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# Some Special Operations and Related Results on Intuitionistic Fuzzy Sets 

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#### Abstract

Operators are playing significant role in intuitionistic fuzzy set theory. There are several operators in literature which are introduced by the researchers. Similarly many operations are also defined time to time. In this paper some new operations are discussed and consequently some related results are established and proved. The main objective of this paper is to study those operations which are already available in the literature along with some newly defined operations and to explain their properties with respect to modal and other operators.


Keywords-Fuzzy sets, Intuitionistic fuzzy sets, Modal operators, Operations in intuitionistic fuzzy sets.

## I. InTRODUCTION

In 1983, Atanassov [1] introduced the concept of intuitionistic fuzzy set as an extension of fuzzy set which was invented by L. A. Zadeh [2] in 1965. Since then many authors and researchers are working for developing intuitionistic fuzzy sets. It is well known to us that every fuzzy set is intuitionistic fuzzy set but the reverse is not true. But more importantly there exist some operators by which we can transform intuitionistic fuzzy sets into fuzzy sets easily. Atanassov [3] has remarkably pointed out about the significance of modal operators which are equivalent of the modal logic operators 'necessity' and 'possibility'. It can be mentioned that the sum of the membership and nonmembership function lies between 0 and 1 in intuitionistic fuzzy set theory. In this concept, another function called hesitation margin also belongs to 0 and 1. Many mathematicians and researchers have their contribution for developing the operators and operations in intuitionistic fuzzy sets. The modal operators ( $\square, \diamond$ ) introduced by Atanassov [4] in 1986. He also introduced ( $\boxplus$, $\boxtimes$ ) and ( $\boxplus_{\alpha}$, $\boxtimes_{\alpha}$ ) where $\alpha \in[0,1]$. The uses of these operators over the basic operations namely union, intersection, addition, multiplication, difference and symmetric difference are rigorously studied and established. In this paper, we concentrate deeply upon the operations $*, \odot, \bowtie, \infty, \triangleright$ and $\triangleleft$ and try to establish some new properties. At the same time the characteristics of the operations $\ominus$ and $\$$ are also discussed.

## II. Related Work

Throughout this paper, intuitionistic fuzzy set and fuzzy set are denoted by IFS and FS respectively.

Definition 2.1 Suppose X be a nonempty set. A fuzzy set $A \in X$ can be defined as $A=\left\{\left\langle x, \mu_{A}(x)\right\rangle: x \in X\right\}$, where $\mu_{\mathrm{A}}: \mathrm{x} \rightarrow[0,1]$ is termed as membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2.2 [4]. Suppose $X$ be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A=\left\{<x, \mu_{A}(x), v_{A}(x)>: x \in X\right\}$, where the functions $\mu_{A}$, $v_{A}: X \rightarrow[0,1]$ define respectively, the degree of membership and non-membership function of the element $x \in X$ to the set $A$, which is a subset of $X$, and for every element $\mathrm{x} \in \mathrm{X}, 0 \leq \mu_{\mathrm{A}}(\mathrm{x})+v_{\mathrm{A}}(\mathrm{x}) \leq 1$.

Furthermore, we have $\pi_{\mathrm{A}}(\mathrm{x})=1-\mu_{\mathrm{A}}(\mathrm{x})-v_{\mathrm{A}}(\mathrm{x})$ which is termed as intuitionistic fuzzy set index or hesitation margin of $x$ in $A . \pi_{A}(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_{\mathrm{A}}(\mathrm{x}) \in[0,1]$,i.e, $\pi_{\mathrm{A}}: \mathrm{X} \rightarrow[0,1]$ and $0 \leq$ $\pi_{\mathrm{A}}(\mathrm{x}) \leq 1$ for every $\mathrm{x} \in \mathrm{X}$.
$\pi_{\mathrm{A}}(\mathrm{x})$ means the lack of knowledge of whether x belongs to IFS A or not.

Definition 2.3 [4]. Let A, B be two IFSs in X. The basic operations are defined as follows:

