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Research Article

Numerical Simulation for an Electron Magnetohydrodynamic (EMHD) Nanofluid with Iron Oxide (Fe_3O_4) under the triple effects of an Electric field, Heat generation/Absorption and Impermeability of the Surface

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Abstract— our exploration is mainly concerned with the heat transfer properties of Fe_3O_4 -water base nanofluid past on an exponential impermeable shrinking/stretching surface. We examined how a boundary layer fluid flow toward a shrinking/stretching surface is affected by the combined actions of an electric field, heat generation/absorption and surface impermeability. Partial differential equations (PDEs) are used to illustrate the flow sensation. Making use of the proper similarity transformation technique, the system of ODEs is derived from the PDEs. The shooting method is then applied to these updated equations. According to the analysis, the Momentum profile is amplified with an increase in heat generation/ absorption, variable viscosity and magnetic field. The reverse is the case with an increased impermeability parameter.
Furthermore, the temperature profile is enhanced with an increase in impermeability and electricity. The results may find use in a variety of technical domains, including the optimization of petroleum pipeline flow. The findings can direct further research in this field and advance our understanding of heat and mass transfer phenomena.

Keywords— Heat generation/ absorption, Electric field, Suction/ injection, Magnetic field, Nanofluid.

1. Introduction

Nanofluids are the novel splitting up of fluids engineered by dispersing nanometer-sized materials (Nanoparticles, Nanofibers, nanotubes, nanowires, Nanorods, Nano sheet, or droplets) in base fluids. In other words, nanofluids are nothing but the nanoscale colloidal suspensions which contains the diluted nanomaterials. There are two-phase systems in which one corresponds to solid phase and the other is related to liquid phase. Nanofluids have been initiated to acquire the improved thermo physical properties such as thermal conductivity, thermal diffusivity, viscosity, and convective heat transfer coefficients which were compared to those of base fluids like oil or water. It has been established with great impending applications in many fields.

The idea behind development of nanofluids is to use them as thermo fluids in heat exchangers for enhancement of heat transfer coefficient and thus to minimize the size of heat transfer equipment. The important parameters which influence the heat transfer characteristics of nanofluids are its properties which include thermal conductivity, viscosity, specific heat and density. The thermo physical properties of nanofluids also depend on operating temperature of nanofluids. Hence, the accurate measurement of temperature dependent properties of nanofluids is essential. Common fluids such as water, ethylene glycol, and heat transfer oil plays a momentous position in many manufacturing processes such as power generation, heating or cooling processes, chemical processes, and microelectronics. Though, these fluids have comparatively stumpy thermal conductivity and thus cannot reach elevated heat substitute rates in thermal engineering devices. An approach in the direction of rise above this impermeability in using ultra-fine solid particles balanced in frequent fluids to advance their thermal conductivity. The suspension of nano-sized particles (1/100 nm) in a conventional base fluid is called a nanofluid. Choi [1] first used the term nanofluidin 1995. Nanofluids, compared to suspensions with particles of millimeter-or-micrometer size, show better stability, rheological properties, and considerably higher thermal conductivities.

Magnetic nanoparticles (MNPs) are nanoscale substances with distinctive magnetic characteristics, which have been extensively employed in a variety of sectors [2-8]. The rapid advancement and an unprecedented number of studies have elevated MNPs to the forefront of nanoscience and nanobiotechnology [9- 15]. Certain difficulties in manufacturing monodisperse magnetic nanostructures, such as dipolar interactions, particle surface effects, and size controlling, are of major importance. However, new chemical synthesis techniques have made it simpler to limit the nucleation and proliferation of such MNPs. MNPs comprise either several metallic substances or their magnetic oxides and composites Imran et al. [16]. Due to its great biocompatibility, high surface area, and low toxicity, super paramagnetic magnetite (Fe_3O_4) is the most popular iron oxide or magnetic oxide [17- 20]. Additionally, magnetite (Fe_3O_4), hematite (-Fe_2O_3), and magnetite (-Fe_2O_3) are the three greatest prevalent iron oxides found naturally. Such oxides are highly essential in the domain of science and technology [21- 24]. Moreover, for particular purposes, magnetic moment, adsorption kinetics, and super paramagnetism can be customized throughout the manufacturing process Imran et al. [25].

The impacts of suspended nanoparticles and non-linear thermal radiation on the convection and heat transfer boundary layer flow of nanofluids on a heated vertical sheet were examined by Mahanthesh et al. [26]. The purpose of heat generation/absorption is to decrease and increase a fluid's thermal conductivity, correspondingly. The temperature of the high-conductivity fluid rises, while the low-conductivity fluid exhibits the reverse behaviour as discussed by Noor et al. [27]. Using the power series technique, Qasim [28] examined the effects of the heat generation/ absorption around the temperature and mass movement on a vertical stretching plate.

Madaki et al [29] discussed extensively on the temperature and energy transmission along heat transmission of the MHD composite nanofluid which flows between the squeezing of a pair of parallel plates acting as thermal radiation sources, with vanadium pentoxide (V_2O_5) acting as the solid material (nanoparticle). Squeezing Jeffrey's hybrid nanofluid flow due to its complexity in terms of governing equations. Therefore, their research was aimed at obtaining the mathematical solutions of the Mass and heat transfer in radiant-MHD vanadium pentoxide (V_2O_5)-based squeezing flow Jeffrey nanofluid hybrids which were equipped with heat generation/ absorption. The prime goal of Hussaini et al. [30] was to analyse the effect of Heat generation/absorption on an existing mathematical model. Asghar et al. [31] analysed the influence of heat source/sink as well as effects of the slip velocity through a vertically shrinking sheet. A two-dimensional magnetic nanofluid is numerically considered for convection. They considered Al_2O_3/Cu/water composite nanofluid, where water is deemed the base liquid and copper (Cu) and alumina (Al_2O_3) are the solid nanoparticles. Modern composite nanofluids improve heat transfer efficiency. Using the Tiwari-Das model, they examined the effects of the solid volume fraction of copper, heat generation/absorption, MHD, mixed convection, and velocity slip parameters on velocity and temperature distributions. Sayed [32] studied a numerical estimation of the double-diffusive peristaltic flow of a non-Newtonian Sisko nanofluid through a porous medium inside a horizontal symmetric flexible channel, analyzing the effects of Brownian motion and the thermophoresis coefficients under the influence of Joule heating, non-linear thermal radiation, viscous dissipation, and heat generation/absorption in the presence of heat and mass convection. Hussaini [33] discussed how MHD nanofluid flow on a stretched surface is affected by heat generation and absorption. He also discussed extensively on the influence of other parameters over the profiles of momentum, temperature, nanoparticle concentration as well as Nusselt number. He focused his study on the effects of heat generation/absorption, magnetic field, sun radiation and other parameters.

