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Research Article

Reliability, Maintainability and Sensitivity Analysis of Poultry Feed Processing Plant

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Abstract— This research examines the availability, maintainability and sensitivity of a poultry feed processing plant to optimize operational performance. Availability is assessed using Mean Time between Failures (MTBF) and Mean Time to Repair (MTTR), revealing a gradual decline over time due to equipment failures. Maintainability analysis focuses on repair efficiency, highlighting the plant's ability to quickly restore machinery after breakdowns. Sensitivity Analysis identifies preventive maintenance frequency and spare parts availability as key factors affecting system performance. The findings provide means to reduce downtime and improve operational reliability in poultry feed processing. Graphical analysis of availability shows a consistent decline over time across all equipment. Maintainability analysis reveals that despite a high initial recovery rate, minor deviations in repair times can significantly impact overall operational uptime.

Keywords— Poultry, Processing Plant, **Availability, Maintainability, Sensitivity,** feed

1. Introduction

Several notable contributions have emerged within system reliability and performance analysis. For instance [1] undertook pioneering work by developing and meticulously comparing reliability models tailored to varying demand cold standby systems. Their research shed light on the intricate dynamics of such systems, providing valuable insights into their reliability characteristics and operational effectiveness. Similarly, [2] made significant strides in the field with their ground breaking models designed for the performance analysis of complex repairable systems under pre-emptive resume repair strategies. By looking into the complexities of repairable systems and incorporating pre-emptive repair strategies into their models, also it advanced the state-of-the-art in understanding and optimizing the performance of such systems [3]. Their work not only expanded the theoretical framework but also offered practical implications for enhancing system reliability and efficiency in real-world applications. These exemplary studies represent just a fraction of the extensive research efforts aimed at advancing the understanding and analysis of system reliability and performance in the context of solar photovoltaic and solar water pumping systems. Collectively, they underscore the interdisciplinary nature of this field and highlight the on-going quest for innovative methodologies and insights to address the pressing challenges and opportunities in sustainable energy systems.

2. Related Work

The evaluation of expenses related to intricate systems managed by human operators. The study looked into the complexities surrounding human involvement in system operation and upkeep, presenting valuable perspectives on the economic implications and cost determinants affecting the overall efficiency and dependability of such systems [4]. Also, [5] made notable advancements in the field through the development and rigorous examination of a dual-objective optimization model, specifically tailored for series-parallel systems. Employing sophisticated mathematical modeling and optimization methodologies, the research introduced an innovative framework aimed at concurrently addressing reliability and cost considerations in the design and administration of series-parallel systems. This work provided tangible solutions for enhancing system performance and economic viability [6]. An exhaustive investigation into the reliability and operational efficacy of series-parallel systems utilizing copula-based methodologies was carried out [7]. Leveraging copula functions, their study presented a robust approach to capturing the interdependence among system elements and evaluating the overall reliability and performance attributes of the series-parallel configurations. Their findings offered valuable insights into the interconnectedness of system components and their impact on system reliability across diverse operational scenarios.

Reliability, availability, maintainability, and dependability (RAMD) represent fundamental pillars in the evaluation and optimization of system performance across industries. These metrics serve as indispensable tools in the arsenal of plant management, enabling them to gauge the efficacy and resilience of their systems and to implement targeted interventions for improvement [8]. RAMD analysis serves as a strategic framework through which plant management can discern the critical components or subsystems within a system requiring prioritized maintenance interventions. By identifying and addressing these areas proactively, plant managers can bolster the overall performance and longevity of the system [9]. This analytical approach entails evaluating the system at various stages of its lifecycle, employing a diverse array of performance modelling methodologies tailored to the specific context and requirements of the system under scrutiny. Through systematic RAMD analysis, significant performance indicators are derived, offering invaluable insights into the operational dynamics of the system. Among the key metrics derived from RAMD evaluation are Mean Time between Repairs (MTBR) and Mean Time To Repair (MTTR), which provide crucial insights into the frequency and duration of downtime experienced by the system. Availability, reliability, and maintainability metrics offer further granularity, shedding light on the system's ability to consistently deliver optimal performance and to swiftly recover from potential failures [10].

Ensuring the reliability and availability of systems while enhancing their features represents a paramount goal for engineers, and the RAMD (Reliability, Availability, Maintainability, and Dependability) approach stands as a cornerstone in achieving this objective. Building upon this premise, researchers have diligently pursued the development of diverse maintenance models and strategies aimed at optimizing system performance and bolstering RAMD metrics [11].

