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Regression-type Estimators Based on Two Auxiliary Variables of a Finite Population Mean in Two-phase Sampling

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Abstract--This paper suggests a class of regression-type estimators of a finite population mean incorporating auxiliary information on two variables at estimation stage in two-phase sampling. This class of estimators includes many known estimators. Up to the first order of approximation the mean square error (MSE) and optimal MSE are obtained and compared with the MSEs of the estimators included in this paper. Also, an empirical comparison is carried out using a Monte Carlo simulation over three natural populations.

Keywords-- *Auxiliary variables, mean square error, optimum estimation, regression-type estimator, simulation, two-phase sampling.*

I. INTRODUCTION

Two-phase sampling is a powerful and cost-effective technique. It was first proposed by Neyman [1] and subsequently discussed by many authors. The basic results on simple ratio and regression estimators of the population mean using single auxiliary variable in two-phase sampling is found in Cochran [2]. Many authors have extended these estimators using the known parameters of *x*. Further extension has been done using two auxiliary variables, see Mohanty [3], Khare and Srivastava [4], Sahoo et al.[5], Singh and Upadhyaya [6], Upadhyaya and Singh [7], Samiuddin and Hanif [8,9], among others.Many often even if $\overline{X}$ is unknown, information on a cheaply ascertainable variable z, closely related to x but compared to x remotely related to y, is available on all population units. In such situations, various estimators have been proposed, see, e.g., Chand [10], Kiregyera [11,12], Mukerjee et al. [13], Srivastava et al. [14], Sahoo et al. [15], Tracy et al.[16], Singh and Espejo [17], Gupta and Shabbir [18], Singh et al. [19], Shukla et al. [20], Choudhury and Singh [21] among others, where some of them have used the knowledge on mean $\overline{Z}$, standard deviation σ_z, coefficient of variation C_z, coefficient of skewness $\beta_1(z)$, coefficient of kurtosis $\beta_2(z)$ or combinations of these parameters. Motivated from this work, in this paper we suggest a class of estimators that includes many known estimators and obtain the lower bound of the approximate variance of this class. We also suggest one optimal estimator and compare it with some available estimators using real data.

Consider a finite population $U = \{1, \dots, i, \dots, N\}$ of N identifible units. Let y be the study variable and, x and z be auxiliary variables taking the values (y_i, x_i, z_i) respectively for the population unit i. When the two variables y and x are strongly positively related but no information on $\overline{X}$, the population mean of x, is available a two-phase sampling can be used. In this scheme the first-phase sample s' of size n' is selected from U, according to a simple random sampling design and witout replacement (srswor), to obtain a good estimator of $\overline{X}$. Given s', a second-phase sample s of size n is selected from s' acording to srsrwor. Let $\overline{u}' = \sum_{s'} u_i/n'$, $\overline{u} = \sum_s u_i/n$ and $\overline{v} = \sum_s v_i/n$ where, $u = (x, y, z), b_{uv}$ (b'_{uv}) denotes the regression coefficient of u and v for subsample (preliminary sample)

Our objective is to estimate the population mean $\overline{Y} = \sum_U y_i/N$, where Σ_A deonets $\Sigma_{i\in A}$ for any arbitrary set of units $A \subseteq U$. A more extensive use of available auxiliary information is achived through the regression estimators. The two-phase ordinary regression estimator (Cochan [2]) is given by

$$\hat{\hat{\overline{Y}}}_{Regd} = \overline{y} + b_{yx}(\overline{x}' - \overline{x}) \qquad (1)$$

where b_{yx} denotes the regression coefficient of y and x for sub-sample s. The variance of $\hat{\hat{\overline{Y}}}_{Regd}$ to the first order of approximation is

$$V\left(\hat{\hat{\overline{Y}}}_{Regd}\right) \approx \overline{Y}^2 C_y^2\left[f_2 + f_3\left(1 - \rho_{yx}^2\right)\right] \qquad (2)$$

where, $f_1 = n^{-1} - N^{-1}$, $f_2 = n'^{-1} - N^{-1}$, and $f_3 = f_1 - f_2$

and, C_y and ρ_{yx} are the population coefficient of variation of y and regression coefficient of y and x respectively.