Wang et al. [34] discussed extensively about the computational analysis designed to examine the effect of alumina and copper on the flow of a nanofluid based on engine oil through a Darcy Forchheimer porous media. The flow model incorporates the generalized radiative heat and mass transport rules. The Darcy Forchheimer terms in the momentum equation and radiation term in the energy equation are integral parts of the governing nonlinear Navier Stokes equations, which are two-dimensional partial differential equations. Under the constraint of the convective boundary, the stated PDEs are transformed into highly non-linear versions of the ordinary differential equations. In order to solve the final ODEs, the numerical RK-45 approach is combined with the shooting methodology. Important aspects of the governing model include porosity, magnetohydrodynamics (MHD), a convective boundary, thermal radiation, and viscous dissipation. The final findings shown that when the Forchheimer number increases, the velocity decreases because of the inertial effect included in the flow model. In addition, the velocity profile is improved due to the increased volume percentage of both kinds of nanoparticles. The temperature varies greatly depending on the volume fraction. A higher Biot number and the resulting convective border cause a greater heat flow than a non-convective barrier. For two particular examples, with and without MHD influence, interesting streamlines and contour graphs are produced. A numerical investigation was conducted by Mahfoud et al. [35] to explore the influence of magnetic field and the electric conductivity of container walls on the swirling flow of a hybrid nanofluid. In this study, a stationary inner wall and a rotating outer wall with a fixed Ω were considered within the annular between coaxial cylinders. Radial application of a magnetic field was utilized to assess its impact on the average Nusselt number. The mathematical model, formulated by differential equations, was solved using the finite volume method. The study examined the variations in azimuthal velocity, temperature, and Nusselt number with increasing magnetic intensity. Waheed et al. [36] investigated the unsteady magnetohydrodynamics (MHD) flow with Darcy-Forchheimer effect and heat transportation. The system incorporates a Casson nanofluid (CNF) with heat transfer during melting and slip velocity, influenced by heat source/sink and thermal radiation. The motivation of investigating the current topic came because of the squeeze flow is of practical physics. The mathematical process involves converting nonlinear partial differential equations

(PDEs) into nonlinear ordinary differential equations (ODEs). The nonlinear ODEs are analytically solved via the Homotopy perturbation method (HPM), while considering the appropriate boundary conditions (BCs). Through a non-dimensional procedure, many dimensionless physical quantities are achieved. The primary results of velocity, temperature profiles, local skin-friction, and the local Nusselt number are shown and analyzed based on several non-dimensional parameters.

The purpose of the study by Mahmood et al. [37] is to inquire entropy generation on viscous $\mathrm{Ti}O_2{-}C_2H_6O_2$ nanofluid through a permeable exponentially surface with porous media and effect that nanoparticle aggregation with thermal radiation, mixed convective stagnation point flow. The controlling partial differential equations were simplified by using an appropriate similarity transformation, resulting in a set of ordinary differential equations. After that, we used Mathematica's shooting technique and fourth order Runge-Kutta integration to get a numerical answer to these equations. Mahmood et al. [38] determine the effect that NP shape has on the entropy produced by $Al_2O_3{-}H_2O$ nanofluid across a permeable MHD stretching sheet under the conditions of quadratic velocity, thermal radiation, and viscous dissipation. H_2O is the cold fluid, while $Al_2O_3{-}H_2O$ nanofluid, which includes five various NP forms (oblate spheroid, platelet, blade, brick, and cylinder), is the hot fluid. Nanofluid containing $Al_2O_3{-}H_2O$ sees widespread use in industrial production because of its remarkable capacity to boost heat transfer. Via a sequence of similarity transformations, the controlling PDEs are converted into a nonlinear differential system of linked ODEs. Madaki et al. [39] discussed on the immense applications of Electro magneto- hydrodynamics (EMHD) in machine building and industries. Their research is based on Magnetohydrodynamics fluid flow over a stretched sheet whose thickness varies which is non-linear. The electric field is also incorporated. They considered base fluid is considered to be water, with nanometer-sized Copper (Cu) particles inside as nanoparticles. they were mainly concerned about the influence caused by some appropriate variables among which are Chemical reaction, induction heat, Dufour diffusivity, Joule heating, variable fluid viscosity, impermeability of the surface, non-uniform heat flux, as well as Viscous dissipation among other parameters on the model.

In view of the previously discussed literature, it is clear that the aim of this research is to explore the nature of electro-magnetohydrodynamic (EMHD) mixed convective nanofluid flow over an impermeable exponential shrinking/stretching surface with heat generation/ absorption effects and the suction/ injection parameters. Conventional base fluid is water with Fe_3O_4 solid nanoparticles. The temperature and velocity profiles are achieved over the numerical results. The present results are appropriately matched with earlier published Kumar et al. [40]. Other relevant researches in this area includes [42- 46]. According to literature survey, there exists no such research which is reported up to yet in line with the mentioned effects above. It is expected that the current investigation will be beneficial for new researchers to apprehend the dynamical and thermal nature of nanofluid.

2. Experimental Procedure

A continuous flow of a fluid that resembles a velocipede across a stretching/ shrinking sheet is taken into consideration, and the momentum, temperature and Nusselt number of the entire surface are kept constant. The fluid has a temperature of T_∞ at the open channels. A homogeneous first-order reaction with a steady pace kc is responsible for the chemical interaction between the liquid and the diffusing components.

It is thought that the fluid's characteristics never change. We induced a magnetic field and electric field by taking a weak magnetic Reynolds number. They are continually applied at maximal intensity. Viscous dissipation is thought to have a major effect. The surface temperature of the stretching/ shrinking sheet is thought to fluctuate, as seen in Fig. 1 below. Assuming the boundary layer approximation is valid, the following are the governing equations for the Boussinesq approximation.