1. $[$ inclution $] \mathrm{A} \subseteq \mathrm{B} \Leftrightarrow \mu_{\mathrm{A}}(\mathrm{x}) \leq \mu_{\mathrm{B}}(\mathrm{x})$ and $v_{\mathrm{A}}(\mathrm{x}) \geq$ $v_{\mathrm{B}}(\mathrm{x}) \forall \mathrm{x} \in \mathrm{X}$.
2. [equality] $\mathrm{A}=\mathrm{B} \Leftrightarrow \mu_{\mathrm{A}}(\mathrm{x})=\mu_{\mathrm{B}}(\mathrm{x})$ and $v_{\mathrm{A}}(\mathrm{x})=$ $v_{\mathrm{B}}(\mathrm{x}) \forall \mathrm{x} \in \mathrm{X}$.
3. $\left[\right.$ complement] $A^{c}=\left\{<x, v_{A}(x), \mu_{A}(x)>: x \in X\right\}$.
4. [union] $\mathrm{A} \cup \mathrm{B}=\left\{<\mathrm{x}, \max \left(\mu_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right), \min \left(v_{\mathrm{A}}(\mathrm{x})\right.\right.$, $\left.\left.v_{B}(x)\right)>: x \in X\right\}$.
5. [intersction] $\mathrm{A} \cap \mathrm{B}=\left\{<\mathrm{x}, \min \left(\mu_{\mathrm{A}}(\mathrm{x}), \quad \mu_{\mathrm{B}}(\mathrm{x})\right)\right.$, $\left.\max \left(v_{\mathrm{A}}(\mathrm{x}), v_{\mathrm{B}}(\mathrm{x})\right)>: \mathrm{x} \in \mathrm{X}\right\}$.
6. [addition] $\mathrm{A} \oplus \mathrm{B}=\left\{<\mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x})+\mu_{\mathrm{B}}(\mathrm{x})-\mu_{\mathrm{A}}(\mathrm{x})\right.$ $\left.\mu_{\mathrm{B}}(\mathrm{x}), v_{\mathrm{A}}(\mathrm{x}) \nu_{\mathrm{B}}(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\right\}$.
7. [multiplication] $\mathrm{A} \otimes \mathrm{B}=\left\{<\mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x}) \mu_{\mathrm{B}}(\mathrm{x}), v_{\mathrm{A}}(\mathrm{x})+\right.$ $\left.v_{B}(x)-v_{A}(x) v_{B}(x)>: x \in X\right\}$
8. [difference] A-B $=\left\{<x, \quad \min \left(\mu_{A}(x), \quad v_{B}(x)\right)\right.$, $\left.\max \left(v_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right)>: \mathrm{x} \in \mathrm{X}\right\}$.
9. [symmetric difference] A $\Delta \quad B \quad=\{<x$, $\max \left[\min \left(\mu_{\mathrm{A}}(\mathrm{x}), v_{\mathrm{B}}(\mathrm{x})\right), \min \left(\mu_{\mathrm{B}}(\mathrm{x}), \nu_{\mathrm{A}}(\mathrm{x})\right)\right]$,
$\min \left[\max \left(v_{\mathrm{A}}(\mathrm{x}) \mu_{\mathrm{B}}(\mathrm{x}), \max \left(v_{\mathrm{B}}(\mathrm{x}) \mu_{\mathrm{A}}(\mathrm{x})\right]>: \mathrm{x} \in \mathrm{X}\right\}\right.$.
Definition 2.4 Let A and B be two IFSs in a nonempty set X. Then
(a) $[5] \quad \mathrm{A} \ominus \mathrm{B}=\left\{\left\langle\mathrm{x}, 1 / 2\left[\mu_{\mathrm{A}}(\mathrm{x})+\mu_{\mathrm{B}}(\mathrm{x})\right], 1 / 2\left[v_{\mathrm{A}}(\mathrm{x})+\right.\right.\right.$ $\left.\left.v_{\mathrm{B}}(\mathrm{x})\right]>: \mathrm{x} \in \mathrm{X}\right\}$.
(b) $[5]$ A $\$ \mathrm{~B}=\left\{<\mathrm{x}, \quad\left(\mu_{\mathrm{A}}(\mathrm{x}) \cdot \mu_{\mathrm{B}}(\mathrm{x})\right)^{1 / 2}, \quad\left(\mathrm{v}_{\mathrm{A}}(\mathrm{x})\right.\right.$. $\left.\left.v_{\mathrm{B}}(\mathrm{x})\right)^{1 / 2}>: x \in \mathrm{X}\right\}$.

Definition 2.5 [6]. $A$ is said to be a proper subset of $B$ i.e. $\mathrm{A} \subset \mathrm{B}$ if $\mathrm{A} \subseteq \mathrm{B}$ and $\mathrm{A} \neq \mathrm{B}$. It means $\mu_{\mathrm{A}}(\mathrm{x}) \leq \mu_{\mathrm{B}}(\mathrm{x})$ and $v_{\mathrm{A}}(\mathrm{x}) \geq v_{\mathrm{B}}(\mathrm{x})$ but $\mu_{\mathrm{A}}(\mathrm{x}) \neq \mu_{\mathrm{B}}(\mathrm{x})$ and $v_{\mathrm{A}}(\mathrm{x}) \neq v_{\mathrm{B}}(\mathrm{x})$ for $\mathrm{x} \in \mathrm{X}$.

