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Research Paper

Coordinate Transformation of GPS Measurement Results using the Cartesian-to-Ellipsoidal Transformation System

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Abstract—GPS is generally applied in geodetic techniques for determining the three-dimensional location of points on the earth's surface. While the points are fixed in a global geocentric coordinate system, most work requires positional data based on the local coordinate system in terms of geodetic or local plane coordinates. As a result, conversion of coordinates of points from the geocentric coordinate system to the geodetic coordinate system becomes a necessity. It is normally achieved by using the method of coordinate transformation based on a mathematical model that creates a geometrical link between the coordinates of points in different reference frames. This study is aimed at the transformation of Global Positioning System measurement results based on the Cartesian-to-Ellipsoidal transformation model. Specifically, the iterative and closed-form solutions are presented. It was revealed that using the closed-form systems are fast and effective because the used parameters are directly recovered by using precise formulae without solving any nonlinear equation. However, the iterative algorithms has shown more advantages as it can produce precise information about the unknowns.

Keywords—Closed solution, iterative solution, local coordinate system, transformation parameters, WGS-84, reference frame

1. Introduction

The coordinates of points on the Earth are essential for accurate mapping and assessment of resources. Hence the need for accurate determination of locations on the Earth. In that regard, the Global Navigation Satellite Systems (GNSS), predominantly, the GPS (Global Positioning System) is generally embraced [1]. The GPS uses satellites for rapid positioning virtually anywhere on Earth at any time. Thus, it is used to create the base for current positioning applications [2]. The initial reference system adopted for the GPS was the World Geodetic System 1972 (WGS-72). Currently, the World Geodetic System 1984 (WGS-84) is used [3-7].

Though the GPS obtains observations on the WGS-84 ellipsoid, computing coordinates of terrestrial points is not usually preferred in a global framework. The results are rather required in geodetic or local plane coordinates. Therefore, the conversion of coordinates from one system to another using coordinate transformation becomes essential. Numerous transformation models exist including the small-rotation-angle transformation model [8], and arbitrary-rotation-angle transformation model [9, 10]. Similarly, many approaches for assessing model parameters have been suggested including the 7-parameter linear adjustment technique, 8-parameter linear adjustment scheme [8], Procrusters-based direct solutions [11], analytical close-form solutions [12], ill-condition model [13], and total least squares method based on the errors-in-variables model [14]. Furthermore, many researchers have used Artificial Neural Network (ANN) for coordinate transformation [15-20]. The focus of this study is on the method used for coordinate transformation concerning the Global Positioning System measurement results using Cartesian-to-Ellipsoidal transformation systems.

The study is organized in six sections. The first section contains the introduction. The second section deals with related studies. The third section presents the theoretical background of the study. In section four, methodology of the study is explained. Furthermore, the results and discussion of the study are presented in section five. Finally, the conclusion is presented in section six.

2. Related Work

Several pieces of research have been conducted on coordinate transformation in recent times. For instance, Ziggah et al. [1] investigated and compared the transformation models for Ghana. The results indicated that the accuracy of Bursa-Wolf, Molodensky-Badekas and Veis models parameters is ±1 m and that of the abridged Molodensky parameters is ±1.9 m. Similarly, Kumi-Boateng and Ziggah [4] conducted an accuracy assessment by evaluating the performance of seven approaches in transforming from Cartesian to geodetic coordinates in Ghana. Paul's technique was shown to be a better fit for the Ghana geodetic reference net.