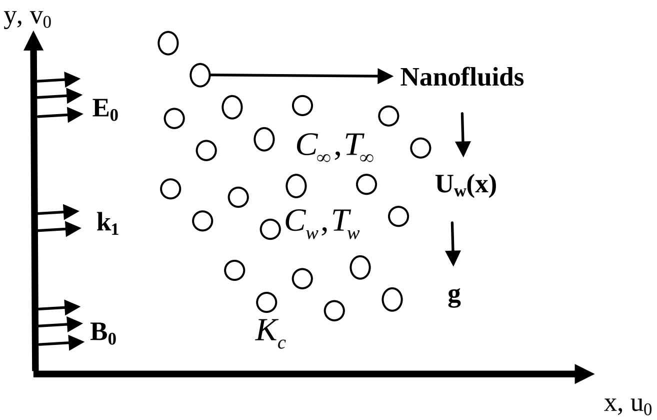

Figure1 The physical model of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho_{nf}}\left[\begin{array}{l}\mu_{nf}\dfrac{\partial^2 u}{\partial y^2}-\sigma_{nf}(E(x)B(x)-\\ B(x)^2u)+g(\rho\beta)_{nf}(T-T_\infty)\end{array}\right]$$
$$+\frac{\mu_e}{k}u, \tag{2}$$

$$(\rho Cp)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k_{nf}+\frac{16T_\infty^3\sigma}{3k_f k_f^*}\right)\frac{\partial^2 T}{\partial y^2}$$
$$+\sigma_{nf}(uB(x)-E(x))^2+\frac{Q}{\rho_{nf}}\left(T-T_\infty\right) \tag{3}$$

The constraints are stated as follows:

$$u=\lambda_1 u_w(x)+A_1\frac{du}{dy}, v=v_w(x), T=T_w(x)$$

$$+B\frac{\partial T}{dy}\ \ at\, y=0,$$

$$u=0, T=T_\infty\ \ as\, y\rightarrow\infty \tag{4}$$

In this case, the fluid density is represented by $\rho_{nf}=(1-\varphi)_{\rho f}+\varphi_{\rho s}\ \rho,\ \ \mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}$ is the dynamic viscosity, $\sigma_{nf}=1+\frac{3\varphi\left(\frac{\sigma_s}{\sigma_f}-1\right)}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)}\sigma_f$ denotes the electric conductivity, $\alpha_{nf}=\frac{k_{nf}}{(\rho Cp)_{nf}}$ denotes the thermal conductivity of the fluid, $(\rho\beta)_{nf}$ represents the thermal expansion of nanofluid, $\frac{\mu_e}{k}u$ and stands for impermeability of the stretching / shrinking surface. $(\rho Cp)_{nf}$ is the nanofluid heat capacitance, is the Q is the radiative heat transfer, $\lambda=\frac{Q}{(\rho Cp)_{nf}T_w\rho_{nf}}$ is the heat generation/ absorption parameter, T is the fluid's temperature. In addition, u and v represent the fluid's velocities in the x and y directions, while B_0 represents the electromagnetic induction. The coefficient of chemical interactions is shown by k_c, and the heat at the free stream is indicated by T_∞, the wall's heat is indicated by T_w. Assuming that the surface temperature varies in the manner described below.

$$T_w(x)-T_\infty=Ax^n \tag{5}$$

The Momentum $f(\eta)$ and Temperature $\theta(\eta)$ are used as a stream term ψ and a similarity term η

$$\eta=y\sqrt{\frac{U_w}{2\vartheta L}}e^{x/2L},\psi=\sqrt{2\vartheta L u_w}\,e^{x/2L}f(\eta),$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

Here, $\psi(x, y)$ denotes stream function that is demarcated into

$$u=\frac{\partial\psi}{\partial y}=Uf'\ and\, v=-\frac{\partial\psi}{\partial x}=\frac{1}{2}\sqrt{\frac{uv}{x}}(\eta f'-f) \tag{7}$$

It is noteworthy that prime denotes differentiation about η. The following is the result of reducing Eq. (1) to Eq. (4) using Eqs. (5) to (7):

$$f'''+(1-\varphi)^{2.5}\left\{\begin{array}{l}(1-\varphi)+\varphi\left(\frac{\rho_s}{\rho_f}\right)\left(-2f'^2+f''f\right)\\ -2\left(1+\frac{3\varphi\left(\frac{\sigma_s}{\sigma_f}-1\right)}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\varphi}\sigma_f\right)\\ (A_2M(E_1-f'))-k_1f'\end{array}\right\}=0, \tag{8}$$

$$\left(\frac{k_{nf}}{k_f}+\frac{4}{3}Rd\right)\theta''+\Pr\left\{(1-\varphi)+\varphi\frac{(\rho Cp)_s}{(\rho Cp)_f}\left(f\theta'-4f'\theta\right)\right\}$$

$$+A_2EcM\left(f'-E_1\right)^2+\lambda\theta=0, \tag{9}$$

The following are the boundary conditions:

$$f'(0)=\lambda_1,\ f(0)=S,\theta(0)=\delta(1-\theta(0))$$

$$f'(\infty)=0,\theta'(\infty)=0, \tag{10}$$

In addition, $A_2=2\vartheta U_w(T_w-T_\infty)$ is the variable viscosity parameter, $M=\frac{\sigma B_0^2L}{\rho U_w}$ is the magnetic field, $E_1=\frac{2LE_0}{\sigma B_0^2U_w}$ is the electric field parameter, $k_1=\frac{2LU_w\mu_e}{k}$ represents the dimensionless impermeability parameter, $\xi=\frac{Gr}{\mathrm{Re}_x^2}$ is the mixed convection parameter such that $Gr=\frac{L^3g\beta_fT_0}{\vartheta f^2}$. $Rd=\frac{4T_\infty^3\sigma^*}{k_fk_f^*}$ Stands as the non-dimensional radiation parameter, $\Pr=\frac{\vartheta(\rho Cp)_f}{k_f}$ is the

Prandtl number, $Ec = \dfrac{U_w^2}{\vartheta\left(T_w(x) - T_\infty\right)}$ stands for the total amount of Eckert number and $\lambda = \dfrac{\vartheta Q}{U_w T_w \rho_{nf}}$ is the heat generation/ absorption parameter. The suction/ injection parameter is given by $S = -\sqrt{\dfrac{2L}{\vartheta U_w}}$, $\lambda_1 = A\sqrt{\dfrac{\vartheta U_w}{2L}}$ is the stretching/ shrinking parameter, $\delta = B\sqrt{\dfrac{U_w}{2\vartheta L}}$ is the thermal slip parameter.

