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Research Paper

Break Points in Life Table Mean Residual Life Times and Estimated Hazard Rates

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Abstract — The paper is notably focused on statistics and technical aspects of Mean Residual Life Time (MRLT) and Hazard Rate (HR). A polynomial with appropriate degree is fit to the life table MRLT data of countries belonging to different income groups and geological regions. Break points of MRLTs at various age of human life cycle are studied through segmented regression. HRs are estimated using respective MRLTs, break points in estimated HRs are analyzed.

Keywords — Mean Residual Life Time, Hazard Rate, Life Table, Segmented Regression, Polynomial Regression, Break Points.

1. Introduction

We delve into the problems origin and highlight the unique contributions of this work. We begin by explaining how the problem at hand was identified, providing context for its significance within the field. Furthermore, this section will emphasize the novelty of this research, shedding light on the fresh approaches, methodologies, and insights it brings to the domain.

MRLT and HR can be regarded as tools to characterize survival distributions, which provide the statistician with additional information to analyze consequences of using a particular model. In particular, break points in MRLTs and HRs influence many real-world phenomena of Reliability Engineering, Actuarial Science and Insurance, Quality Control and Process Improvement, Finance and Investment, Health and Epidemiology, etc. Some examples of use of MRLT, HR and their breakpoints are:
• In reliability engineering, the break points in MRLTs and HRs help in development of infrastructure, building models, transportation system, maintenance requirements, warranty analysis, etc.
• In product quality control, to eliminate the initial failures, products are often burn in until they reach a low HR before they leave the factory. The helpful tools for analyzing burn-in process are MRLTs and HRs and knowing their break points aid the decision making regarding the length of burn-in process.
• Deterioration and ageing explain decreasing mean residual life (DMRL) and increasing failure rate (IFR) stage. Obviously MRLTs and HRs and their break points are used by insurers to design insurance portfolios in assessment of premiums, annuities, claims, etc.
• In biomedical setting, researchers analyze survivorship studies through the MRL function, Hazard function and their break points.
• In Health care industry, these are used in terms of medical field, as preventive care, specialized health care services and technology, chronic disease management.
• In economics, MRL and their break points are applied for investigating landholding, optimal time for disposal of an asset.
• Finance is another field where these concepts have found niche. Financial instruments such as loans, credits, borrowings, interest rates, etc. use the knowledge of MRLTs, HRs and their break points.
• In service sector, to provide efficient service to customers, one should have a good team of employees. The retention policies of employees are based on their MRLT into that company and their inflection points.

Thus, the inflections in MRLTs and HRs are multifaceted, and it requires interdisciplinary collaboration among various stakeholders, including policymakers, researchers, healthcare professionals, insurers, engineers, and economists to address the challenges and opportunities presented by break points in MRLTs and HRs of longer life spans.

The MRLT, $m(t)$ and HR, $h(t)$ are defined respectively as

$$m(t) = \int_t^{\infty} \frac{S(a)da}{S(t)} \quad (1)$$

$$m'(t) = m(t).h(t) + 1 \quad (2)$$

where, $S(a)$ is survival function and the integral is taken over the range of remaining lifetimes. HR at 't' provides information about a small interval after 't', whereas MRLT at 't' considers information about whole interval after age 't'.

The demographic interpretation of MRLT is the average number of years the persons in cohort could expect to live after age 'x' and is termed as Life Expectancy at age 'x'. It is presented in the last column of Life Tables. It is computed as the ratio of T_x to l_x, where l_x is the expected number of survivors out of hypothetical cohort of 100,000 persons at exact age 'x' or number of survivors at age 'x' and T_x is the expected total number of years lived beyond exact age 'x' by l_x persons alive at that age. It is a projected estimate that applies today's age-specific death rates to forecast the future survival of a specific group or cohort. And, the demographic interpretation of hazard rate is an instantaneous rate of mortality at a certain age measured on an annualized basis which is popularly known as force of mortality. It is calculated as the ratio of number of deaths in one year to the number of survivors at age t. One can compute this $h(t)$ for human population using functions of Life Tables.

Review of Literature

The review of literature follows a chronological narrative, tracing the journey of researchers as they embarked on this research endeavour. The aim is to illustrate how each stage of research contributes to the broader narrative and relates to our own study. To enhance clarity, we will organize this section into a table format. This table will include References to the key papers, their significant findings, and the ensuing discussions.