An analysis centered on the reliability, availability, and maintainability of a cement plant. Their investigation illuminated the operational intricacies inherent in cement production processes, providing valuable strategies to optimize system performance and elevate RAMD metrics within industrial contexts [12].

A performance measure decision-making approach tailored for T-spherical operators, thereby enriching decision-making processes aimed at enhancing system performance and reliability [13]. The optimization of profit and availability within a single-unit system featuring imperfect switchover was deliberated upon [14]. Their discussion provided actionable strategies for maximizing system profitability while ensuring optimal availability.

A comprehensive study on the Reliability, Availability, Maintainability, and Dependability Analysis of Cold Standby Series-Parallel System. Their research significantly contributed to advancing the understanding and optimization of system reliability and availability within intricate industrial setups [15].

3. Theory/Calculation

Let δ_i and μ_i represent the failure rate and repair rate of the system for some $i = 1,2,3,\ldots,7$

$S_0(t)$: Probability that the system is operating at maximum capacity

$S_i(t);\ i = 0,1,2,3,\ldots 7$: Steady-state probability that the system is in i^{th} state.

$\delta_i;\ i = 1,2,3,\ldots 7$: Failure rate of the subsystem A,B,C,D,E,F and G

$\mu_i;\ i = 1,2,3,\ldots 7$: Repair rate of the subsystem A,B,C ,D,E,F and G

3.1 Assumptions

The study considers a repairable system made up of seven component/subsystems connected in series: A, B, C, D, E, F and G. All the Subsystem A consists of single units, in which all the seven subsystems must be in operating for the entire system to work if at least one subsystem fail the whole system will collapse. The system might be repaired in any case. Thus, every subsystem is as good as new after the repair.

3.2 Description of the System

Subsystem A (Crushing)**:** It's the first step after receiving the raw material. Any grain go through this process to undergo size reduction and increase the surface area for the greater nutritional value for the poultry.

Subsystem B (Mixer)**:** The main objective of this component is to combine the ingredients together to ensure they are distributed in the mixture properly.

Subsystem C (Pelleting Machine)**:** This is the process of transforming soft dusty feed into a hard pellet. This process involves passing the feed mixture through a conditioning chamber where steam is added.

Subsystem D (Cooling)**:** This process aims to reduce the temperature of the pellets resulting from the pelleting process and it also results in increasing the hardness of the pellets.

Subsystem E (Crumbling)**:** This process is to break down the pellets into small pieces that can be easily consumed by the chicks.

Subsystem F (Sieving)**:** Sifting is required when producing pellets. Usually, small fragment are produced as a result when the hot, moist pellets are cut off from the die inside the pelleting chamber, and as produced pellets pass through the cooling and conveying process.

Subsystem G (Coating)**:** Fats and oil can be added in this process to further improve the nutritional value of the pellets. This aims to add the remaining amount of oils that could not be added before the pelleting process.

4. Experimental Method/Procedure/Design

The equipment for the computation of RAMD measures for the model are as follows, when all the failure and repair rate obey the exponential distribution.

I. Exponential distribution

A random variable X is said to follow an exponential distribution with $\omega > 0$ as parameter, if its probability density function is given by

$$f(x,\omega) = \begin{cases} \omega e^{-\omega x}, if\ x \geq 0 \\ 0, \quad o,w \end{cases} \quad (1)$$

II. Constant failure rate

The constant failure rate function can be written as follows:

$$f(t,\omega) = \begin{cases} \omega e^{-\omega t}, if\ t \geq 0 \\ 0, \quad o,w \end{cases} \quad (2)$$

Where ω is a constant with probability density function, with $H(t) = 1 - e^{-\omega t}$ and $R(t) = 1 - e^{-\omega t}$ (3)

III. Reliability

The ability of a device to perform its function within the stipulated time is termed as reliability

$$R(t) = e^{-\int_0^{\infty} f(t)dt} \quad (4)$$

For a component with an exponential distributed failure rate equation (4) can be reduced to

$$R(t) = e^{-\delta t} \quad (5)$$

IV. Availability

The chance that a device will operates in a specific state within a specific period of time is known as availability

$$\text{Availability} = \frac{MTBF}{MTBF+MTTR} \quad (6)$$

V. Maintainability

When the maintenance is observed fluently to the need level is refer to as system maintainability

$$M(t) = P(T \leq t) = 1 - e^{-\mu t} \quad (7)$$

Where λ is the repair rate of the system.