3. Results and Discussion

Numerical solutions of equations 2 and 3 are obtained subject to specified conditions defined in equation 3 by Maple software using the shooting method. This software solves boundary value problems numerically by default using the fourth-fifth order Runge-Kutta- Fehlberg approach, by considering different initial guesses for shrinking and stretching cases of surface. Table 1 is designed to illustrate the physical properties in the numerical form of the base fluid water and solid nanoparticles (Fe_3O_4). The obtained numerical results are numerically compared with already obtained results for correctness and robustness which are available at [40] and [41]. These numerical results show a good agreement (see Table 2). A scenario in which the fluid is a regular fluid devoid of nanoparticles and the stretching/ shrinking surface is kept at a constant temperature in the absence of any heat source, sink, or mass suction effects. Furthermore, the profiles of velocities and temperatures are achieved at different values of physical parameters. Figs. 2–20 demonstrate the computed numerical results to elaborate the silent features related to the presence of various flow parameters used in velocity, skin friction, the heat transfer equations and the Nusselt number profiles.

Momentum Profile

Fig. 2 is plotted to depict the influence of heat generation/ absorption parameter over the momentum profile. In this case irrespective heat generation $(\lambda > 0)$ or heat absorption $(\lambda < 0)$, any increase in the parameter increases the momentum of the fluid. It can be observed on fig. 3 which is about the influence of impermeability parameter (k_1) over the momentum profile, it is seen that a large impermeability parameter enhances the velocity profile. The presence of k_1 allows the exchange of fluid particles among regions within the boundary layer. Now, increasing the values for k_1 expands the pore size, hence providing space for more movement of fluid particles. Naturally, the wall thickness is placed within the range of 0.5mm to 4mm. A rudimentary scheme standard is to retain wall thicknesses as tinny as probable.

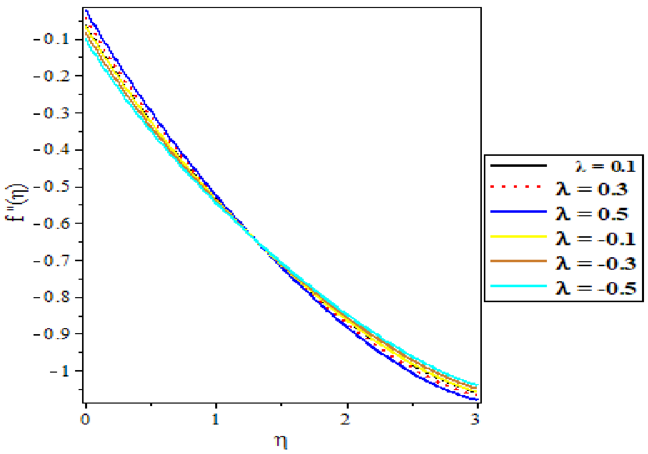

Fig. 2: Influence of heat generation/ absorption parameter on Momentum Profile

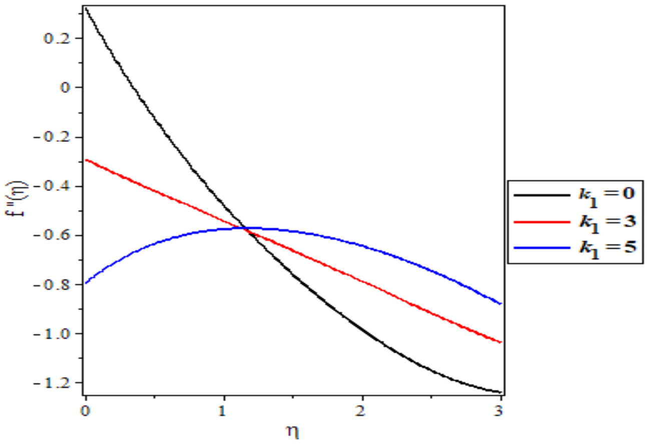

Fig. 3: Influence of Impermeability Parameter on Momentum Profile

The variation of momentum under the impact of the Electric field parameter (E_1) is exhibited in Fig. 5; it is observed that the growth in the parameter lessens the momentum for both the ground fluid and the nanofluid. The underlying mechanism of this observation could be that the increased electric field parameter enhances the efficiency of heat dissipation through radiative processes, affecting the fluid's thermal energy and, consequently, its momentum. Similarly, fig. 6 indicates the influence of the stretching/shrinking parameter (λ_1) over the momentum profile. It can be observed that for any increment in the parameter, the momentum profile is decreased. On the contrary fig. 7, shows the effect of the magnetic field parameter (M), on the momentum profile above the stretching/shrinking sheet. Like the elasticity number, an increase in the parameter is seen to increase velocity components at any point above the stretching/ shrinking surface. This is because of the retarding effects of the Lorentz force set forth by the magnetic field. Interestingly, the effect of the magnetic number on flow kinematics is seen to be more pronounced than the effect of the elasticity number. An increase in the velocity components, u and v, should decrease the amount of heat transferred from the surface to the fluid.