Furthermore, Adewale, Emenari, and Amusuk [21] carried out the coordinate transformation of Birnin Kebbi from local to the Universal Transverse Mercator (UTM - WGS84). The Affine 5-parameter datum transformation was employed. Hmam [22] presented some approximation systems for transforming Cartesian coordinates to geodetic coordinates. Ziggah, Akwensi, and Annan, [23] examined and compared the efficiency of general least squares (GLS) with ordinary least squares (OLS) using a four-parameter similarity transformation model. The results based on test data revealed that both the GLS and the OLS models attained identical coordinate transformation results. Yang et al. [24] used many experimental data to authenticate the effectiveness and applicability of the method for assessing the coordinate transformation precision of various transformation schemes. Hart and Okeke [25] evaluated the major abilities of the National Transformation Version 2 (NTv2) in addressing the problem of distortion in the Nigerian Datum Transformation. They show the significance of the accuracy, simplicity, and capacity of this grid transformation procedure to cater for prevailing distortion in the Nigerian Geodetic Network.

3. Theoretical Background

WGS-84 is the reference system adopted for the GPS. As shown in figure 1, its coordinate origin is in the center of Earth's mass. Also, the IERS reference meridian is at zero longitude. The X-axis is in direction of the intersection of equatorial and zero meridian. The Z-axis goes towards the Conventional Terrestrial Pole (CTP) while the Y-axis complements the right handed coordinate system.

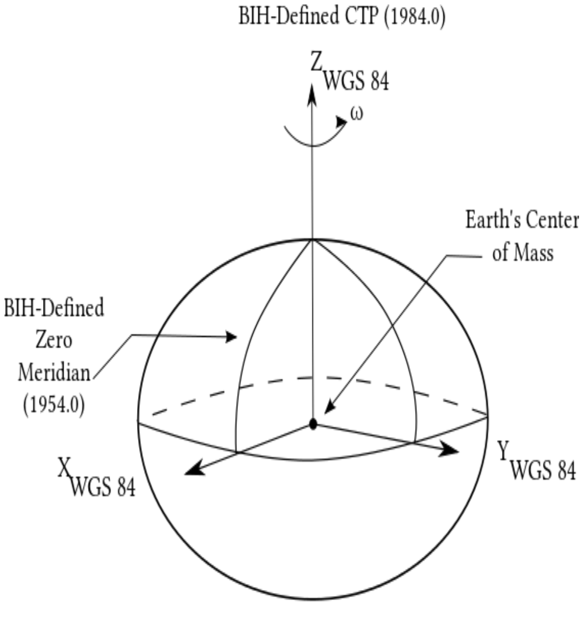

Figure 1: WGS-84 reference frame. Source: Subirana [26]

WGS-84 is the reference system applied for GPS readings based on a 3-d rectangular geocentric coordinate system. Though the geocentric coordinate frame is really linked to the Earth, it is associated with some issues. Consequently, they are frequently transformed to the generally used and locally oriented geodetic coordinates through the process of coordinate transformation. The process of coordinate transformation normally takes the coordinates of a location in one coordinate system and returns the coordinates of the same place in another coordinate system [27]. Transformation parameters have been derived between WGS-84 and local coordinate systems for various countries (see [28]) to be used for coordinate transformation.

4. Methodology

The relationship between the Cartesian and ellipsoidal coordinate systems will be established. This will be followed by the process of coordinate transformation between the two coordinate systems by iterative, and closed-form solutions.

5. Results and Discussion

5.1 Cartesian to Ellipsoidal Coordinate Transformation

A point P in space can be stated in Cartesian (rectangular) coordinates (X, Y, Z). If an ellipsoid of revolution with the same origin, the same point P can be expressed by ellipsoidal (geodetic) coordinates [φ, λ, h]) as shown in Figure 2. In the figure, P is a point on the physical surface of the ellipsoid, φ is its geodetic latitude while λ is its geodetic longitude.

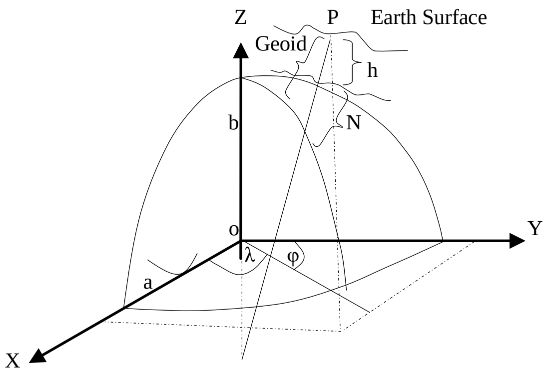

Figure 2: Cartesian and ellipsoidal coordinate schemes.