Definition 2.6 [6]. Let X be a nonempty set. If A is an IFS drawn from X , then
(i) $\quad \square \mathrm{A}=\left\{<\mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x}), 1-\mu_{\mathrm{A}}(\mathrm{x})>: \mathrm{x} \in \mathrm{X}\right\}$
(ii) $\quad \Delta \mathrm{A}=\left\{\left\langle\mathrm{x}, 1-v_{\mathrm{A}}(\mathrm{x}), v_{\mathrm{A}}(\mathrm{x})\right\rangle: \mathrm{x} \in \mathrm{X}\right\}$

For a proper IFS, $\square \mathrm{A} \subset \mathrm{A} \subset \diamond \mathrm{A}$ and $\square \mathrm{A} \neq \mathrm{A} \neq \diamond \mathrm{A}$.
Definition 2.7 [7]. Let X be a nonempty set. If A is an IFS drawn from $X$, then,

$$
\begin{aligned}
& \boxplus A=\left\{\left\langle x, \frac{\mu_{A}(x)}{2}, \frac{v_{A}(x)+1}{2}\right\rangle\right\} \text { and } \\
& \boxtimes A=\left\{\left\langle x, \frac{\mu_{A}(x)+1}{2}, \frac{v_{A}(x)}{2}\right\rangle\right\}
\end{aligned}
$$

Clearly $\boxplus \mathrm{A} \subset \mathrm{A} \subset \boxtimes \mathrm{A}$.

Definition 2.8 [7]. Let $\alpha \in[0,1]$ and $A$ be an IFS drawn from $X$, then
(i) $\boxplus_{\alpha} \mathrm{A}=\left\{<\mathrm{x}, \alpha \cdot \mu_{\mathrm{A}}(\mathrm{x}), \alpha \cdot v_{\mathrm{A}}(\mathrm{x})+1-\alpha>: \mathrm{x} \in \mathrm{X}\right\}$
(ii) $\boxtimes_{\alpha} \mathrm{A}=\left\{\left\langle\mathrm{x}, \alpha . \mu_{\mathrm{A}}(\mathrm{x})+1-\alpha, \alpha \cdot v_{\mathrm{A}}(\mathrm{x})\right\rangle: \mathrm{x} \in \mathrm{X}\right\}$

## III. SOME SPECIAL OPERATIONS AND THEIR PROPERTIES IN IFS

Definition 3.1 Let $X$ be a nonempty set. If $A$ and $B$ be two IFSs drawn from $X$, then,

$$
\begin{equation*}
A * B=\left\{\left\langlex, \frac{\mu_{A}(x)+\mu_{B}(x)}{2\left(\mu_{A}(x)+\mu_{B}(x)+1\right)}\right.\right. \tag{i}
\end{equation*}
$$

$$
\left.\frac{v_{A}(x)+v_{B}(x)}{2\left(v_{A}(x)+v_{B}(x)+1\right)}>: x \in X\right\}
$$

(ii)
$A \odot B=\left\{\left\langle x, \frac{\mu_{A}(x) \mu_{B}(x)}{2\left(\mu_{A}(x) \mu_{B}(x)+1\right)}\right.\right.$, $\left.\frac{v_{A}(x) v_{B}(x)}{2\left(v_{A}(x) v_{B}(x)+1\right)}>: x \in X\right\}$
(iii) $\quad \mathrm{A} \bowtie \mathrm{B}=\left\{\left\langle\mathrm{x}, \frac{\mu_{A}(\mathrm{x})+\mu_{\mathrm{B}}(\mathrm{x})}{2\left(\mu_{A}(\mathrm{x})+\mu_{\mathrm{B}}(\mathrm{x})\right)+1}\right.\right.$, $\left.\frac{v_{A}(x)+v_{B}(x)}{2\left(v_{A}(x)+v_{B}(x)\right)+1}>: x \in X\right\}$
(iv) $\quad A \infty B=\left\{<x, \frac{\mu_{A}(x) \mu_{B}(x)}{2\left(\mu_{A}(x) \mu_{B}(x)\right)+1}\right.$, $\left.\frac{v_{A}(x) v_{B}(x)}{2\left(v_{A}(x) v_{B}(x)\right)+1}>: x \in X\right\}$
(v) $\quad A \triangleright B=\left\{<x, \frac{\mu_{A}(x)+\mu_{B}(x)}{\mu_{A}(x)+\mu_{B}(x)+1}\right.$, $\left.\frac{v_{A}(x)+v_{B}(x)}{v_{A}(x)+v_{B}(x)+1}>: x \in X\right\}$
(vi) $\quad A \triangleleft B=\left\{<x, \frac{\mu_{A}(x) \mu_{B}(x)}{\mu_{A}(x) \mu_{B}(x)+1}\right.$,
$\left.\frac{v_{A}(x) v_{B}(x)}{v_{A}(x) v_{B}(x)+1}>: x \in X\right\}$
Here, $\mathrm{A} * \mathrm{~B}, \mathrm{~A} \odot \mathrm{~B}, \mathrm{~A} \bowtie \mathrm{~B}, \mathrm{~A} \infty \mathrm{~B}, \mathrm{~A} \triangleright \mathrm{~B}$, and $\mathrm{A} \triangleleft \mathrm{B}$ are all intuitionistic fuzzy sets.