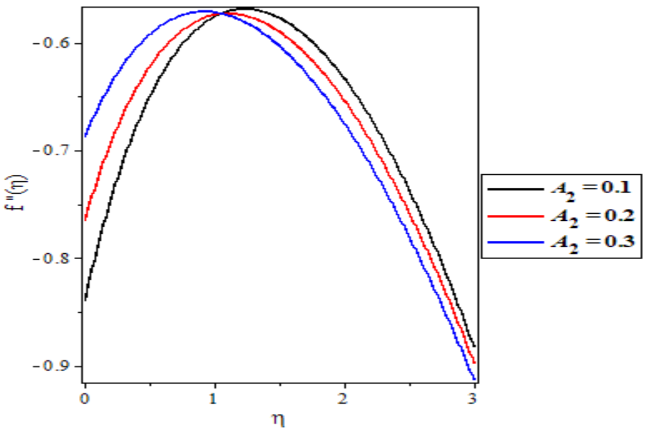

Fig. 4: Influence of Variable Viscosity parameter on Momentum Profile

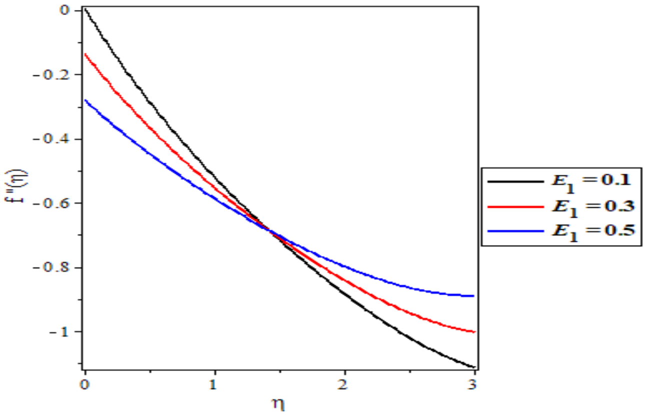

Fig. 5: Influence of Electric Field on Momentum

The influence of the Suction/ injection parameter (S) on the profile of momentum is illustrated in Fig. 8. One conclusion we can get from these results is that the wall mass suction/ injection is very significant in maintaining the steady boundary layer near the surface by delaying the separation. In this figure, we can observe that as the values of the suction parameter ($S > 0$) increase it decreases the momentum of the fluid. On the other hand, as the values for the injection parameter ($S < 0$) are increased it causes an increase in the momentum profile. The effect of the mixed convection parameter (ξ) on the profile of momentum is illustrated in Fig. 9. It is vividly observable that an increase in the parameter causes an instant decrease in the momentum profile as well as its boundary layers' thicknesses

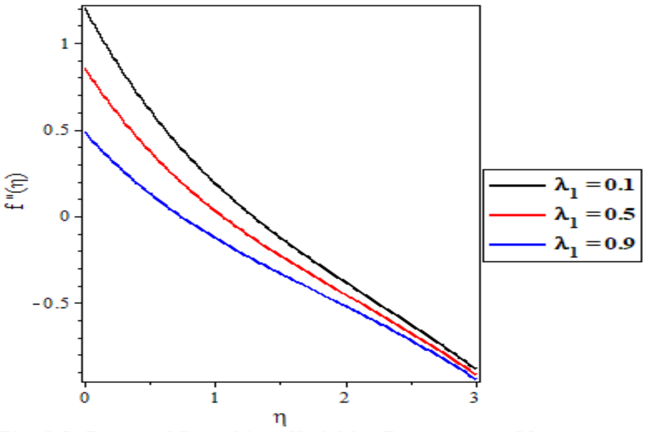

Fig. 6: Influence of Stretching/ Shrinking Parameter on Momentum Profile

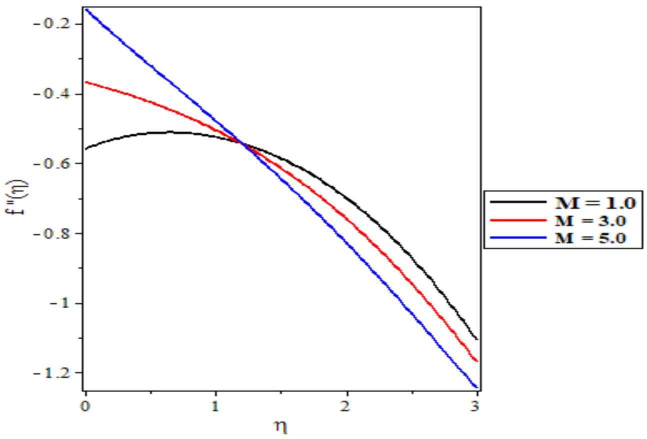

Fig. 7: Influence of Magnetic Field Parameter on Momentum Profile

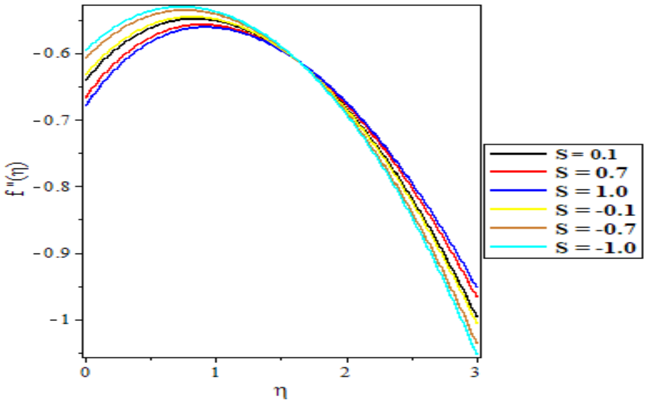

Fig. 8: Influence of Suction/ Injection Parameter on Momentum Profile

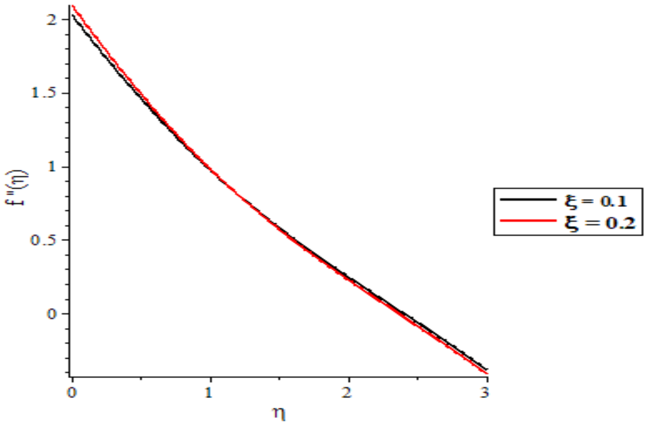

Fig. 9: Influence of Mixed Convective Parameter on Momentum Profile

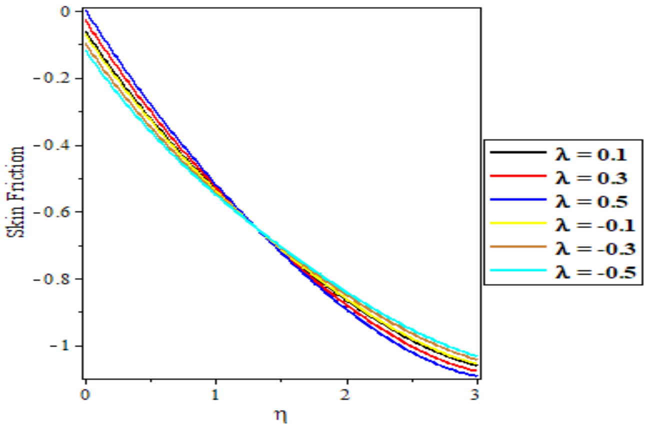

Fig. 10: Influence of Heat generation/ Absorption on Skin Friction Profile

The variation of skin friction under the impact of the heat generation parameter $(\lambda > 0)$ and heat absorption parameter $(\lambda < 0)$ is depicted in Fig. 10; it is observed that the growth in the $(\lambda < 0)$ parameter lessens the skin friction for both the ground fluid and the nanofluid. On the other hand, in the case of $(\lambda > 0)$ underlying mechanism of this observation could be that the increased parameter enhances the efficiency of skin friction, affecting the fluid's thermal energy and, consequently, its skin friction.