Some of the previous works on MRLT(or Life expectancy), HR, interrelationship between these two, inference related to these two concepts and their break points in human population and their applications in different fields are:

Table 1

Reference	Key Findings	Discussion
[1]	Acquisition of Resistance in Guinea Pigs infected with Different Doses of Virulent Tubercle Bacilli.	The infecting bacilli dose primarily influences survival time, whereas differences in bacilli virulence modify the quantitative dynamics of initial bacilli numbers and survival duration i.e., Survival time increases as the infecting bacilli dose decreases.
[2]	Testing for a constant hazard rate against a Change-Point Alternative Method	For the Leukemia researchers concern the comparison between a constant failure rate and an alternative failure rate with a single change-point is made.
[3]	Estimation of parameters in hazard rate models with a change-point	A consistent estimator of the change-point is obtained by examining the properties of the density represented as a mixture. The effectiveness of the estimator is validated via simulation.
[4]	MRL function defined as a good decision-making criterion for replacement policies.	The relationship between asymptotic values of the MRL function and the HR of a generic continuous distribution function obtained and established that derivative of the MRL function of a positive random variable always tends to zero as $t \to \infty$.
[5]	Inference for change points in HR.	The likelihood ratio process for detecting change points in hazard rates. By transforming the empirical process into a Poisson process through a random time scale change, the authors have derived ways to estimate significance levels and develop confidence regions for change points and the size of change.
[6]	Estimation of regression parameters in exponential and Cox proportional hazards models.	They investigated two methods for estimation: one is based on exponential regression model's maximum likelihood equation, and the other one using Cox's proportional hazard's structure. The performance of estimators is validated through simulation.
[7]	Life distribution which shows a trend change in MRL.	A new procedure is proposed for testing the exponentiality against IDMRL or DIMRL, assuming that the proportion of the population, p, that dies at or before the change point of MRL is known.
[8]	Life expectancy at birth as a measure of the health status for the populations of England and Wales.	The study has established a methodology which will allow Office for National Statistics (ONS) to calculate life expectancy at birth, with 95% confidence intervals, for populations of 5,000 and more.
[9]	Estimation of regression models with unknown break-points.	The study deals with fitting piecewise terms in regression models where one or more break points are true parameters of the model.
[10]	Estimation of future mortality rates and Life Expectancy in chronic medical conditions.	Projection of old-age mortality in the construction of Life tables and its usefulness in the area of life insurance of elderly population is studied.
[11]	Population longevity in the United States, as measured by life expectancy.	The work describes the demographic context of change in life expectancy of United States.
[12]	Estimation of change-point in hazard and regression parameters.	A parametric survival regression model is studied. The generalizations of hazard change point models are considered and estimators for change points are constructed.
[13]	Theory and	A review of MRL theory and its

	Applications of MRL.	applications are considered. Many useful results concerning MRL have been derived.
[14]	Fitting piecewise terms of regression models.	A simple method to fit segmented relationships in regression models has discussed.
[15]	Detection of multiple change points in piecewise constant hazard functions.	Estimating the underlying hazard function with possible time change points to understand the impact of medical breakthroughs, treatments, or interventions, on the survival experience of a population for reduction in prostate cancer mortality rates.
[16]	Significant relationship between health expenditures and life expectancy.	The paper aims to analyze the relationship between the dynamics of the inputs of the health care system expressed by health care expenditures per capita and the output of the health care systems expressed by life expectancy at birth (years).
[17]	Modelling and analysis for time to failures of aircraft glass.	The lifetime of aircraft glass is assessed through survival modelling technique. The aircraft glass failure data has been taken from the National Institute of Standards and Technology (NIST).
[18]	Estimation in the single change-point hazard function for interval-censored data with a cure fraction.	In this paper, the authors assume that the hazard function is piecewise constant with a single jump at an unknown time. Authors proposed the single change point model for interval-censored survival data with a cure fraction. Estimation methods for proposed model and large-sample properties of the estimators are established.
[19]	Life Expectancy from the point of view of its applicability to epidemiological and public health research.	The author has established that Life Expectancy is not the measure of choice in aetiological research or in research with the aim to identify risk factors of death, but that Life expectancy may be a compelling choice in public health contexts.
[20]	E-Bayesian estimations for the cumulative hazard rate and mean residual life based on exponential distribution and record data.	This study employed a new method, E-Bayesian, for estimating the parameters of exponential distribution using record data. A comparison among the E-Bayesian estimations and earlier methods are made, using a real data and the Monte Carlo simulation.
[21]	Estimation of change-points in HRs using life table data.	Hazard rate at various ages of human life cycle and change points of hazard rate are estimated by using quadratic hazard rate model and mortality index of life tables. Change points in HRs are studied for countries belonging to different income groups and geographical locations.
[22]	Predicting the Expected Time of Diabetes Mellitus Using Exponentiated Gamma Distribution.	The exponentiated gamma model is used to calculate the diabetes mellitus anticipated time. The analytical findings are supported by numerical examples.