VI. Dependability

This is the design criterion, according to Wohl. It's mathematically defined as

$$d = \frac{\mu}{\delta} = \frac{MTBF}{MTTR} \quad (8)$$

The following formula calculate the minimum value of dependability

$$D_{min} = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d\ln d}{d-1}\right)}\right) \quad (9)$$

VII. MTTR

Mean Time To Repair is mathematically defined as

$$MTTR = \mu^{-1} \quad (10)$$

Where μ is the repair rate.

VIII. MTBF

Mean Time Between Failure for an exponentially distributed system is as follows

$$MTBF = \int_0^{\infty} R(t)\,dt = \int_0^{\infty} e^{-\delta t}dt = \delta^{-1} \quad (11)$$

Where δ is the failure rate.

Table 1 Failure and repair rate

Subsystem	Failure rate (δ)	Repair rate (μ)
Crushing (A)	$\delta_1 = 0.015$	$\mu_1 = 0.35$
Mixer (B)	$\delta_2 = 0.025$	$\mu_2 = 0.20$
Pelleting machine (C)	$\delta_3 = 0.010$	$\mu_3 = 0.15$
Cooling (D)	$\delta_4 = 0.035$	$\mu_4 = 0.40$
Crumbling (E)	$\delta_5 = 0.050$	$\mu_5 = 0.55$
Sieving (F)	$\delta_6 = 0.025$	$\mu_6 = 0.15$
Coating (G)	$\delta_7 = 0.011$	$\mu_7 = 0.41$

4.1 Formulation of Mathematical Models for RAMD

In this section, Chapman Kolmogorov differential equations for each subsystem have been constructed using the Markov birth-death process for mathematical modelling of poultry feed processing plant. System performance measures such as reliability, availability, maintainability and dependability have been derived by solving the appropriate Chapman-Kolmogorov differential equations in a steady-state and employing normalization conditions recursively.

$$\frac{dS_0(t)}{dt} = -\delta_k S_0 + \mu_k S_1 \quad (12)$$

$$\frac{dS_1(t)}{dt} = -\delta_k S_1 + \mu_k S_0 \quad (13)$$

For $k = 1,2,3,\ldots,7.$

Solving (12) and (13) in a stable state $\frac{dS_i(t)}{dt} = 0, i = 0,1.$

We have

$$-\delta_k S_0 + \mu_k S_1 = 0 \quad (14)$$

$$-\delta_k S_1 + \mu_k S_0 = 0 \quad (15)$$

By applying the normalizing condition $\sum S_i = 1; i = 0,1.$ We have

$$S_0 = \frac{\mu_k}{\mu_k + \delta_k}, \quad S_1 = \frac{\delta_k}{\mu_k} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{sys}(t) = e^{-\delta_k t} \quad (16)$$

$$A_{sys}(t) = \left(1 + \frac{\delta_k}{\mu_k}\right)^{-1} \quad (17)$$

$$M(t) = 1 - e^{-\mu t} \quad (18)$$

$$D_{min} = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d\ln d}{d-1}\right)}\right) \quad (19)$$

4.2 RAMD Analysis for Subsystem A (Crushing unit)

In subsystem crushing unit, there is only one unit consist in series configuration with other subsystems and failure of it cause the failure of complete system. Differential- difference equations for the subsystem is derived using birth–death processes on the basis of the recurrence relations are as follows:

For $k = 1$

$$\frac{dS_0(t)}{dt} = -\delta_1 S_0 + \mu_1 S_1 \quad (20)$$

$$\frac{dS_1(t)}{dt} = -\delta_1 S_1 + \mu_1 S_0 \quad (21)$$

Solving (12) and (13) in a stable state $\frac{dS_i(t)}{dt} = 0, i = 0,1.$

We have;

$$-\delta_1 S_0 + \mu_1 S_1 = 0 \quad (22)$$

$$-\delta_1 S_1 + \mu_1 S_0 = 0 \quad (23)$$

By applying the normalizing condition, $\sum S_i = 1; i = 0,1.$ We have

$$S_0 = \frac{\mu_1}{\mu_1 + \delta_1}, \quad S_1 = \frac{\delta_1}{\mu_1} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{S_A}(t) = e^{-\delta_1 t} = e^{-0.012t}$$
(24)

$$A_{S_A}(t) = \left(1 + \frac{\delta_1}{\mu_1}\right)^{-1} = 0.9090 \quad (25)$$

$$M_{S_A}(t) = 1 - e^{-\mu_1 t} = 1 - e^{-0.12t} \quad (26)$$

$$D_{min_A}(t) = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d \ln d}{d-1}\right)}\right) = 0.9140$$
(27)