Temperature Profile

A closer look at Fig. 11 will illuminate how the temperature distribution is rising together with the enhancement of the heat generation parameter $(\lambda > 0)$ as well as the heat absorption parameter $(\lambda < 0)$ values. Physically, the average kinetic energy of the fluid particles causes an increase in the temperature distribution field transit irrespective of heat generation/absorption, this is because the temperature affects the kinetic energy involved in the movement of the fluid molecules and nanoparticles, which also results in tiny gaps between the fluid molecules, which causes an increase in the movement and speed of the fluid molecules within the channel. The influence of impermeability parameter (k_1) is depicted in Fig. 12, a large impermeability parameter enhances the velocity profile. The presence of k_1 allows the exchange of fluid particles among regions within the boundary layer. Now, increasing the values for k_1 expands the pore size, hence providing space for more movement of fluid particles. Naturally, the wall thickness is placed within the range of 0.5mm to 4mm. A rudimentary scheme standard is to retain wall thicknesses as tinny as possible. The effects of the variable viscosity parameter (A_2) on the energy profile are displayed in Fig. 13, it is clearly visible that an increase in the values of the parameter causes a rise in the thickness of the thermal boundary layer and hence the temperature of the system is enhanced. This is because the energy variance among the fluid surface and the neighbouring air decreases.

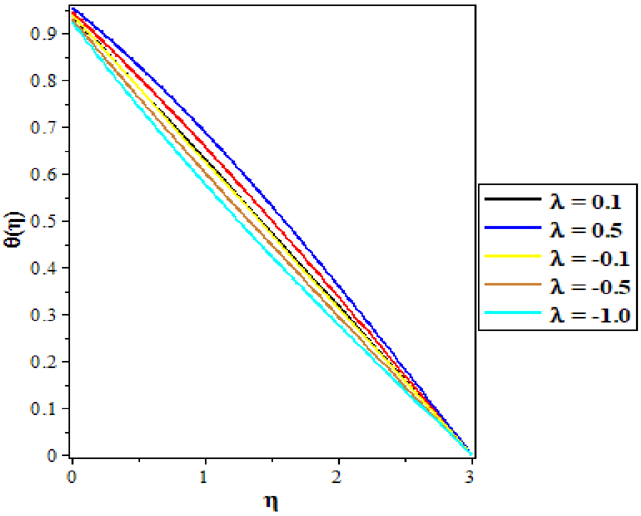

Fig. 11: Influence of Heat generation/Absorption on Temperature Profile.

The Electricity parameter (E_1) influence on the boundary layer temperature is shown in Fig. 14: where it is worth mentioning that, as the fluid considered is indeed at high viscosity, thus, the electricity with both small and large values were chosen, so as to see the nature of their impact upon the thermal boundary layer. It is found that as E_1 increases, the temperature of the fluid within the boundary layer decreases rapidly. The variation of temperature under the impact of the Magnetic field (M) is exhibited in Fig. 15; it is observed that the growth in the parameter lessens the temperature distribution for both the ground fluid and the nanofluid. The underlying mechanism of this observation could be that the increased magnetic field parameter enhances the efficiency of heat dissipation through radiative processes, affecting the fluid's thermal energy and, consequently, its temperature distribution.

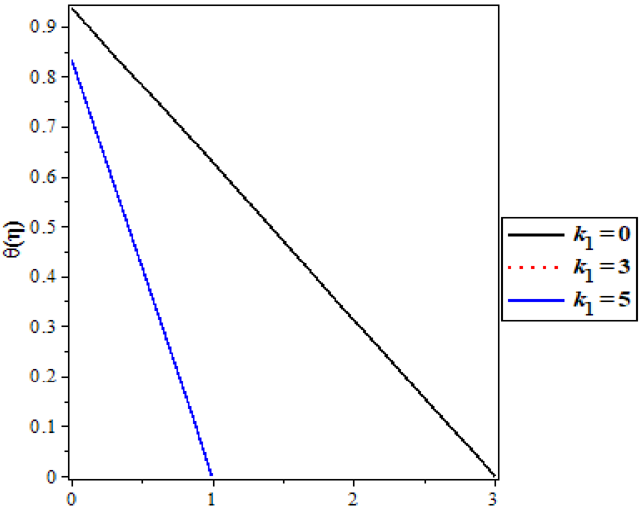

Fig. 12: Influence of Impermeability parameter on Temperature Profile

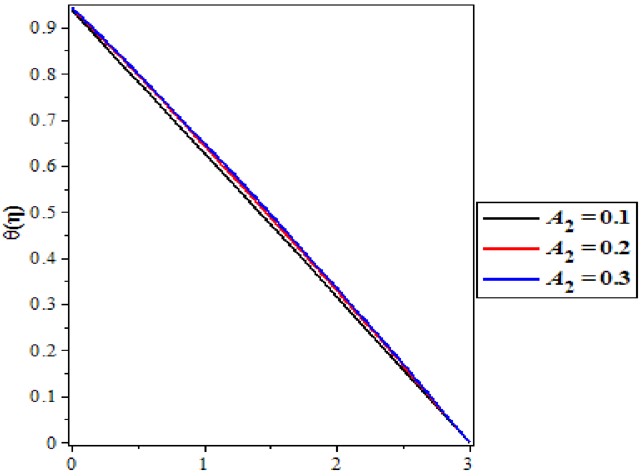

Fig. 13: Influence of Variable Viscosity parameter on Temperature Profile

However, Fig. 16, depicts the impact of varying Prandtl number parameters (Pc) on the temperature profile. An increment in the Prandtl number implies an increase in the kinetic energy of the fluid particles, which increases the vibration of particles and leads to collisions. Due to this collision, the dissipation of heat in the boundary layer's region increases the fluid temperature.