Theorem 3.2 Let $X$ be a nonempty set. If A and B be any two IFSs drawn from $X$, then
(a) $[$ ( $\mathrm{A} \cup \mathrm{B})]^{\mathrm{C}}=\boxtimes\left(\mathrm{A}^{\mathrm{C}}\right) \cap \boxtimes\left(\mathrm{B}^{\mathrm{C}}\right)$
(b) $[\boxtimes(A \cup B)]^{C}=\boxplus\left(\mathrm{A}^{\mathrm{C}}\right) \cap \boxplus\left(\mathrm{B}^{\mathrm{C}}\right)$
(c) $[\boxplus(\mathrm{A} \cap \mathrm{B})]^{\mathrm{C}}=\boxtimes\left(\mathrm{A}^{\mathrm{C}}\right) \cup \boxtimes\left(\mathrm{B}^{\mathrm{C}}\right)$
(d) $[\boxtimes(A \cap B)]^{C}=\boxplus\left(\mathrm{A}^{\mathrm{C}}\right) \cup \boxplus\left(\mathrm{B}^{\mathrm{C}}\right)$
(e) $[\boxplus(\mathrm{A} \oplus \mathrm{B})]^{\mathrm{C}} \supset \boxtimes\left(\mathrm{A}^{\mathrm{C}}\right) \otimes \boxtimes\left(\mathrm{B}^{\mathrm{C}}\right)$
(f) $[\boxtimes(A \oplus B)]^{C} \supset \boxplus\left(\mathrm{~A}^{\mathrm{C}}\right) \otimes \boxplus\left(\mathrm{B}^{\mathrm{C}}\right)$
(g) $[\boxplus(\mathrm{A} \otimes \mathrm{B})]^{\mathrm{C}} \subset \boxtimes\left(\mathrm{A}^{\mathrm{C}}\right) \oplus \boxtimes\left(\mathrm{B}^{\mathrm{C}}\right)$
(h) $[\boxtimes(\mathrm{A} \otimes \mathrm{B})]^{\mathrm{C}} \subset \boxplus\left(\mathrm{A}^{\mathrm{C}}\right) \oplus \boxplus\left(\mathrm{B}^{\mathrm{C}}\right)$