Other performance indicators of subsystem A are given as follows:

$MTBF = 83.333h$, $MTTR = 8.333h$, $d = 10.000$, $D_{min_{SA}} = 0.9140$

4.3 RAMD Analysis for Subsystem B (Mixer)

In subsystem mixer unit, there is only one unit consist in series configuration with other subsystems and failure of it cause the failure of complete system.

For $k = 2$

$$\frac{dS_0(t)}{dt} = -\delta_2 S_0 + \mu_2 S_1 \quad (28)$$

$$\frac{dS_1(t)}{dt} = -\delta_2 S_1 + \mu_2 S_0 \quad (29)$$

Solving (12) and (13) in a stable state $\frac{dS_i(t)}{dt} = 0, i = 0,1.$

We have;

$$-\delta_2 S_0 + \mu_2 S_1 = 0 \quad (30)$$

$$-\delta_2 S_1 + \mu_2 S_0 = 0 \quad (31)$$

By applying the normalizing condition, $\sum S_i = 1; i = 0,1.$

We have;

$$S_0 = \frac{\mu_2}{\mu_2 + \delta_2}, \quad S_1 = \frac{\delta_2}{\mu_2} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{S_B}(t) = e^{-\delta_2 t} = e^{-0.014t} \quad (32)$$

$$A_{S_B}(t) = \left(1 + \frac{\delta_2}{\mu_2}\right)^{-1} = 0.9028 \quad (33)$$

$$M_{S_B}(t) = 1 - e^{-\mu_2 t} = 1 - e^{-0.13t} \quad (34)$$

$$D_{min_B}(t) = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d \ln d}{d-1}\right)}\right) = 0.9972$$
(35)

Other performance indicators of subsystem B are given as follows:

$MTBF = 76.923h$, $MTTR = 7.692h$, $d = 9.286$, $D_{min_{SB}} = 0.9972$

4.4 RAMD Analysis for Subsystem C (Pelleting Machine)

In subsystem Pelleting Machine, there is only one unit consist in series configuration with other subsystems and failure of it cause the failure of complete system.

For $k = 3$

$$\frac{dS_0(t)}{dt} = -\delta_3 S_0 + \mu_3 S_1 \quad (36)$$

$$\frac{dS_1(t)}{dt} = -\delta_3 S_1 + \mu_3 S_0 \quad (37)$$

Solving (1-3) in a stable state $\frac{dS_i(t)}{dt} = 0, i = 0,1.$

$$-\delta_3 S_0 + \mu_3 S_1 = 0 \quad (38)$$

$$-\delta_3 S_1 + \mu_3 S_0 = 0 \quad (39)$$

By applying the normalizing condition, $\sum S_i = 1; i = 0,1.$ We have

$$S_0 = \frac{\mu_3}{\mu_3 + \delta_3}, \quad S_1 = \frac{\delta_3}{\mu_3} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{S_C}(t) = e^{-\delta_3 t} = e^{-0.009t} \quad (40)$$

$$A_{S_C}(t) = \left(1 + \frac{\delta_3}{\mu_3}\right)^{-1} = 0.9244 \quad (41)$$

$$M_{S_C}(t) = 1 - e^{-\mu_3 t} = 1 - e^{-0.11t} \quad (42)$$

$$D_{min_C}(t) = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d \ln d}{d-1}\right)}\right) = 0.9972$$
(43)

Other performance indicators of subsystem C are given as follows:

$MTBF = 111.111h$, $MTTR = 12.222h$, $d = 9.091$, $D_{min_{SC}} = 0.9972$

4.5 RAMD Analysis for Subsystem D (Cooling)

In subsystem Cooling unit, there is only one unit consist in series configuration with other subsystems and failure of it cause the failure of complete system.

