-metric p on a S-metric space (X,S), observe that $p(x,y,z)=0$ need not imply $x=y=z$. Therefore $p(x,x,y)$ and $p(y,y,x)$ need not be equal for $x,y\in X$.

2.15 Example [12]: Suppose (X,S) is a S-metric space. Define $p: X^3\to[0,\infty)$ by $p(x,y,z)=S(x,y,z)$ for $x,y,z\in X$, then p is a weak S-metric.

Using the notion of weak S-metric, K.Sarada, V.Jhansi Rani, V.Siva Rama Prasad proved the following theorem as in [12].

2.16 Theorem [12]: Suppose (X,S) is a complete S-metric space with a weak S-metric p on it. Suppose $f: X\to X$ is a continuous function such that $p(fx,fx,fy)\le Lp(x,x,y)$ for all $x,y\in X$ and for some $L\in[0,1)$. Then f has a fixed point $z\in X$. Also if $u\in X$ is another fixed point of f then $p(u,u,z)=0$.
From this result, Banach Contraction principle is obtained as a corollary.

2.17 Corollary [12]: (Banach Contraction principle) If (X,d) is a complete metric space and $f: X\to X$ is a mapping such that $d(fx,fy)\le Ld(x,y)$ for all $x,y\in X$, for some $L\in[0,1)$ then f has a unique fixed point $z\in X$.
Note: Incidentally we observe that continuity is used in theorem 2.16, while we do away with continuity in corollary 3.3.

III. MAIN RESULTS

In this section, we introduce weak S_b-metric on a S_b-metric space and give two examples for weak S_b-metric. We prove a fixed point theorem for weak S_b-metric on complete S_b-metric spaces.

3.1 Definition: Let X be a non empty set. Suppose (X,S_b) is a S_b-metric space. A weak S_b-metric on X is a function $p: X^3\to[0,\infty)$ satisfying the following conditions:
(3.1.1) $p(x,y,z)\le p(a,a,x)+p(a,a,y)+p(a,a,z)$ for $x,y,z,a\in X$
(3.1.2) p is continuous
(3.1.3) For each $\epsilon>0$, there exists $\delta>0$ such that $p(a,a,x)<\delta$, $p(a,a,y)<\delta$ and $p(a,a,z)<\delta$ for some $a\in X\Longrightarrow S_b(x,y,z)<\epsilon$.

Now we give an example of a weak S_b-metric.

3.2 Example: Let $X=[0,1]$. Suppose $S_b: X^3\to[0,\infty)$ is defined by $S_b(x,y,z)=(|y+z-2x|+|y-z|)^2$ for $x,y,z\in X$. Then (X,S_b) is a S_b-metric space with $b\ge 3$.
Define $p: X^3\to[0,\infty)$ by

$p(x,y,z) = |x-y| + |y-z| + |z-x|$**.** Then p is a weak S_b-metric on X.

Proof: Clearly (X, S_b) is a S_b-metric space with $b \geq 3$ (by example (2.6.2)).

We now show that p is a weak S_b-metric on X.

We have, for any $x, y, z, a \in X$,

$$p(a,a,x) + p(a,a,y) + p(a,a,z) = 2|x-a| + 2|y-a| + 2|z-a| = 2(|x-a| + |y-a| + |z-a|) \quad (3.2.1)$$

Also

$$p(x,y,z) = |x-y| + |y-z| + |z-x| = \begin{pmatrix} |x-a+a-y| + |y-a+a-z| + \\ |z-a+a-x| \end{pmatrix} \leq \begin{pmatrix} |x-a| + |a-y| + |y-a| + |a-z| \\ +|z-a| + |a-x| \end{pmatrix} = 2(|x-a| + |y-a| + |z-a|) \quad (3.2.2)$$

∴ From (3.2.1) and (3.2.2) follows that

$p(x,y,z) \leq p(a,a,x) + p(a,a,y) + p(a,a,z)$.

Clearly $p: X^3 \to [0,\infty)$ is continuous.

Let $\epsilon > 0$. Let $0 < \delta < \frac{\sqrt{\epsilon}}{3}$.