Proof (a) to (d) are obvious.
(e) Now L.H.S $=[\boxplus(A \oplus B)]^{C}=\left[\boxplus\left(\mu_{A}(x)+\mu_{B}(x)-\mu_{A}(x)\right.\right.$
$\left.\mu_{\mathrm{B}}(\mathrm{x}), v_{\mathrm{A}}(\mathrm{x}) \mathrm{v}_{\mathrm{B}}(\mathrm{x})\right]^{\mathrm{c}}$
$=\left\{\left\langle\frac{\mu_{A}(x)+\mu_{B}(x)-\mu_{A}(x) \mu_{B}(x)}{2}, \frac{v_{A}(x) v_{B}(x)+1}{2}\right\rangle\right\}^{c}=$ $\left\{\left\langle\frac{v_{A}(x) v_{B}(x)+1}{2}, \frac{\mu_{A}(x)+\mu_{B}(x)-\mu_{A}(x) \mu_{B}(x)}{2}\right\rangle\right\}$
and R.H.S $=\boxtimes\left(\mathrm{A}^{\mathrm{C}}\right) \otimes \boxtimes\left(\mathrm{B}^{\mathrm{C}}\right)=\boxtimes\left(v_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{A}}(\mathrm{x})\right) \otimes$
$\boxtimes\left(v_{B}(x), \mu_{B}(x)\right)$
$=\left\{\left\langle\frac{v_{A}(x)+1}{2}, \frac{\mu_{A}(x)}{2}\right\rangle\right\} \otimes\left\{\left\langle\frac{v_{B}(x)+1}{2}, \frac{\mu_{B}(x)}{2}\right\rangle\right\}$
$=\left\{\left\langle\frac{v_{A}(x)+1}{2} \frac{v_{B}(x)+1}{2}, \frac{\mu_{A}(x)}{2}+\frac{\mu_{B}(x)}{2}-\frac{\mu_{A}(x)}{2} \frac{\mu_{B}(x)}{2}\right.\right.$
>\}
Clearly it can be checked that
$\frac{v_{A}(x)+1}{2} \frac{v_{B}(x)+1}{2}<\frac{v_{A}(x) v_{B}(x)+1}{2}$
and $\frac{\mu_{A}(x)}{2}+\frac{\mu_{B}(x)}{2}-\frac{\mu_{A}(x)}{2} \frac{\mu_{B}(x)}{2}>$
$\frac{\mu_{A}(\mathrm{x})+\mu_{B}(\mathrm{x})-\mu_{A}(\mathrm{x}) \mu_{B}(\mathrm{x})}{2}$
Hence $[\boxplus(\mathrm{A} \oplus \mathrm{B})]^{\mathrm{C}} \supset \boxtimes\left(\mathrm{A}^{\mathrm{C}}\right) \otimes \boxtimes\left(\mathrm{B}^{\mathrm{C}}\right)$
Similarly（f）to（h）can be proved．
Theorem 3．3 Let $X$ be a nonempty set．If A and B be any two IFSs drawn from $X$ ，then
（a）$[\boxplus(A \cup B)]^{C}=\boxtimes(A \cup B)^{C}$
（b）$[\boxtimes(A \cup B)]^{C}=\boxplus(A \cup B)^{C}$
（c）$[\boxplus(\mathrm{A} \cap \mathrm{B})]^{\mathrm{C}}=\boxtimes(\mathrm{A} \cap \mathrm{B})^{\mathrm{C}}$
（d）$[\boxtimes(A \cap B)]^{C}=\boxplus(A \cap B)^{C}$
（e）$[\boxplus(A \oplus B)]^{C}=\boxtimes(A \oplus B)^{C}$
（f）$[\boxtimes(A \oplus B)]^{C}=\boxplus(A \oplus B)^{C}$
（g）$[\boxplus(\mathrm{A} \otimes \mathrm{B})]^{\mathrm{C}}=\boxtimes(\mathrm{A} \otimes \mathrm{B})^{\mathrm{C}}$
（h）$[\boxtimes(A \otimes B)]^{C}=\boxplus(A \otimes B)^{C}$
（i）$[\boxplus(\mathrm{A}-\mathrm{B})]^{\mathrm{C}}=\boxtimes(\mathrm{A}-\mathrm{B})^{\mathrm{C}}$
（j）$[\boxtimes(A-B)]^{C}=\boxplus(A-B)^{C}$
（k）$[\boxplus(\mathrm{A} \Delta \mathrm{B})]^{\mathrm{C}}=\boxtimes(\mathrm{A} \Delta \mathrm{B})^{\mathrm{C}}$
（l）$[\boxtimes(\mathrm{A} \Delta \mathrm{B})]^{\mathrm{C}}=\boxplus(\mathrm{A} \Delta \mathrm{B})^{\mathrm{C}}$

Proof（a）Here $\boxplus(A \cup B)=\boxplus\left[\max \left(\mu_{A}(x), \mu_{B}(x)\right)\right.$ ， $\left.\min \left(v_{A}(x), v_{B}(x)\right)\right]$
$=\left\{\left\langle\frac{\max \left(\mu_{A}(\mathrm{x}) \mu_{B}(\mathrm{x})\right)}{2}, \frac{\min \left(v_{A}(\mathrm{x}) v_{B}(\mathrm{x})\right)+1}{2}\right\rangle\right\}$
Now L．H．S $=[\boxplus(\mathrm{A} \cup \mathrm{B})]^{\mathrm{C}}=$
$\left\{\left\langle\frac{\min \left(v_{A}(x) v_{B}(x)\right)+1}{2}, \frac{\max \left(\mu_{A}(x) \mu_{B}(x)\right)}{2}\right\rangle\right\}$
and R．H．S $=\boxtimes(\mathrm{A} \cup \mathrm{B})^{\mathrm{C}}=\boxtimes\left(\min \left(v_{\mathrm{A}}(\mathrm{x}), v_{\mathrm{B}}(\mathrm{x})\right)\right.$ ，
$\left.\max \left(\mu_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right)\right)$

$$
=\left\{\left\langle\frac{\min \left(v_{A}(x)_{i} v_{B}(x)\right)+1}{2}, \frac{\max \left(\mu_{A}(x)_{\mu} \mu_{B}(x)\right)}{2}\right\rangle\right\}
$$