For $k = 4$

$$\frac{dS_0(t)}{dt} = -\delta_4 S_0 + \mu_4 S_1 \quad (44)$$

$$\frac{dS_1(t)}{dt} = -\delta_4 S_1 + \mu_4 S_0 \quad (45)$$

Solving (1-3) in a stable state $\frac{dS_i(t)}{dt} = 0, i = 0,1.$

$$-\delta_4 S_0 + \mu_4 S_1 = 0 \quad (46)$$

$$-\delta_4 S_1 + \mu_4 S_0 = 0 \quad (47)$$

By applying the normalizing condition, $\sum S_i = 1; i = 0,1.$ We have

$$S_0 = \frac{\mu_4}{\mu_4 + \delta_4}, \quad S_1 = \frac{\delta_4}{\mu_4} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{S_D}(t) = e^{-\delta_4 t} = e^{-0.015t}$$
(48)

$$A_{S_D}(t) = \left(1 + \frac{\delta_4}{\mu_4}\right)^{-1} = 0.9244 \quad (49)$$

$$M_{S_D}(t) = 1 - e^{-\mu_4 t} = 1 - e^{-0.35t} \quad (50)$$

$$D_{min_D}(t) = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d \ln d}{d-1}\right)}\right) = 0.9972$$
(51)

Other performance indicators of subsystem D are given as follows:

$MTBF = 3333.34\ h$, $MTTR = 57.4\ h$ $d = 345$, $D_{min_{SD}} = 0.9995$

4.6 RAMD Analysis for Subsystem E (Crumbling)

In subsystem Crumbling, this is only one unit consist in series configuration with other subsystems and failure of it cause the failure of complete system.

For $k = 5$

$$\frac{ds_0(t)}{dt} = -\delta_5 S_0 + \mu_5 S_1 \quad (52)$$

$$\frac{ds_1(t)}{dt} = -\delta_5 S_1 + \mu_5 S_0 \quad (53)$$

Solving (52 - 53) in a stable state $\frac{ds_i(t)}{dt} = 0, i = 0,1.$

$$-\delta_5 S_0 + \mu_5 S_1 = 0 \quad (54)$$

$$-\delta_5 S_1 + \mu_5 S_0 = 0 \quad (55)$$

By applying the normalizing condition, $\sum S_i = 1; i = 0,1.$ We have

$$S_0 = \frac{\mu_5}{\mu_5 + \delta_5}, \quad S_1 = \frac{\delta_5}{\mu_5} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{S_E}(t) = e^{-\delta_5 t} = e^{-0.016t} \quad (56)$$

$$A_{S_E}(t) = \left(1 + \frac{\delta_5}{\mu_5}\right)^{-1} = 0.9244 \quad (57)$$

$$M_{S_E}(t) = 1 - e^{-\mu_5 t} = 1 - e^{-0.20t} \quad (58)$$

$$D_{min_E}(t) = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d \ln d}{d-1}\right)}\right) = 0.9972 \quad (59)$$

Other performance indicators of subsystem E are given as follows:

$MTBF = 3456\ h,\quad MTTR = 2d\ h,\quad d = 23456,$
$D_{min_{SE}} = 0.9875$

4.7 RAMD Analysis for Subsystem F (Sieving)

In subsystem Sieving unit, there is only one unit consist in series configuration with other subsystems and failure of it cause the failure of complete system.

For $k = 6$

$$\frac{ds_0(t)}{dt} = -\delta_6 S_0 + \mu_6 S_1 \quad (60)$$

$$\frac{ds_1(t)}{dt} = -\delta_6 S_1 + \mu_6 S_0 \quad (61)$$

Solving 60 -61 in a stable state $\frac{ds_i(t)}{dt} = 0, i = 0,1.$ We've

$$-\delta_6 S_0 + \mu_6 S_1 = 0 \quad (62)$$

$$-\delta_6 S_1 + \mu_6 S_0 = 0 \quad (63)$$

By applying the normalizing condition, $\sum S_i = 1; i = 0,1.$ We have

$$S_0 = \frac{\mu_6}{\mu_6 + \delta_6}, \quad S_1 = \frac{\delta_6}{\mu_6} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{S_F}(t) = e^{-\delta_6 t} = e^{-0.014t} \quad (64)$$

$$A_{S_F}(t) = \left(1 + \frac{\delta_6}{\mu_6}\right)^{-1} = 0.9244 \quad (65)$$

$$M_{S_F}(t) = 1 - e^{-\mu_6 t} = 1 - e^{-0.15t} \quad (66)$$

$$D_{min_F}(t) = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d \ln d}{d-1}\right)}\right) = 0.9972 \quad (67)$$