Suppose there is a $a \in X$ such that

$p(a,a,x) = 2|a-x| < \delta$

$p(a,a,y) = 2|a-y| < \delta$

$p(a,a,z) = 2|a-z| < \delta$

$\therefore p(a,a,x) + p(a,a,y) + p(a,a,z)$

$= 2(|a-x| + |a-y| + |a-z|)$

$< 3\delta < \sqrt{\epsilon}$

Now

$$S_b(x,y,z) = (|y+z-2x| + |y-z|)^2 = (|(y-x)+(z-x)| + |y-z|)^2 \leq (|(y-x)+(z-x)| + |y-z|)^2 = \begin{pmatrix} |(y-a+a-x)+(z-a+a-x)| + \\ |y-a+a-z| \end{pmatrix}^2 \leq \begin{pmatrix} |y-a| + |a-x| + |z-a| + \\ |a-x| + |y-a| + |a-z| \end{pmatrix}^2 = (2|a-x| + 2|a-y| + 2|a-z|)^2 < (\delta+\delta+\delta)^2 = (3\delta)^2$$

∴ We get $S_b(x,y,z) < \epsilon$

Thus, it follows that p is a weak S_b-metric on X.

In the following example we show that every complete metric space can be regarded as a S_b-metric space with a weak S_b-metric with $b = 1$.

3.3 Example: Suppose (X,d) is a complete metric space. Define $S_b: X^3 \to [0,\infty)$ by $S_b(x,y,z) = d(x,z) + d(y,z)$ for $x,y,z \in X$. Then it can be easily verified that (X,S_b) is a complete S_b-metric space with $b = 1$.

Also define $p: X^3 \to [0,\infty)$ by

$p(x,y,z) = S_b(x,y,z) = d(x,z) + d(y,z)$ for all $x,y,z \in X$. Then p is a weak S_b-metric on (X,S_b), with $b = 1$.

Now we prove a lemma which we use later.

3.4 Lemma: Suppose (X,S_b) is a S_b-metric space and p is a weak S_b-metric on X. Let $\{x_n\}$ and $\{y_n\}$ be sequences in X. Then for $x,y,z \in X$

(3.4.1) $p(x_n,x_n,y) \to 0$ and $p(x_n,x_n,z) \to 0 \Longrightarrow y = z$.

In particular, $p(x,x,y) = p(x,x,z) = 0 \Longrightarrow y = z$

(3.4.2) $p(x_n,x_n,y_n) \to 0$ and $p(x_n,x_n,z) \to 0$ imply that $y_n \to z$ as $n \to \infty$ in (X,S_b)

(3.4.3) $p(x_m,x_m,x_n) \to 0$ for all $m,n \to \infty \Longrightarrow \{x_n\}$ is a Cauchy sequence in (X,S_b)

(3.4.4) $p(y,y,x_n) \to 0$ as $n \to \infty \Longrightarrow \{x_n\}$ is a Cauchy sequence in (X,S_b)

Proof: (3.4.1)

Let $\epsilon > 0$. Take $= \frac{\epsilon}{2}$. Then there exists N, such that

$p(x_n,x_n,y) < \delta$, $p(x_n,x_n,z) < \delta$ for $n \geq N$.

Therefore $p(x_N,x_N,y) < \delta$, $p(x_N,x_N,z) < \delta$

∴ (by (3.1.3)) $S_b(y,z,z) < \epsilon$

This is true for every $\epsilon > 0$

$\therefore S_b(y,z,z) = 0$

Therefore $y = z$.

In particular suppose $p(x,x,y) = 0$ and $p(x,x,z) = 0$.

Take $x_n = x$ for all n.

Then $p(x_n,x_n,y) = 0$, $p(x_n,x_n,z) = 0$

Therefore for any $\epsilon > 0$, by taking $\delta = \epsilon$, follows that

$p(x_n,x_n,y) < \delta$, $p(x_n,x_n,z) < \delta$

Therefore $y = z$.

(3.4.2) We have $p(x_n,x_n,y_n) \to 0$ and $p(x_n,x_n,z) \to 0$

Let $\epsilon > 0$. Take $\delta = \epsilon$. Then there exists N, such that $p(x_N,x_N,y_N) < \delta$ and $p(x_N,x_N,z) < \delta$

$\therefore S_b(y_n,z,z) < \epsilon$.

This is true for every $n \geq N$

$\therefore$ $y_n \to z$ as $\to \infty$.