The goal of the present paper is to study the break points in MRLTs and HRs for populations of different countries. MRLTs at different ages of life tables of considered countries are used to study the break points in MRLTs. Further, HRs at different ages of populations are computed using MRLTs of Life Tables. Break points in estimated HRs are obtained through segmented regression. The rest of the paper is organized as follows: Section 2 presents data source. Methodology used is discussed in Section 3. Results are summarized in Section 4. Section 5 deals with conclusions, limitations and future scope of the study.

2. Data and its source

The life table represents a hypothetical cohort of 100,000 persons born at the same time who experience the rate of mortality represented by q_x, the probability that a person aged 'x' will die within one year, for each age 'x' throughout their lives. The data is extracted from World Health Organization (WHO) website, from which life tables of 19 countries for the year 2019 are considered. Nineteen countries considered belong to four income groups viz: low income (i1), lower middle income (i2), upper middle income (i3), and high income (i4). The geological regions of these countries are African Region (R1), Region of the Americas (R2), South East Asia Region (R3), European Region (R4), Eastern Mediterranean Region (R5), Western Pacific Region (R6). The countries considered along with their income group and geological region are:

Central African Republic (CAR) (R1,i1), Democratic People's Republic of Korea (DPRK)(R3,i1), Nepal (R3,i1), Afghanistan (R5,i1), Nigeria (R1,i2), Bangladesh (R3,i1), India (R3,i2), Egypt (R5,i2), South Africa (R1,i3), Mexico (R2,i3), Thailand (R3,i3), China (R6,i3), Canada (R2,i4), United States of America (USA) (R2,i4), France (R4,i4), Germany (R4,i4), United Arab Emirates (UAE) (R5,i4), Australia (R6,i4), Japan (R6,i4).

3. Methodology

In order to estimate the points of inflections in MRLTs segmented polynomial MRLTs are used. From life tables of the considered countries, $m(t)$, life expectancy at age t (which are MRLTs at age t) for age groups $0-1, 1-4, 5-9, 10-14, \dots, 80-84$ are used. For all the countries considered, number of class intervals are 18. The plot of $m(t)$ against 't' indicates, there is no linear relationship between 't' and $m(t)$. Hence, the polynomial

$$m(t) = a_0 + a_1 t + a_2 t^2 + \cdots + a_n t^n \qquad (3)$$

is fitted. The required assumption for fitting (3) to the dataset is 'n' should be less than the number of pairs of data values of $(t, m(t))$. Polynomial regression is a unique least-squares solution. The plot of $m(t)$ against 't' is studied in order to fit the polynomial of appropriate degree. The factors like directions of endpoints, flex points, number of turnings of graph of $m(t)$ against 't' are considered in the decision of polynomial degree.

Breakpoints of polynomial are the values of independent variables (in our study ages) at which the polynomial function changes its behaviour. The break points in MRLTs are studied using segmented regression, segmented regression models are used to analyze data with non-linear relationships. The segmented regression models have been used in various fields, including engineering, biomedical, ecology and economics. The segmented regression divides the data into different segments or regions and fits separate linear regression models to each segment. Breakpoints, known as points of inflections are the locations in the data where the relationship between the variables (in our case age and MRLTs) change. Segmented regression method bypasses the non-linear and non-differentials as suggested by [9]. This method uses simple iterative procedure. The one break point model with break point at 'y' as given in [9] is

$$m(t) = m(t) + d(t - t^0)t_1$$

where,

$$t_1 = \begin{cases} 0 & if\ t^0 < y \\ 1 & if\ t^0 > y \end{cases} \quad (4)$$

Similarly, the polynomial with two or more segments or break points can be constructed. R-package "segmented" is used with the data on $(t, m(t))$ to find the break points of $m(t)$, which is developed by [14].

The hazard rates $h(t)$ are estimated using the data on $m(t)$, the MRLTs with the help of the relationship between the two as defined in (2). The procedures of fitting polynomials with appropriate degree and obtaining the break points in MRLTs are repeated in the study of HRs also.