Other performance indicators of subsystem F are given as follows:

$MTBF = \quad h,\quad MTTR = \quad h,\quad d = \quad ,$
$D_{min_{SF}} =$

4.8 RAMD Analysis for Subsystem G (Coating)

In subsystem Coating, there is only one unit consist in series configuration with other subsystems and failure of it cause the failure of complete system.

For $k = 7$

$$\frac{ds_0(t)}{dt} = -\delta_7 S_0 + \mu_7 S_1 \quad (68)$$

$$\frac{ds_1(t)}{dt} = -\delta_7 S_1 + \mu_7 S_0 \quad (69)$$

Solving 68 - 69 in a stable state $\frac{ds_i(t)}{dt} = 0, i = 0,1.$

$$-\delta_7 S_0 + \mu_7 S_1 = 0 \quad (70)$$

$$-\delta_7 S_1 + \mu_7 S_0 = 0 \quad (71)$$

By applying the normalizing condition, $\sum S_i = 1; i = 0,1.$ We have

$$S_0 = \frac{\mu_7}{\mu_7 + \delta_7}, \quad S_1 = \frac{\delta_7}{\mu_7} S_0$$

The RAMD measures of the system can be determine by the following equations:

$$R_{S_G}(t) = e^{-\delta_6 t} = e^{-0.017t} \quad (72)$$

$$A_{S_G}(t) = \left(1 + \frac{\delta_7}{\mu_7}\right)^{-1} = 0.9244 \quad (73)$$

$$M_{S_G}(t) = 1 - e^{-\mu_7 t} = 1 - e^{-0.40t} \quad (74)$$

$$D_{min_G}(t) = 1 - \left(\frac{1}{d-1}\right)\left(e^{-\left(\frac{\ln d}{d-1}\right)} - e^{-\left(\frac{d \ln d}{d-1}\right)}\right) = 0.9972 \quad (75)$$

Other performance indicators of subsystem G are given as follows:

$MTBF = 233.5h,\quad MTTR = 34\ h,\quad d = 8,$

$D_{min_{SG}} = 0.9857$

5. Results and Discussion

5.1 System description

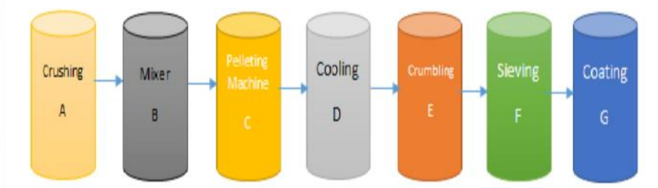


Fig1. System description

5.1.1 System reliability

Because all four subsystems are linked in series, the failure of one causes the entire system to fail. The whole system's reliability is determined by:

$$R_{sys} = R_{S_A}(t) * R_{S_B}(t) * R_{S_C}(t) * R_{S_D}(t) * R_{S_E}(t) * R_{S_F}(t) * R_{S_G}(t)$$
$$= (e^{-\delta_1 t})(e^{-\delta_2 t})(e^{-\delta_3 t})(e^{-\delta_4 t})(e^{-\delta_5 t})(e^{-\delta_6 t})(e^{-\delta_7 t})$$
$$= (e^{-0.012t})(e^{-0.014t})(e^{-0.009t})(e^{-0.015t})(e^{-0.016t})(e^{-0.018t})(e^{-0.017t})$$
$$R_{sys} = e^{-0.101t}$$

The variation in reliability with respect to time is analysed using above equation.