Hence the proof．
Similarly（b）to（l）can be proved．
Theorem 3．4 Let X be a nonempty set．If A and B be two IFSs drawn from $X$ ，then
（a）$\boxplus(\mathrm{A} \ominus \mathrm{B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} \theta \mathrm{B})]^{\mathrm{C}}$
（b）$\boxtimes(A \ominus B)^{C}=[\boxplus(A \ominus B)]^{C}$
（c）$\boxplus(\mathrm{A} \$ \mathrm{~B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} \$ \mathrm{~B})]^{\mathrm{C}}$
（d）$\triangle(\mathrm{A} \$ \mathrm{~B})^{\mathrm{C}}=[$（ $\left.\mathrm{A} \$ \mathrm{~B})\right]^{\mathrm{C}}$
（e）$\boxplus(\mathrm{A} * \mathrm{~B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} * \mathrm{~B})]^{\mathrm{C}}$
（f）$\boxtimes(\mathrm{A} * \mathrm{~B})^{\mathrm{C}}=[\boxplus(\mathrm{A} * \mathrm{~B})]^{\mathrm{C}}$
（g）$\boxplus(\mathrm{A} \odot \mathrm{B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} \odot \mathrm{B})]^{\mathrm{C}}$
（h）$\boxtimes(A \odot B)^{C}=[\boxplus(A \odot B)]^{C}$
（i）$\boxplus(\mathrm{A} \bowtie \mathrm{B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} \bowtie \mathrm{B})]^{\mathrm{C}}$
（j）$\boxtimes(A \bowtie B)^{C}=[\boxplus(A \bowtie B)]^{C}$
（k）$\boxplus(\mathrm{A} \infty \mathrm{B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} \infty \mathrm{B})]^{\mathrm{C}}$
（l）$\boxtimes(\mathrm{A} \infty \mathrm{B})^{\mathrm{C}}=[\boxplus(\mathrm{A} \infty \mathrm{B})]^{\mathrm{C}}$
（m）$\quad \boxplus(\mathrm{A} \triangleright \mathrm{B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} \triangleright \mathrm{B})]^{\mathrm{C}}$
（n）$\boxtimes(\mathrm{A} \triangleright \mathrm{B})^{\mathrm{C}}=[\boxplus(\mathrm{A} \triangleright \mathrm{B})]^{\mathrm{C}}$
（o）$\boxplus(\mathrm{A} \triangleleft \mathrm{B})^{\mathrm{C}}=[\boxtimes(\mathrm{A} \triangleleft \mathrm{B})]^{\mathrm{C}}$
（p）$\otimes(\mathrm{A} \triangleleft \mathrm{B})^{\mathrm{C}}=[⿴ 囗 十(\mathrm{~A} \triangleleft \mathrm{~B})]^{\mathrm{C}}$
Proof（a）L．H．S $=\boxplus(\mathrm{A} \theta \mathrm{B})^{\mathrm{C}}$
$=\boxplus\left\{\left\langle\frac{v_{A}(\mathrm{x})+\mathrm{v}_{B}(\mathrm{x})}{2}, \frac{\mu_{A}(\mathrm{x})+\mu_{B}(\mathrm{x})}{2}\right\rangle\right\}$
$=\left\{\left\langle\frac{\left(v_{A}(x)+v_{B}(x)\right.}{4}, \frac{1}{2}\left(\frac{\mu_{A}(x)+\mu_{B}(x)}{2}+1\right)\right\rangle\right\}$
R．H．S $=[\boxtimes(\mathrm{A} \ominus \mathrm{B})]^{\mathrm{C}}$
$\left.=\left[\boxtimes<\frac{\mu_{A}(x)+\mu_{B}(x)}{2}, \frac{v_{A}(x)+v_{B}(x)}{2}\right\rangle\right] C$
$=\left\{\left\langle\frac{1}{2}\left(\frac{\mu_{A}(x)+\mu_{B}(x)}{2}+1\right), \frac{\left(v_{A}(x)+v_{B}(x)\right.}{4}\right\rangle\right\}^{C}$
$=\left\{\left\langle\frac{\left(v_{A}(x)+v_{B}(x)\right.}{4}, \frac{1}{2}\left(\frac{\mu_{A}(x)+\mu_{B}(x)}{2}+1\right)\right\rangle\right\}$
Hence the proof．
Similarly（b）to（p）can be proved．
Theorem 3．5 Let $X$ be a nonempty set．If $A$ and $B$ be two IFSs drawn from X ，then
（a）$\boxplus_{\alpha}(\mathrm{A} \theta \mathrm{B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} \theta \mathrm{B})\right]^{\mathrm{C}}$
（b）$\boxtimes_{\alpha}(\mathrm{A} \ominus \mathrm{B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} \ominus \mathrm{B})\right]^{\mathrm{C}}$
（c）$\boxplus_{\alpha}(\mathrm{A} \$ \mathrm{~B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} \$ \mathrm{~B})\right]^{\mathrm{C}}$
（d）$\boxtimes_{\alpha}(\mathrm{A} \$ \mathrm{~B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} \$ \mathrm{~B})\right]^{\mathrm{C}}$
（e）$\boxplus_{\alpha}(\mathrm{A} * \mathrm{~B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} * \mathrm{~B})\right]^{\mathrm{C}}$
（f）$\boxtimes_{\alpha}(\mathrm{A} * \mathrm{~B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} * \mathrm{~B})\right]^{\mathrm{C}}$
（g）$\boxplus_{\alpha}(\mathrm{A} \odot \mathrm{B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} \odot \mathrm{B})\right]^{\mathrm{C}}$
（h）$\boxtimes_{\alpha}(\mathrm{A} \odot \mathrm{B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} \odot \mathrm{B})\right]^{\mathrm{C}}$
（i）$\boxplus_{\alpha}(\mathrm{A} \bowtie \mathrm{B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} \bowtie \mathrm{B})\right]^{\mathrm{C}}$
（j）$\boxtimes_{\alpha}(\mathrm{A} \bowtie \mathrm{B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} \bowtie \mathrm{B})\right]^{\mathrm{C}}$
（k）$\boxplus_{\alpha}(\mathrm{A} \infty \mathrm{B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} \infty \mathrm{B})\right]^{\mathrm{C}}$
（l）$\boxtimes_{\alpha}(\mathrm{A} \infty \mathrm{B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} \infty \mathrm{B})\right]^{\mathrm{C}}$
（m）$\boxplus_{\alpha}(\mathrm{A} \triangleright \mathrm{B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} \triangleright \mathrm{B})\right]^{\mathrm{C}}$
（n）$\boxtimes_{\alpha}(\mathrm{A} \triangleright \mathrm{B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} \triangleright \mathrm{B})\right]^{\mathrm{C}}$
（o）$\boxplus_{\alpha}(\mathrm{A} \triangleleft \mathrm{B})^{\mathrm{C}}=\left[\boxtimes_{\alpha}(\mathrm{A} \triangleleft \mathrm{B})\right]^{\mathrm{C}}$
（p）$\boxtimes_{\alpha}(\mathrm{A} \triangleleft \mathrm{B})^{\mathrm{C}}=\left[\boxplus_{\alpha}(\mathrm{A} \triangleleft \mathrm{B})\right]^{\mathrm{C}}$
Proof（a）L．H．S $=\boxplus_{\alpha}(\mathrm{A} \ominus \mathrm{B})^{\mathrm{C}}$
$=\boxplus_{\alpha}\left\{\left\langle\frac{v_{A}(x)+v_{B}(x)}{2}, \frac{\mu_{A}(x)+\mu_{B}(x)}{2}\right\rangle\right\}$