Chand [10] suggested a chain ratio estimator

$$\hat{\hat{\overline{Y}}}_c = \left(\overline{x}'\overline{Z}/\overline{z}'\right)\overline{y}/\overline{x} \qquad (3)$$

Kiregyera [11,12] proposed the following three estimators:

$$\hat{\hat{\overline{Y}}}_{k1} = \left[\overline{x}' + b'_{xz}\left(\overline{Z} - \overline{z}'\right)\right]\overline{y}/\overline{x} \qquad (4)$$

$$\hat{\hat{\overline{Y}}}_{k2} = \overline{y} + b_{yx}\left(\overline{x}'\overline{Z}/\overline{z}' - \overline{x}\right) \qquad (5)$$

$$\hat{\hat{\overline{Y}}}_{k3} = \overline{y} + b_{yx}\left[(\overline{x}' - \overline{x}) + b'_{xz}\left(\overline{Z} - \overline{z}'\right)\right] \qquad (6)$$

This paper is organized as follows. In next section we propose the class of estimators. Its approximate variance and the optimum variance are obtained in Section III. An empirical comparison is carried out in Section IV. The conclusion is given in Section V.

II. PROPOSED ESTIMATOR AND THE CLASS OF REGRESSION-TYPE ESTIMATORS

Here we propose the class of regression-type estimators which is defined as

$$t_{Regcd} = \overline{y} + b_{yx}\left[\overline{x}'\frac{c\overline{Z} + d}{c\overline{z}' + d} - \overline{x}\right] \qquad (7)$$

where $c \neq 0$ and d are either real numbers or the known population summary statistics of the auxiliary variable z. The subclass of (7) for specific choice $b_{yx} = \overline{y}/\overline{x}$ is

$$t_{Rcd} = \bar{y}\frac{\bar{x}'}{\bar{x}}\left[\frac{c\bar{Z}+d}{c\bar{z}'+d}\right] \quad (8)$$

These two classes of estimators include the following estimators for various choices of a and b:

$$t_{R1} = \frac{\bar{y}}{\bar{x}}\bar{x}'\frac{\bar{Z}}{\bar{z}'}$$

(Chand [10])

$$t_{Reg1} = \bar{y} + b_{yx}\left[\bar{x}'\frac{\bar{Z}}{\bar{z}'} - \bar{x}\right] = \hat{\hat{\bar{Y}}}_{k2}$$

(Kiregyera [11])

$$t_{R2} = \frac{\bar{y}}{\bar{x}}\bar{x}'\left[\frac{\bar{Z}+C_z}{\bar{z}'+C_z}\right] \qquad t_{Reg2} = \bar{y} + b_{yx}\left[\bar{x}'\frac{\bar{Z}+C_z}{\bar{z}'+C_z} - \bar{x}\right]$$

(Singh and Upadhyaya [6])

$$t_{R3} = \frac{\bar{y}}{\bar{x}}\bar{x}'\left[\frac{\beta_2(z)\bar{Z}+C_z}{\beta_2(z)\bar{z}'+C_z}\right] \qquad t_{Reg3} = \bar{y} + b_{yx}\left[\bar{x}'\frac{\beta_2(z)\bar{Z}+C_z}{\beta_2(z)\bar{z}'+C_z} - \bar{x}\right]$$

(Upadhyaya and Singh [7])

$$t_{R4} = \frac{\bar{y}}{\bar{x}}\bar{x}'\left[\frac{C_z\bar{Z}+\beta_2(z)}{C_z\bar{z}'+\beta_2(z)}\right] \qquad t_{Reg4} = \bar{y} + b_{yx}\left[\bar{x}'\frac{C_z\bar{Z}+\beta_2(z)}{C_z\bar{z}'+\beta_2(z)} - \bar{x}\right]$$

(Upadhyaya and Singh [7])

$$t_{R5} = \frac{\bar{y}}{\bar{x}}\bar{x}'\left[\frac{\bar{Z}+\sigma_z}{\bar{z}'+\sigma_z}\right] \qquad t_{Reg5} = \bar{y} + b_{yx}\left[\bar{x}'\frac{\bar{Z}+\sigma_z}{\bar{z}'+\sigma_z} - \bar{x}\right]$$

(Singh [22])

$$t_{R6} = \frac{\bar{y}}{\bar{x}}\bar{x}'\left[\frac{\beta_1(z)\bar{Z}+\sigma_z}{\beta_1(z)\bar{z}'+\sigma_z}\right] \qquad t_{Reg6} = \bar{y} + b_{yx}\left[\bar{x}'\frac{\beta_1(z)\bar{Z}+\sigma_z}{\beta_1(z)\bar{z}'+\sigma_z} - \bar{x}\right]$$

(Singh [22])

$$t_{R7} = \frac{\bar{y}}{\bar{x}}\bar{x}'\left[\frac{\beta_1(z)\bar{Z}+C_z}{\beta_1(z)\bar{z}'+C_z}\right] \qquad t_{Reg7} = \bar{y} + b_{yx}\left[\bar{x}'\frac{\beta_1(z)\bar{Z}+C_z}{\beta_1(z)\bar{z}'+C_z} - \bar{x}\right]$$