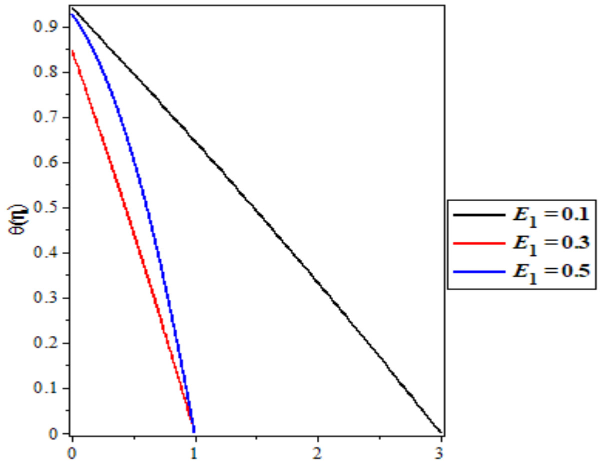

Fig. 14: Influence of Electric Field on Temperature Profile

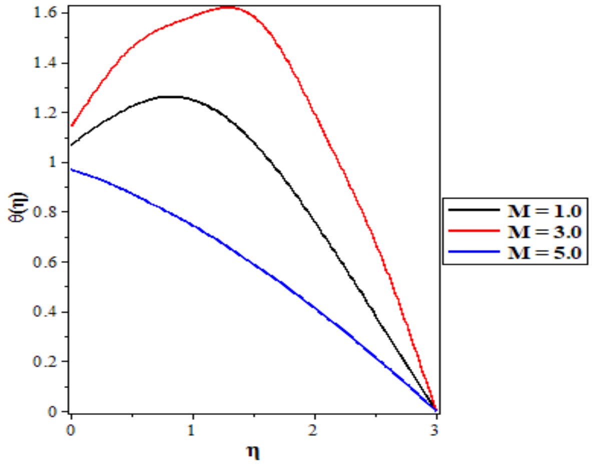

Fig. 15: Influence of Magnetic Field on Temperature Profile

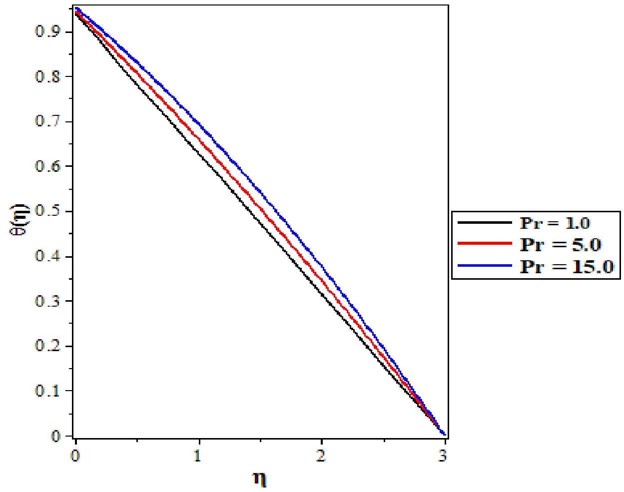

Fig. 16: Influence of Prandtl number on Temperature Profile

Nusselt Number Profile

On the other hand, a closer look at Fig. 17, illustrates how the Nusselt number distribution is rising together with the enhancement of the electric field parameter (E_1) values. However, in Fig. 18 we can observe how the Nusselt number distribution is rising together with the enhancement of the heat generation parameter (λ >0) values. Physically, the average kinetic energy of the fluid particles causes a decrease in the Nusselt number distribution field transit from the heat absorption parameter (λ1 < 0), this is because the Nusselt number parameter affects the kinetic energy involved in the movement of the fluid molecules and nanoparticles, which also results in tiny gaps between the fluid molecules, which causes a decrease in the movement and speed of the fluid molecules within the channel. Moreover, Fig. 19 is plotted to explain the effect of the Prandtl number parameter (Pr) over the local Nusselt number profile. Here we have studied that any increment in the values of the parameter brings about a decrease in skin friction coefficient and a reverse for the case of the local Nusselt number. Physically speaking, this seems to be reasonable. It's noteworthy to notice that for the Prandtl number parameter, the local Nusselt number for the fluid in the presence of nanoparticles is larger than the fluid in its absence.

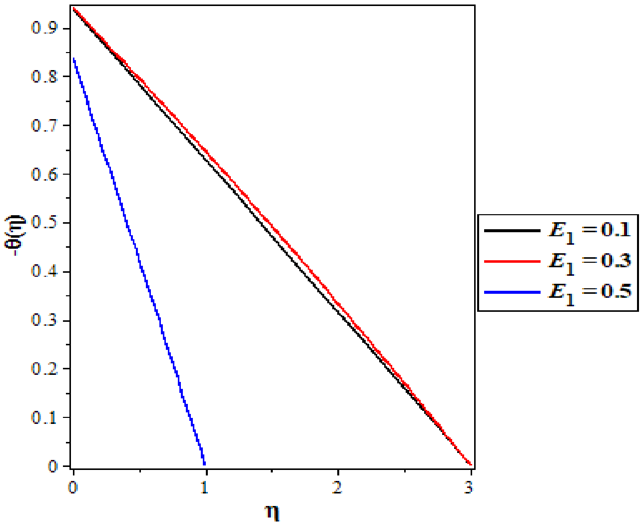

Fig. 17: Influence of Electric Field on Nusselt Number Profile

The impacts of mixed convection parameter (ξ) on Nusselt number profile. The profile of the Nusselt number behaves erratically as the parameter increases. The Nusselt number exhibits extemporaneous comportment. The presence of hot air close to the powder could potentially be a contributing factor. The Nusselt number profile is heightened by the mixed convection parameter when the external temperature is zero. Figure 20 displays the fluctuation of the Nusselt number profile with mixed convection parameter (ξ). While the fluid's temperature and velocity do not significantly change when the mixed convection parameter (ξ) grows, it is noticed that an increase in mixed convection parameter (ξ) value increases the Nusselt number of species in the boundary layer. This is due to the fact that the mixed convection parameter (ξ) in this system causes the chemical to be consumed, which lowers Nusselt number profile. The primary outcome is that the first-order chemical reaction tends to reduce overshoot in the Nusselt Number profile in the solutal border layer.