$$
=\left\{\left\langle\alpha\left(\frac{v_{A}(x)+v_{B}(x)}{2}\right), \alpha\left(\frac{\mu_{A}(x)+\mu_{B}(x)}{2}\right)+1-\alpha\right\rangle\right\}
$$

Again R．H．S $=\left[\boxtimes_{\alpha}(\mathrm{A} \theta \mathrm{B})\right]^{\mathrm{C}}$
$=\left\{\left\langle\alpha\left(\frac{\mu_{A}(\mathrm{x})+\mu_{B}(\mathrm{x})}{2}\right)+1-\alpha, \alpha\left(\frac{v_{A}(\mathrm{x})+v_{B}(\mathrm{x})}{2}\right)\right\rangle\right\}^{\mathrm{C}}$
$=\left\{\left\langle\alpha\left(\frac{v_{A}(x)+v_{B}(x)}{2}\right), \alpha\left(\frac{\mu_{A}(x)+\mu_{B}(x)}{2}\right)+1-\alpha\right\rangle\right\}$
Hence the proof．
Similarly（b）to（p）can be proved．

Theorem 3.6 Let $X$ be a nonempty set. If A and B be any two IFSs drawn from $X$, then
(a) $\left[\boxplus_{\alpha}(\mathrm{A} \cup \mathrm{B})\right]^{\mathrm{C}}=\boxtimes_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \cap \boxtimes_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
(b) $\left[\boxtimes_{\alpha}(\mathrm{A} \cup \mathrm{B})\right]^{\mathrm{C}}=\boxplus_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \cap \boxplus_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
(c) $\left[\boxplus_{\alpha}(\mathrm{A} \cap \mathrm{B})\right]^{\mathrm{C}}=\boxtimes_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \cup \boxtimes_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
(d) $\left[\boxtimes_{\alpha}(\mathrm{A} \cap \mathrm{B})\right]^{\mathrm{C}}=\boxplus_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \cup \boxplus_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
(e) $\left[\boxplus_{\alpha}(\mathrm{A} \oplus \mathrm{B})\right]^{\mathrm{C}} \supset \boxtimes_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \otimes \boxtimes_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
(f) $\left[\boxtimes_{\alpha}(\mathrm{A} \oplus \mathrm{B})\right]^{\mathrm{C}} \supset \boxplus_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \otimes \boxplus_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
(g) $\quad\left[\boxplus_{\alpha}(\mathrm{A} \otimes \mathrm{B})\right]^{\mathrm{C}} \subset \boxtimes_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \oplus \boxtimes_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
(h) $\left[\boxtimes_{\alpha}(\mathrm{A} \otimes \mathrm{B})\right]^{\mathrm{C}} \subset \boxplus_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \oplus \boxplus_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$