III. APPROXIMATE VARIANCE

To obtain the approximate variance of t_{Regcd}, let us write

$$t_{Regcd} = g\left(\bar{y}, \mathrm{b_{yx}}, \bar{x}', \bar{z}', \bar{x}\right) = g\left(\hat{\underline{\theta}}\right) \quad \text{and} \quad \bar{Y} = g(\bar{Y}, \beta_{yx}, \bar{X}, \bar{Z}, \bar{X}) = g\left(\underline{\theta}\right)$$

where

$$\hat{\theta}_1 = \bar{y}, \quad \hat{\theta}_2 = \mathrm{b_{yx}}, \quad \hat{\theta}_3 = \bar{z}', \quad \hat{\theta}_4 = \bar{x}', \quad \hat{\theta}_5 = \bar{x}$$

and

$$\theta_1 = \bar{Y}, \quad \theta_2 = \beta_{yx}, \quad \theta_3 = \bar{Z}, \quad \theta_4 = \bar{X}, \quad \theta_5 = \bar{X}$$

Now expanding $g(\underline{\hat{\theta}})$ around $g(\underline{\theta})$ using Taylor linearization technique and using Equation (10.12) given in Stuart and Ord [23] , viz

$$V\left(g(\underline{\hat{\theta}})\right) \approx \sum_i \left[\frac{\partial g(\underline{\hat{\theta}})}{\partial \hat{\theta}_i}\right]^2_{\underline{\hat{\theta}}=\underline{\theta}} V(\hat{\theta}_i)$$

$$+\sum\sum_{i\neq j}\left[\frac{\partial g(\underline{\hat{\theta}})}{\partial \hat{\theta}_i}\cdot\frac{\partial g(\underline{\hat{\theta}})}{\partial \hat{\theta}_j}\right]_{\underline{\hat{\theta}}=\underline{\theta}} Cov(\hat{\theta}_i,\hat{\theta}_j) + O(n^{-3})$$

we obtain

$$V(t_{Regcd}) \approx V(\overline{y}) + \beta_{yx}^2 V(\overline{x}') + \beta_{yx}^2\left(\frac{\overline{X}}{\overline{Z}}\right)^2 \theta^2 V(\overline{z}') + \beta_{yx}^2 V(\overline{x}) + 2\beta_{yx} Cov(\overline{y},\overline{x}')$$

$$-2\beta_{yx}\frac{\overline{X}}{\overline{Z}}\theta Cov(\overline{y},\overline{z}') - 2\beta_{yx} Cov(\overline{y},\overline{x}) - 2\beta_{yx}^2 Cov(\overline{x}',\overline{x})$$

where

$$\theta = c\overline{Z}/(c\overline{Z}+d) \tag{9}$$

Inserting variances and covariance under two-phase sampling (Singh et al. [24]) we obtain after simplification

$$V(t_{Regcd}) \approx \overline{Y}^2 C_y^2\left[f_1(1-\rho_{xy}^2) + f_2\rho_{xy}^2\right] + f_2\left(\Delta^2 C_z^2 - 2\Delta\overline{Y}\rho_{yz}C_yC_z\right) \tag{10}$$

where

$$\Delta = \beta_{yx}\overline{X}\theta = \beta_{yx}\overline{X}(c\overline{Z})/(c\overline{Z}+d) \tag{11}$$

and

$$f_1 = n^{-1} - N^{-1},\ f_2 = n'^{-1} - N^{-1},\ f_3 = f_1 - f_2$$

Minimization of (10) gives the optimum value of Δ as

$$\Delta_{opt} = \rho_{yz}\overline{Y}C_y/C_z \tag{12}$$

Equivalently (from (11) and (12))

$$\theta_{opt} = \rho_{yz}C_x/\rho_{yx}C_z = \beta_{yz}\overline{Z}/\beta_{yx}\overline{X} = \Lambda, \text{say}$$

which gives

$$d_{opt} = c\overline{Z}(1-\Lambda)/\Lambda \quad \text{or} \quad c_{opt} = d\Lambda/\overline{Z}(1-\Lambda) \tag{13}$$

The minimum variance is then given by

$$minV(t_{Regcd}) \approx \overline{Y}^2 C_y^2\left[f_3(1-\rho_{xy}^2) + f_2(1-\rho_{yz}^2)\right] \tag{14}$$

Remark 1. Usually Λ is unknown and must be estimated using both the samples.

Inserting $\widehat{\Lambda} = b_{yz}\overline{Z}/b_{yx}\overline{x}'$ in (13) we obtain the optimal value of d and consequently the optimal estimator as

$$t_{opt} = \bar{y} + b_{yx}\left[\overline{x}'\frac{\bar{Z}+d_1}{\overline{z}'+d_1} - \overline{x}\right] = \bar{y} + b_{yx}\left[\frac{\overline{x}'}{1+\frac{b_{yz}}{b_{yx}}\left(\frac{\overline{z}'-\overline{Z}}{\overline{x}'}\right)} - \overline{x}\right] \quad (15)$$

where $d_1 = \overline{Z}(1-\widehat{\Lambda})/\widehat{\Lambda}$. (for $c = 1$)

IV. EMPIRICAL COMPARISON

This section deals with the empirical comparison of the estimators included in this paper.