The Cartesian coordinates and ellipsoidal coordinates are connected in matrix form as follows:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} (N + h) \cos \varphi \cos \lambda \\ (N + h) \cos \varphi \sin \lambda \\ (\frac{b^2}{a^2} N + h) \sin \lambda \end{pmatrix} \qquad [1.1]$$

where X, Y, and Z are Cartesian latitude, longitude, and height respectively; N is the radius of curvature in the prime vertical; a, and b are semi-major, and semi-minor axes of the equator.

Equation [1.1] can be re-written in linear form as follows:

$$X = (N + h) \cos \varphi \cos \lambda \quad [1.2a]$$

$$Y = (N + h) \cos \varphi \sin \lambda \quad [1.2b]$$

$$Z = (\frac{b^2}{a^2} N + h) \sin \lambda \quad [1.2c]$$

where $N = a^2 / (a^2 \cos^2 \varphi + b^2 \sin^2 \varphi)^{1/2}$

The equation [1.2] transforms ellipsoidal coordinates into Cartesian coordinates. However, the reverse transformation is required for GPS observations as positioning produces Cartesian coordinates instead of ellipsoidal coordinates. This inverse problem can be solved by iterative or closed solutions.

5.2 Iterative Solution

A radial line (r) from the origin can be obtained from a distance equation in a two-dimensional plane. From equations [1.2a] and [1.2b], we have:

$$r = (X^2 + Y^2)^{1/2} = (N + h) \cos \varphi \quad [1.3]$$

By re-arranging [1.3], we define the equation for ellipsoidal height (h) as follows:

$$h = (r / \cos \varphi) - N \quad [1.4]$$

We know that the first eccentricity (e) of an ellipse is given by:

$$e^2 = (a^2 - b^2)/a^2 \quad [1.5]$$

$$:- \quad b^2/a^2 = 1 - e^2 \quad [1.6]$$

Substituting [1.6] in [1.2c] will give:

$$Z = [(1-e^2) N + h) \sin \varphi \quad [1.7]$$

$$Z = (N - Ne^2) + h) \sin \varphi \quad [1.8]$$

$$Z = (N + h - e^2 N - h) \sin \varphi \quad [1.9]$$

Equation [1.9] can be re-written as:

$$Z = (N + h) (1 - e^2 N / (N + h)) \sin \varphi \quad [1.10]$$

Dividing [1.10] by [1.3] will yield:

$$Z/r = (N + h) [1 - e^2 N / (N + h)] \sin \varphi / (N + h) \cos \varphi \quad [1.11]$$

$$Z/r = (1 - e^2 N/(N + h)) \tan \varphi \quad [1.12]$$

$$\tan \varphi = Z/r (1 - e^2 N/(N + h))^{-1} \quad [1.13]$$

The longitude can be obtained directly by dividing equation [1.2b] by [1.2a], hence,

$$\frac{Y}{X} = \frac{(N + h) \cos \varphi \cos \lambda}{(N + h) \cos \varphi \sin \lambda} \quad [1.14]$$

$$\frac{Y}{X} = \frac{\cos \lambda}{\sin \lambda}$$

$$\frac{Y}{X} = \tan \lambda \quad [1.15]$$

At first glance, it appeared as if h and φ can be computed from equations [1.4] and [1.13]. The problem here is that the former contains an unknown quantity φ directly and indirectly in N while in the later, the preferred φ is implicitly contained on the right-hand side. Hence using these equations, solution for h and φ can be obtained by an iterative process using the following steps proposed by Hofman-Wellennhof et al. [29].