4. Results

Tables 1 and 2 present the results of polynomial regression fit given in (3) along with respective AIC, BIC and break points for MRLTs and HRs respectively. The tables also provide the information of degree (n) of polynomial of best fit. It is observed that for all the countries considered, the degree of polynomial of best fit is less than the number of data values of each country (Class Intervals considered). The break points are the values that correspond to minimum BIC.

The analysis of MRLTs reveals intriguing trends in the break points for different nations:

1. **First breakpoint:**
   - For countries such as Thailand, Nigeria, South Africa, the initial breakpoints in MRLTs occurs at approximately one year or slightly more. Notably, Central African Republic (CAR) exhibits a unique breakpoint at 0.07 years.
   - Countries like the United Arab Emirates (UAE), China, Japan, Australia, and Canada experience their first breakpoint within the age range of 1.5 to 2 years.
   - For the remaining nations, the first break point in MRLTs materializes between ages 2 and 3. The maximum age at which the first break point in MRLT is observed for Egypt.

2. **Second breakpoint:**
   - Among the observed countries, CAR exhibits the lowest recorded second breakpoint at 5 years of age.
   - Nigeria and Thailand witness the second break point around the age of 22, whereas the range slightly extends to 25-28 years for the UAE, China, South Africa, and Canada.
   - A distinct shift is found in the age range of 30 to 32 years for countries like Japan, Egypt, France, Nepal, USA, and Australia. Meanwhile, the same break point is observed at age 41.5 for India, Mexico, Afghanistan, and the Democratic People's Republic of Korea (DPRK).

3. **Third breakpoint:**
   - Notably, most countries manifest a third break point between the ages of 41 and 42, which is the second break point in MRLT for India, Mexico, Afghanistan and DPRK.
   - A few exceptions, including India, Mexico, Afghanistan, Bangladesh, and Democratic People's Republic of Korea (DPRK), exhibit the third break point in a slightly later age range of 46 to 47 years.

4. The degree of polynomial that is best fit to the data is either 3 or 4 except for CAR which records a polynomial of degree 6 to be a best fit.

In case of HRs, the analysis reveals the following:

1. **First breakpoint:**
   The first break point in HRs is observed during the later part of the first year of age or the early part of the second year of age with an exception for Japan, France, Germany, Australia, CAR, India and Afghanistan, where this break point occurs within the age interval of 1.5 to 2 years. Mexico, Egypt and SA have the first break point in HRs at age less than 0.5 year.

2. **Second breakpoint:**
   - Nigeria, Mexico, and South Africa exhibit the second transition in HRs in the age interval of 11 to 12 years, China and Bangladesh both experience their second transition point at 15 years of age, for India it is at age 19, while UAE, Thailand, USA, Canada, DPRK, CAR, and Nepal exhibit the second transition point in HRs in the age range of 25 to 27 years. Japan, France, and Australia all have their second transition points at 35 years of age. Germany and Egypt experience their second transition points at the ages of 32 and 39, respectively. Thus, the second break point in HR is exhibited at various ages by different countries.

3. **Third breakpoint:**
   - The third break point in HRs is notably consistent across the majority of countries, occurring at 39 years of age

with an exception of Egypt, for which the same is being at 71 years.

4. The degree of polynomial that is a best fit to the data of HRs is either 5 or 6.

Figures 1 and 2 are MRLTs plots of Central African Republic (CAR) and South Africa (SA) respectively. Figures 3 and 4 are plots of HRs for Egypt and Nigeria respectively.

5. Conclusions

By examining the plots of MRLTs and studying the breakpoints in MRLTs, an increase in MRLT from birth to first breakpoint for the countries Afghanistan, South Africa, India, CAR, Bangladesh, Nigeria and Nepal can be seen. This may be attributed to the fact that neonatal and perinatal mortality is more in these countries. All these countries belong to either low income or lower middle-income groups except South Africa. MRLTs are stable from birth to first break point for countries Egypt, Mexico and DPRK implying that the mortality risk at birth is less for these countries. A decrease in MRLT is observed from birth to first breakpoint for Thailand, Australia, Japan, UAE, Germany, USA, China, France, and Canada.

The study of HR plots reveals that for all considered countries HR plots are 'J' shaped (tilted towards the right). HRs increase in straight line up to the age of 10 years, then up to the age of 60 years a concave form and from 60 to 80 years an increasing straight line. For Nigeria and CAR, the plots of HR is 'V' shaped during (0, 10) age interval and the least HR being at 5 years of age. The HRs at the age of 80 years are 0.38 for Nepal, India, Egypt, Afghanistan, Bangladesh, South Africa, CAR, and China. Most of these countries belong to the low-income or lower-middle-income groups and/or they have higher population densities. The least HR at the age of 80 years is observed for Mexico, Thailand, Germany, Canada, Japan and Nigeria. HRs at early ages of life and at later ages of life (dependent population) indicate the social setup and health care system of the country.