Proof (a) L.H.S $=\left[\boxplus_{\alpha}(A \cup B)\right]^{C}$
$=\boxplus_{\alpha}\left\{<\left(\max \left(\mu_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right), \min \left(\mathrm{v}_{\mathrm{A}}(\mathrm{x}), \mathrm{v}_{\mathrm{B}}(\mathrm{x})\right)>\right\}\right.$
$=\left\{<\alpha \cdot \max \left(\mu_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right), \alpha \cdot \min \left(v_{\mathrm{A}}(\mathrm{x}), v_{\mathrm{B}}(\mathrm{x})\right)+1-\alpha>\right\}^{\mathrm{C}}$
$\left.=\left\{<\alpha \cdot \min \left(v_{A}(x), v_{B}(x)\right)+1-\alpha, \alpha \cdot \max \left(\mu_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right)\right\rangle\right\}$
Again R.H.S $=\boxtimes_{\alpha}\left(\mathrm{A}^{\mathrm{C}}\right) \cap \boxtimes_{\alpha}\left(\mathrm{B}^{\mathrm{C}}\right)$
$\left.\left.=\boxtimes_{\alpha}\left\{<\left(v_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{A}}(\mathrm{x})\right)\right\rangle\right\} \cap \boxtimes_{\alpha}\left\{<\left(v_{\mathrm{B}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right)\right\rangle\right\}$
$=\left\{\left\langle\alpha \cdot\left(v_{\mathrm{A}}(\mathrm{x})\right)+1-\alpha, \alpha \cdot \mu_{\mathrm{A}}(\mathrm{x})\right\rangle\right\} \cap\left\{\left\langle\alpha .\left(v_{\mathrm{B}}(\mathrm{x})\right)+1-\alpha\right.\right.$,
a. $\left.\mu_{\mathrm{B}}(\mathrm{x})>\right\}$
$=\left\{<\min \left[\left[\alpha \cdot\left(v_{\mathrm{A}}(\mathrm{x})\right)+1-\alpha, \alpha \cdot\left(v_{\mathrm{B}}(\mathrm{x})\right)+1-\alpha\right], \max \left[\alpha \cdot \mu_{\mathrm{A}}(\mathrm{x})\right.\right.\right.$,
a. $\left.\mu_{\mathrm{B}}(\mathrm{x})>\right\}$
$=\left\{\left\langle\alpha \cdot \min \left(v_{\mathrm{A}}(\mathrm{x}), v_{\mathrm{B}}(\mathrm{x})\right)+1-\alpha, \alpha \cdot \max \left(\mu_{\mathrm{A}}(\mathrm{x}), \mu_{\mathrm{B}}(\mathrm{x})\right)\right\rangle\right\}$
Hence the proof.
Similarly (b) to (h) can be proved.

## IV. CONCLUSION

Here we discussed on some special operators in intuitionistic fuzzy sets and investigated few results. The results which are established in this paper will certainly develop the literature. These will be very helpful for extending other new area of intuitionistic fuzzy sets in near future. The real life problems can also be solved with the help of these operators.

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## References

[1] K.T. Atanassov, "Intuitionistic Fuzzy Sets," VII ITKR's Session, Sofia, 1983.
[2] L.A.Zadeh, "Fuzzy sets," Information and Control,Vol 8, pp 338-353, 1965.
[3] K.T. Atanassov, "Intuitionistic Fuzzy Sets Past, Present and Future," CLBME-Bulgarian Academy of Science, Sofia, 2003.
[4] K.T. Atanassov, "Intuitionistic Fuzzy Sets," Fuzzy Sets and Systems, Vol.20, pp 87-96,1986.
[5] J. Bhattacharya, "A Few More on Intuitionistic Fuzzy Set," Journal of Fuzzy set valued Analysis, Vol 3, pp 214-222, 2016.
[6] K.T. Atanassov, "Some Operators in Intuitionistic Fuzzy Sets," First Int. Conf. on IFS, Sofia, NIFS Vol 3 pp 28-33,1997.
[7] K.T. Atanassov, "New Operations Defind Over Intuitionistic Fuzzy Sets," Fuzzy Sets and Systems, Vol 61, 2 pp 137-142, 1994.

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