1. Compute the radial line

   $r = (X^2 + Y^2)^{1/2}$

2. Compute an approximate value for latitude from:

   $\tan \varphi = Z/r (1-e^2)^{-1}$

3. Compute an approximate value for N from:

   $N_o = a^2 / (a^2 \cos^2 \varphi_o + b^2 \sin^2 \varphi_o)^{1/2}$

4. Compute the ellipsoidal height h from:

   $h = (r / \cos \varphi_o) - N_o$

5. Compute an improved value for latitude from:

   $\tan \varphi = Z/r ((1-e^2) N_o/(N_o+h))$

6. Conduct a logical check on a solution to ensure that $\varphi_1 = \varphi_o$? If not, set $\varphi_1 = \varphi_o$ and go back to step 3. Continue the iteration until $\varphi_1 = \varphi_o$ or a perpendicular limit is reached, which will mark the end of the iteration loop. The solution ends since the longitude coordinate has been obtained directly from equation [1.15].

Many iterative algorithms are in the literature (e.g., [30-36]). For this type of algorithm, there is the usual need for good initial value of unknown and iterative calculation. Nevertheless, a good initial value of the parameter is not often attainable in some circumstances. Thus, much iteration is required by the algorithm. Generally, the iterative methods are faster than the non-iterative methods; they quickly converge to millimeter level right after two iterations; and they produce accuracy which exceeds the requirements of any practical application (see [37]).

5.1.1 Closed

The formulas in closed-form solution for transforming the Cartesian ellipsoidal coordinates are given as:

$$\varphi = \tan^{1} \{(z+e^{'2} b\sin^{3}\varphi)/(r-e^{2}a\cos^{3}\varphi)\} \quad [1.16]$$

$$\lambda = \tan^{1} [Y/X] \quad [1.17]$$

$$h = (r/\cos\varphi)-N \quad [1.18]$$

where $\varphi = \tan^{1} [Z_a/r_b]$

$$e'^2 = (a^2-b^2)/a^2$$

The phi (φ) is an auxiliary quantity while e’ is the numerical value of the second eccentricity.

The benefit of closed-form equations is that the numerical errors are controllable (e.g., by careful computer coding), rather than having to run tedious numerical experiments for all likely (and perhaps unlikely) scenarios [38]. Many studies used this method for geodetic coordinate transformations (see [39-41]). For instance, Wang et al. [42] used a closed-form pairwise registration system of point clouds. The efficiency of the algorithm was apparent. Yet, it cannot deal with the general weight matrix of observations and the precise appraisal of transformation parameters.

6. Conclusion and Future Scope

The GPS normally affords its user the ability to determine his/her three-dimensional location on the Earth in a global geocentric coordinate system. Yet, positional data determined based on a local coordinate system in terms of geodetic or local plane coordinates is more meaningful to most users of the GPS. Consequently, the coordinates of points acquired by the GPS in the geocentric coordinate system are frequently converted to the coordinates of the same points in the geodetic coordinate system. To achieve this, the technique of coordinate transformation are used. In this case, mathematical model that creates a geometrical relationship between the coordinates of points in different reference frames is applied.

There are several method used to solve the fundamental problem in computational geodesy involving mathematical transformation of coordinates. This study has presented the transformation of Global Positioning System measurement results based on the Cartesian-to-Ellipsoidal transformation system. The technique is based on the iterative and close-form solutions. Generally, the closed-form systems have proven to be fast and effective. The reason is that the parameters used are directly gotten by using precise formulae without solving any nonlinear equation. Nevertheless, the iterative algorithms is seemingly characterized by more advantages as it has the capacity to supply accurate information about unknowns.

Data Availability
None.

Conflict of Interest
The author states that he do not have any conflict of interest.

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Authors’ Contributions
T.U.O. researched literature and conceived the study. T.U.O. performed data analysis. T.U.O. wrote the first draft of the manuscript, reviewed and corrected the manuscript, and approved the final version of the manuscript.

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