The study of break points in MRLTs and HRs make it possible to reveal future features of survival and/or mortality experience of a population after certain time (or age).

Limitations and future scope of the Study

Along with the limitations of life tables, use of discrete version of some concepts of continuous r.v.'s negates to accuracy of results. However, one should note limitations of the study while arriving at any conclusions in applications of MRLTs, HRs and their break points. MRLTs, HRs and their break points can be estimated more accurately by modelling the death statistics with appropriate probability laws. Also, the significance of break points needs to tested.

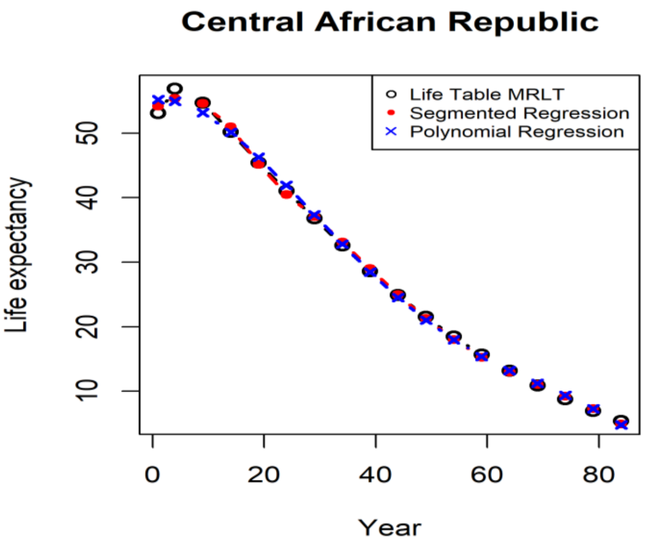

Figure 1. Life table and Fitted MRLTs with Break Points for Central African Republic (CAR)

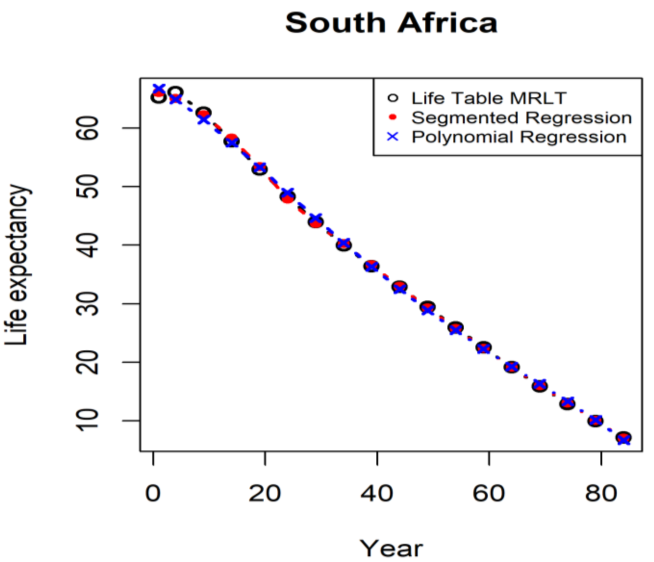

Figure 2. Life table and Fitted MRLTs with Break Points for South Africa (SA)