(3.4.3) We have $p(x_m,x_m,x_n) \to 0$ as $m,n \to \infty$

Let $\epsilon > 0$.

Take $\delta = \epsilon$. Then there exists $N \in \mathbb{N}$, such that

$p(x_m,x_m,x_n) < \delta$ for $m,n \geq N$.

Then $p(x_N,x_N,x_n) < \delta$ and $p(x_N,x_N,x_m) < \delta$

∴ (by (3.1.3)) $S_b(x_m,x_m,x_n) < \epsilon$

This is true, for every $m,n \geq N$.

$\therefore \{x_n\}$ is a Cauchy sequence in X.

(3.4.4) Let $p(y,y,x_n) \to 0$ as $n \to \infty$

$\Longrightarrow \{x_n\}$ is a Cauchy sequence in (X,S_b).

Let $\epsilon > 0$.

Take $\delta = \frac{\epsilon}{3}$. Then there exists N such that $p(y,y,x_n) < \delta$ for all $n \geq N$

For $m,n \geq N$,

$$p(x_m,x_m,x_n) \leq 2\,p(y,y,x_m) + p(y,y,x_n) < 2\delta + \delta = 3\delta = \epsilon$$

Since this is true for each $\epsilon > 0$, it follows that $p(x_m,x_m,x_n) \to 0$

∴ By (3.4.3), we have $\{x_n\}$ is a Cauchy sequence.

Thus, the lemma is established.

Now we need another lemma for subsequent development.

3.5 Lemma: Suppose (X, S_b) is a S_b-metric space, p is a weak S_b-metric on X. Further suppose that $0 \le L < 1$ and $f: X \to X$ satisfies $p(fx, fy, fz) \le Lp(x, y, z)$ for all $x, y, z \in X$.
If u is a fixed point of f, then $p(u, u, u) = 0$.
Proof: $p(u, u, u) = p(fu, fu, fu) \le Lp(u, u, u)$
$\Longrightarrow p(u, u, u) = 0$ (since $0 \le L < 1$).
Now we state and prove our main theorem.

The following theorem establishes the existence of unique fixed point theorem for a function (not necessarily continuous) which is a contraction controlled by a weak S_b-metric.

3.6 Theorem: Suppose (X, S_b) is a complete S_b-metric space with a weak S_b-metric p on it. Suppose $0 \le L < 1$ and $f: X \to X$ is a function such that $p(fx, fx, fy) \le Lp(x, x, y)$ for all $x, y \in X$ (3.6.1)
Then f has unique fixed point $z \in X$.
Proof: Let $x_0 \in X$ and write $x_n = fx_{n-1}$ for $n \ge 1$, so that $\{x_n\} \subseteq X$.
We now prove that $\{x_n\}$ is a Cauchy sequence in (X, S_b).
For any integer $k \ge 0$, let $\alpha_k = p(x_k, x_k, x_k)$, $\beta_k = p(x_k, x_k, x_{k+1})$
and $r_n^{n+k} = p(x_{n+k}, x_{n+k}, x_n)$
We have $\alpha_k = p(x_k, x_k, x_k)$
$= p(fx_{k-1}, fx_{k-1}, fx_{k-1})$
$\le Lp(x_{k-1}, x_{k-1}, x_{k-1}) = L\alpha_{k-1}$ (by (3.6.1))
$\therefore \alpha_k \le L\alpha_{k-1}$
which on repeated use gives
$\alpha_k \le L\alpha_{k-1} \le L^2\alpha_{k-2} \le \cdots \le L^k\alpha_0$
and similarly $\beta_k \le L\beta_{k-1} \le L^2\beta_{k-2} \le \cdots \le L^k\beta_0$ (3.6.2)
Consider $r_n^{n+k} = p(x_{n+k}, x_{n+k}, x_n)$
$\le 2p(x_{n+k-1}, x_{n+k-1}, x_{n+k}) + p(x_{n+k-1}, x_{n+k-1}, x_n)$ (by (3.1.1))
$= 2\beta_{n+k-1} + r_n^{n+k-1}$
$\le 2L^{n+k-1}\beta_0 + r_n^{n+k-1}$ (by (3.6.2))
Hence, by induction, we get

$$r_n^{n+k} \le \begin{pmatrix} 2L^{n+k-1}\beta_0 + 2L^{n+k-2}\beta_0 + 2L^{n+k-3}\beta_0 + \\ \dots + 2L^n\beta_0 + \alpha_n \end{pmatrix}$$