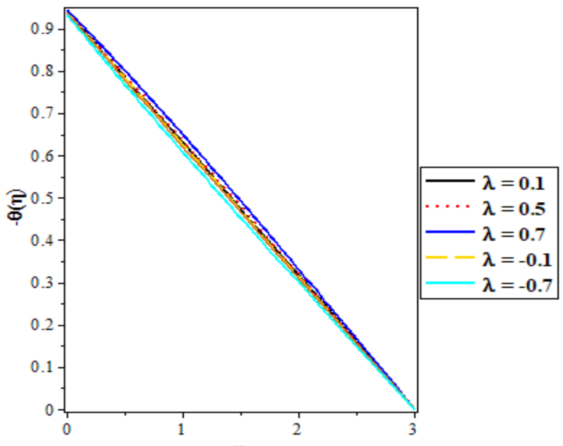

Fig. 18: Influence of Heat generation/absorption on Nusselt Number Profile

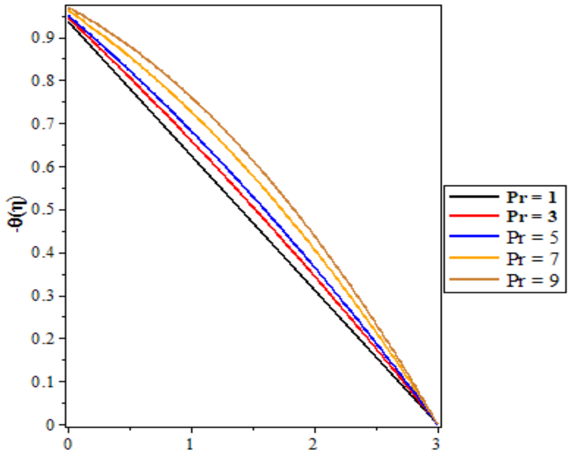

Fig. 19: Influence of Prandtl number on Nusselt Number Profile

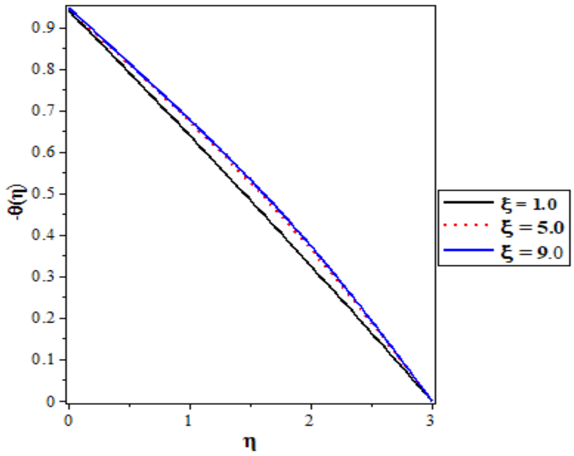

Fig. 20: Influence of Mixed convective parameter on Nusselt Number Profile

Table 1: The thermo-physical properties of water and nanofluid (base fluid) as in Kumar et al [40]

	$\rho(kgm^{-3})$	$Cp(kg^{-1}k^{-1})$	$k(Wm^{-1}k^{-1})$	$\beta\times10^{-5}(k^{-1})$
Water	997.1	4179	0.6130	21
Fe_3O_4	5180	670	80.4	20.6

Table 2: Comparative values of $f''(0)$ and $-\theta'(0)$ with published results of [40].

Varying parameter	**Kumar et al. [40]**		**Present result**	
S	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
0	1.28181	0.767768	1.28181	0.767766
0.3	-	-	1.38501	0.896891
0.6	1.5982	1.014571	1.59828	1.014580
0.8	-	-	1.70173	1.732067
1.0	-	-	1.98016	2.497629

4. Conclusion and Future Scope

A numerical study was conducted on mixed convection EMHD boundary layer laminar flow and the heat and mass transfer of Fe_3O_4-water base nanofluid on stretching/ shrinking surface with heat generation/ absorption parameter and impermeability of the surface. The equations of the present problem are modified from partial differential equations to a set of ordinary differential equations. The similarity solution of governing ordinary differential equations is acquired and solved in the Maple software by shooting technique. For validation of the obtained results, a comparison of the numerically obtained values of local Nusselt numbers and skin friction coefficients are shown in Table 2. Furthermore, based on numerical observations of different physical parameters, the following conclusions are made:

I. The Momentum profile is raised with the increase in both heat generation/absorption, magnetic field, variable viscosity as well as injection parameters. On the other hand, the momentum profile is decreased with an increase in permeability parameter, electric field, and stretching/shrinking as well as mixed convection parameters.
II. Prandtl number and heat generation/absorption parameters enhance the temperature of the system. While the reverse is the case as observed for magnetic field, variable viscosity, impermeability as well as electric field parameters.
III. The skin friction coefficient is increased with heat generation it decreases with heat absorption.
IV. As the Prandtl number rises, so does the Nusselt number profile. Heat generation, electricity and mixed convection parameters. Reverse is the case with heat absorption.

In conclusion, our study has not only filled a significant knowledge vacuum regarding boundary layer flow involving heat generation/ absorption and impermeability effects, but it also has promise for future use in a range of scientific and technical fields. By taking into account the impacts of chemical reactions or diffusion effects, the results direct future study and add to the corpus of information pertaining to both mass and heat transmission. It also provides workable answers for streamlining industrial procedures and fluid flow systems.

Data Availability

None.

Conflict of Interest

Author (s) declare that they do not have any conflict of interest.

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Nomenclature

x, y: Cartesian Coordinate
u, v: Components of velocities in the x and y-axes.
T : Temperature
T_∞ : Ambient fluid temperature
θ: Dimensionless temperature
Cp: Specific heat at the constant temperature
ρ: Density
knf: Thermal conductivity of nanofluid
αnf: Thermal diffusivity of nanofluid
$(\rho\beta)nf$: Thermal expansion of nanofluid
$(\rho Cp)nf$: Nanofluid heat capacitance
μnf: Dynamic viscosity of nanofluid
η: Transformed variable
Fe_3O_4 Iron oxide
B_0 Magnetic field strength
ψ: Stream function
Pr: Prandtl number
ξ: Mixed convective parameter
Uw: Constant of velocity of the nanofluid
λ: *heat generation/ absorption*
λ_1 : Stretching/shirking parameter
δ_T : Thermal slip parameter
Rex: Local Remolds number
Cfx: Skin friction coefficient
Nux: Nusselt number
Rd: Radiation parameter
S: suction/ injection parameter
δ: Velocity slip parameter
M: Magnetic parameter
k_1 : *Impermeability parameter*
E_1 : *Electric field*

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