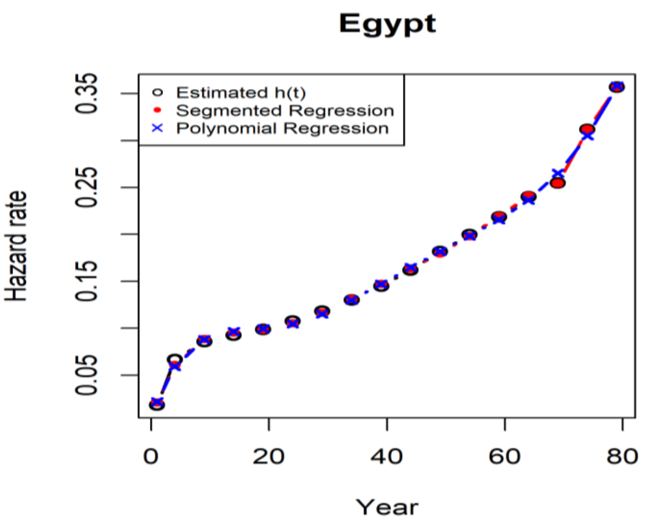

Figure 3. Estimated (using MRLTs) and Fitted HRs with Break Points for Egypt

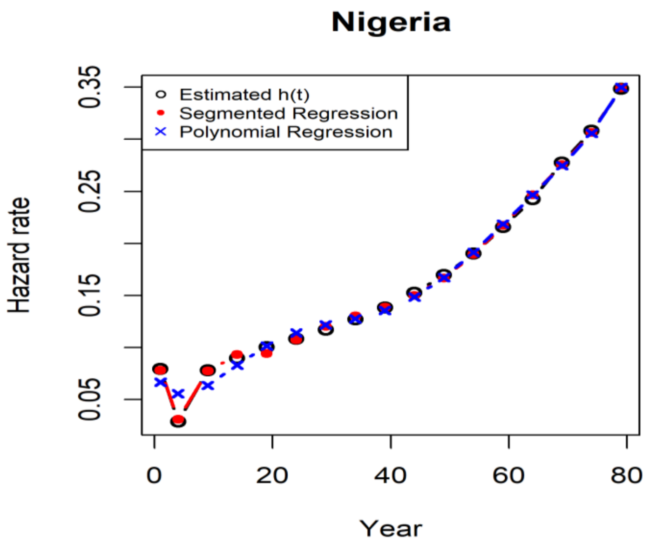

Figure 4. Estimated (using MRLTs) and Fitted HRs with Break Points for Nigeria

Table 1. Breakpoints in MRLTs

Country	AIC	BIC	Break Points			Degrees
CAR	-46.4829	-37.5792	0.07	5.18	41.5	6
UAE	21.36913	27.60174	1.63	26.75	41.5	3
China	22.8726	29.1052	1.69	27.39	41.5	3
Nigeria	46.89731	54.02028	1.18	22.05	41.5	4
Japan	19.60932	25.84192	1.95	30.64	41.5	3
Egypt	35.64678	41.87938	2.77	30.53	41.5	3
France	25.12285	31.35545	2.24	31.37	41.5	3
Thailand	16.94595	24.06893	1.15	21.75	41.5	4
Germany	20.58531	26.81791	2.11	30.44	41.5	3
India	38.68492	45.80789	2.18	41.5	46.87	4
Mexico	29.06567	36.18864	2.14	41.5	47.01	4
Nepal	39.58657	45.81918	2.26	30.64	41.5	3
USA	24.96982	31.20243	2.05	30.29	41.5	3
SA	34.87685	41.99983	1.34	25.73	41.5	4
Afghanistan	47.507	54.62997	2.27	41.5	46.91	4
Australia	20.90807	27.14068	1.91	30.72	41.5	3
Bangladesh	36.166	43.28898	2.35	41.5	46.65	4
Canada	21.804	28.0366	1.79	27.50	41.5	3
DPRK	26.65358	33.77655	2.19	41.5	46.53	4

Table 2. Breakpoints in HRs

Country	AIC	BIC	Break Points			Degrees
UAE	-145.70	-137.37	1.15	26.51	39	6
China	-140.18	-132.68	0.79	15.82	39	5
Nigeria	-133.75	-125.41	0.22	10.56	39	6
Japan	-133.92	-126.42	2.06	35.74	39	5
France	-134.91	-127.42	2.09	35.58	39	5
Thailand	-142.69	-134.36	1.10	26.63	39	6
Germany	-136.10	-128.60	1.64	32.22	39	7
Mexico	-156.72	-148.39	0.4154	11.30	39	6
USA	-145.25	-136.92	1.27	26.26	39	6
Australia	-134.85	-127.35	1.76	35.83	39	5
Canada	-150.57	-142.24	1.09	26.37	39	6
DPRK	-143.70	-135.37	1.20	26.69	39	6
CAR	-104.43	-96.10	1.54	25.60	39	6
Egypt	-139.43	-131.10	0.41	39	71.07	6
India	-149.22	-141.72	1.73	19.44	39	5
SA	-164.32	-156.83	0.45	11.94	39	6
Afghanistan	-149.04	-141.54	2.37	24.38	39	5
Bangladesh	-154.93	-146.60	1.09	15.21	39	6
Nepal	-160.58	-152.25	1.16	27.11	39	6

Conflict of Interest
The authors declare that they have no conflict of interest.

Authors' Contributions
Group efforts at all stages.

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