($\because r_n^n = \alpha_n$)
$\le 2L^n\beta_0(L^{k-1} + L^{k-2} + \cdots + L + 1) + L^n\alpha_0$
$\le 2L^n\beta_0\frac{1}{1-L} + L^n\alpha_0 \to 0$ as $n \to \infty$
$\therefore p(x_{n+k}, x_{n+k}, x_n) \to 0$ as $n \to \infty$
$\therefore$ by lemma 3.4, $\{x_n\}$ is a Cauchy sequence.
Hence it is convergent.
Let $x_n \to z$ (say)
$\therefore r_n^{n+k} = p(x_{n+k}, x_{n+k}, x_n)$
$= p(x_m, x_m, x_n) \to 0$ where $m = n + k$
$\Longrightarrow p(z, z, z) = 0$ (since p is continuous) (3.6.3)
Again since p is continuous, $p(z, z, x_n) \to p(z, z, z)$=0
$\therefore p(z, z, x_n) \to 0$ as $n \to \infty$
Consider $p(x_{n+1}, x_{n+1}, fz) = p(fx_n, fx_n, fz) \le L\,p(x_n, x_n, z) \to L\,p(z, z, z) = 0$
$\therefore p(z, z, fz) = 0$ (3.6.4)
Also $p(fz, fz, z) \le 2p(z, z, fz) + p(z, z, z)$
$\Rightarrow p(fz, fz, z) = 0$
From (3.6.3) and (3.6.4), we have
$p(z, z, z) = 0$, $p(z, z, fz) = 0$
$\therefore$ by lemma 3.4 we have $fz = z$
$\therefore z$ is a fixed point of f.
Suppose u is any fixed point of f, then $fu = u$
Consider $p(z, z, u) = p(fz, fz, fu) \le Lp(z, z, u)$
Since $0 \le L < 1$, we have $p(z, z, u) = 0$
$\therefore$ We have $p(z, z, u) = 0$ and $p(z, z, z) = 0$
$\therefore$ by lemma 3.4 we have $u = z$
Hence f has unique fixed point.

IV. COROLLARIES

We first obtain Banach Contraction Principle as a corollary to our theorem 3.6.

4.1 Corollary: If (X, d) is a complete metric space and $f: X \to X$ is a mapping such that $d(fx, fy) \le Ld(x, y)$ for all $x, y \in X$, for some $L \in [0,1)$ then f has a unique fixed point.
Proof: Let (X, d) be a complete metric space.
Define $S_b: X^3 \to [0, \infty)$ by $S_b(x, y, z) = d(x, z) + d(y, z)$ for $x, y, z \in X$.
Then, by example 3.3, (X, S_b) is a complete S_b-metric space with $p = S_b$ as a weak S_b-metric, $b = 1$.
Also, $S_b(fx, fx, fy) = 2d(fx, fy) \le 2Ld(x, y)$
$= LS_b(x, x, y)$
$= Lp(x, x, y)$ for all $x, y \in X$
Hence, by theorem 3.6, f has unique fixed point.

4.2 Corollary: Suppose (X, S) is a complete S-metric space with a weak S-metric on it. Suppose $f: X \to X$ is such that $p(fx, fx, fy) \le Lp(x, x, y)$ for all $x, y \in X$, for some $L \in [0,1)$. Then f has a fixed point $\in X$. Also if $u \in X$ is another fixed point of f, then $p(u, u, z) = 0$, and hence $u = z$. Consequently f has unique fixed point.
Proof: Since a S-metric space is a S_b-metric space with $b = 1$, the result follows from theorem 3.6.
Now we obtain result of Sarada.K. et al. [5], as a corollary.

4.3 Corollary: Theorem 2.16
Proof: Follows from corollary 4.2. Since a S-metric space is a S_b-metric space with $b = 1$. Here we observe that continuity of f is assumed in 2.16, while it is not necessary here.

V. CONCLUSION

Existence and uniqueness of a fixed point for a self map on a complete S_b-metric space is established when the map is a contraction with respect to a weak S_b-metric. In proving this continuity of the function is not assumed which is a significant achievement.

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