5.1.2 System availability

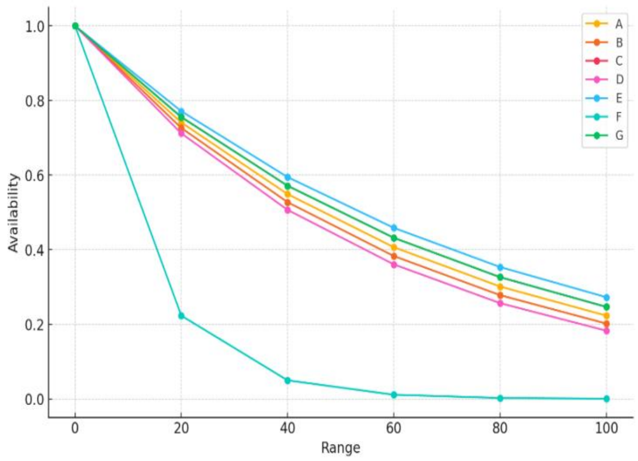
As all the seven subsystems are interconnected in series, the failure of any one subsystem leads to the failure of the entire system.

The overall availability of the system is determined by:

$$A_{Sys}(t) = A_{S_A}(t) * A_{S_B}(t) * A_{S_C}(t) * A_{S_D}(t) * A_{S_E}(t) * A_{S_F}(t) * A_{S_G}(t)$$

Table 2 Variation in subsystem reliability over time

TIME	A	B	C	D	E	F	G
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0.7408	0.7261	0.7557	0.7117	0.7710	0.2231	0.7550
40	0.5488	0.5272	0.5712	0.5066	0.5945	0.0497	0.5712
60	0.4065	0.3828	0.4317	0.3605	0.4584	0.0111	0.4317
80	0.3011	0.2780	0.3262	0.2566	0.3534	0.0024	0.3262
100	0.2231	0.2018	0.2465	0.18268	0.2725	0.0005	0.2465



5.2 System maintainability

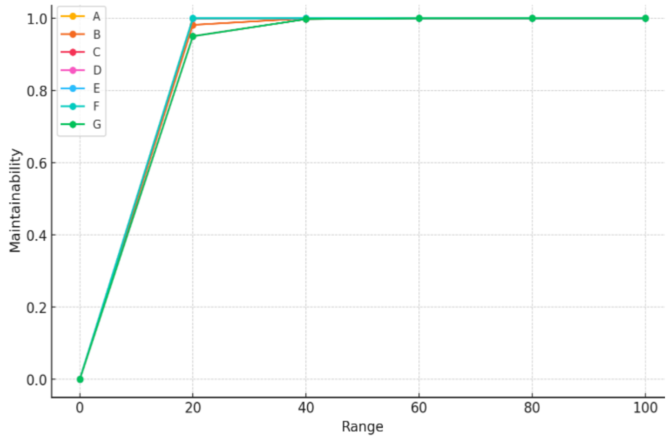
As all four subsystems are interconnected in series, the failure of any one subsystem leads to the failure of the entire system. The overall maintainability of the system is determined by:

$$M_{Sys}(t) = M_{C_A}(t) * M_{C_B}(t) * M_{C_C}(t) * M_{C_D}(t) * M_{C_E}(t) * M_{C_F}(t) * M_{C_G}(t)$$

$$M_{Sys}(t) = 1 - e^{-0.17t}$$

Table 3 Variation in subsystem maintainability over time

	A	B	C	D	E	F	G
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.9990	0.9816	0.9502	0.9996	0.9999	0.9999	0.9502
40	0.9999	0.9996	0.9975	0.9999	0.9999	1.0000	0.9975
60	0.9999	0.9999	0.9998	1.0000	1.0000	1.0000	0.9998
80	1.0000	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999
100	1.0000	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999



The variation in maintainability with respect to time is analysed using above equation.

5.3 System dependability

As all four subsystems are interconnected in series, the failure of any one subsystem leads to the failure of the entire system. The total system resiliency is determined by:

$$D_{min_{Sys}} = D_{min_{SA}} * D_{min_{SB}} * D_{min_{SC}} * D_{min_{SD}} * D_{min_{SE}} * D_{min_{SF}} * D_{min_{SG}}$$

5.4 Sensitivity analysis

It is a technique which is used to identify the impact of independent variable on a specific dependent variable on the basis of some assign assumptions. It determine the effect of the change in parameters and structure of the model. Here, sensitivity analysis for reliability of the subsystems and system with respect to failure rates μ 1 , μ 2 , μ 3, μ 4 , μ 5 , μ 6and μ 7 has been performed. The following expressions have been derived respectively:

5.5 Numerical simulation

Table 4: RAMD indices for subsystem

Indices	Subsystem A	Subsystm B	Subsystem C	Subsystem D	Subsystems E	Subsystem F	Subsystem G
Reliability	$e^{-0.0013t}$	$e^{-0.005t}$	$e^{-0.003t}$	$e^{-0.0052t}$	$e^{-0.0013t}$	$e^{-0.005t}$	$e^{-0.0052t}$
Availability	0.9999 99	0.9965 65	0.9965 65	0.9965	0.9999 9 9	0.9965 6 5	0.9965
Maintainability	$1 - e^{-0.45t}$	$1 - e^{-0.082t}$	$1 - e^{-0.082t}$	$1 - e^{-0.082t}$	$1 - e^{-0.45t}$	$1 - e^{-0.082t}$	$1 - e^{-0.082t}$
Dependability ratio	346.188 88	16.393 93	286.616 16	16.393	346.188 88	16.393 9 3	16.393
MTBF	769.23 23	200 0 0	333.333 33	200	769.23 2 3	200 00	200
MTTR	2.222 22	12.20 20	1.163 63	12.20	2.222 22	12.20 20	12.20

Numerical simulations of reliability, maintainability, and Sensitivity analysis are discussed in this section.

This section discusses the numerical simulations in order to obtain understanding of how the strength, efficacy, and performance of the model under review are evaluated at various levels. Here,
From this table 13 and its corresponding figure 6, we can see that the system reliability's equivalent values for main unit at time $t = 40$ are $Rel._{subsystem\ A} = 0.9872, Rel._{subsystem\ B} = 0.9671, Rel._{subsystem\ C} = 0.6967, Rel._{subsystem\ D} = 0.9403,$ and $Rel._{subsystem\ D} = 0.7964.$
In time $t = 40$, there is $Main._{system} = 0.32632241$ chance of successfully completing maintenance and repairs, and $Main._{subsystem\ A} = 0.9999$, $Main._{subsystem\ B} = 0.9996$, $Main._{subsystem\ C} = 0.9975$, $Main._{subsystem\ D} = 0.9999$ and $Main._{subsystem\ E} = 0.9999$. The system is 0.3363 times reliable at $t = 60$ due to a form decline. This is brought on by the low reliability value of subsystem C. This demonstrates that subsystem C is the main unit's key subsystem. The value of availability is another indicator of how important subsystem C is to the main unit.

Subsystems with the lowest reliability value among the other subsystems need adequate attention of the management for proper maintenance in order to avoid system breakdown and subsequent loss of production and revenue as the tables and figures make sufficient evident. This demonstrates that critical subsystems are the most important and delicate part of the system and needs careful consideration.

6. Conclusion and Future Scope

6.1 Conclusion

This study has looked into the intricacies of RAMD analysis to scrutinize the reliability and maintainability of individual components and subsystems within the system. Through a meticulous examination of RAMD measures, including failure rates, repair rates, reliability, and maintainability, we have identified the most sensitive components that significantly impact the overall system performance. The expressions associated with RAMD measures for each subsystem were derived and rigorously validated through numerical simulations, ensuring the accuracy and reliability of our findings. Our analysis, as depicted in Tables 1, 2, 5, and 6, along with corresponding Figures 2 and 3, has shed light on the influence of varying failure rates on subsystems and system reliability. Notably, our numerical observations underscore a critical insight: the reliability of the entire system is intricately linked to the maintainability of the system. This highlights the pivotal role of maintainability in ensuring sustained system reliability and operational efficiency over time.

Drawing from our findings, we advocate for the adoption of the RAMD approach as a strategic framework to enhance system performance and mitigate the risk of subsystem failures. By implementing proactive maintenance strategies informed by RAMD analysis, stakeholders can pre-emptively address reliability issues, optimize system operation, and minimize downtime. Additionally, prioritizing the enhancement of subsystem maintainability not only fosters the smooth operation of individual components but also safeguards the integrity of the entire system. In essence, the RAMD approach offers a robust methodology to bolster system resilience, promote operational continuity, and mitigate the adverse effects of component failures.

6.2 Future Scope

By leveraging the insights gleaned from RAMD analysis, organizations can optimize resource allocation, streamline maintenance practices, and ultimately ensure the sustained performance and reliability of complex systems in diverse operational environments.

Data Availability

The research is based on a differential equations and numerical simulation has provided all the data needed for clarity.

Study Limitations

None.

Conflict of Interest

All Authors declare that they do not have any conflict of interest.

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Authors' Contributions

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