A. Empirical comparison under optimality condition

The minimum variance of the proposed class of estimators given in (14) was compared with the MSEs of the estimators discussed in this paper using three natural populations. A relative efficiency in percentage of each estimator was computed by considering $\hat{\hat{Y}}_{rd}$ as the bench mark estimator.

Data set I: Jobson [25] **(**The observations are replicated 2 times)

y: Highway Rate
x: Weight
z: Engine size

$N = 194$, $n' = 80$, $n = 30$, $\overline{Y} = 68.37$, $\overline{X} = 2973.71$, $\overline{Z} = 27.60$,

$\sigma_z = 12.1286$, $\rho_{yx} = 0.7790$, $\rho_{yz} = 0.7464$, $\rho_{xz} = 0.8862$

$C_y = 0.1869$, $C_x = 0.1761$, $C_z = 0.4395$, $\beta_1(z) = 0.9441$, $\beta_2(z) = 2.5386$

Data set II: Murthy [26]

y: Output for 80 factories in a region
x: Fixed capital
z: Data on number of workers

$N = 80$, $n' = 40$, $n = 20$, $\bar{Y} = 5182.6$, $\bar{X} = 1126.46$, $\bar{Z} = 285.13$

$\sigma_z = 270.43$, $\rho_{yx} = 0.9413$, $\rho_{yz} = 0.9149$, $\rho_{xz} = 0.9884$

$C_y = 0.3542$, $C_x = 0.7507$, $C_z = 0.9485$, $\beta_1(z) = 1.2761$, $\beta_2(z) = 3.5808$

Data set III: Fisher [27]

y = Petal width
x = Sepal width
z = Sepal length

$N = 200$, $n' = 80$, $n = 50$, $\bar{Y} = 1.152$, $\bar{X} = 3.208$, $\bar{Z} = 5.78$,

$\sigma_z = 0.9049$, $\rho_{yx} = -0.5855$, $\rho_{yz} = 0.8303$, $\rho_{xz} = -0.4287$, $C_y = 0.8229$,

$C_x = 0.1267$, $C_z = 0.1570$, $\beta_1(z) = 0.2753$, $\beta_2(z) = 2.022$

Table I. RE (%) of different estimators with respect to $\bar{Y}_{rd}$

Estimator	Data I	Data II	Data III
$\hat{\hat{Y}}_{rd}$	100.00	100.00	100.00
$\hat{\hat{Y}}_{regd}$	103.55	326.06	132.99
t_{R1}	52.07	63.81	114.57
$\hat{\hat{Y}}_{k1}$	133.49	87.83	95.08
$t_{Reg1} = \hat{\hat{Y}}_{k2}$	72.77	949.62	60.44
$\hat{\hat{Y}}_{k3}$	**140.31**	**1024.21**	**169.09**
t_{R2}	53.56	64.13	114.19
t_{R3}	52.66	63.90	114.38
t_{R4}	71.42	65.08	104.49
t_{R5}	90.55	118.54	112.60
t_{R6}	92.48	112.71	109.27
t_{R7}	68.16	86.82	113.52
t_{Reg2}	74.58	952.47	61.61
t_{Reg3}	74.58	950.43	61.02
t_{Reg4}	76.86	952.62	67.37
t_{Reg5}	113.55	774.74	66.78
t_{Reg6}	115.26	844.10	79.45
t_{Reg7}	74.69	951.86	64.54
t_{Regcd}	**140.59**	**1024.84**	**227.73**

B. Comparison using a Monte Carlo simulation

The estimators t_{opt} given in (15) and the estimators listed in Sections I and II were compared empirically on three populations given above. For comparison of the estimators, a preliminary sample s' of size n' was drawn using srswor and a second-phase sample s of size n was drawn using srswor from each of the populations and these estimators were computed. This procedure was repeated $M = 5000$ times.For each estimator t its relative percentage bias was calculated as

$$RB(t) = 100 * \left(\bar{t} - \bar{Y}\right)/\bar{Y}$$

and the relative efficiency (in percentage) as

$$RE(t) = MSE_{sim}\,(\hat{\hat{Y}}_{reg})/MSE_{sim}(\,t)\;X\;100$$

where

$$\bar{t} = \sum_{j=1}^{M} t_j/M \quad \text{and} \quad MSE_{sim}(t) = \sum_{j=1}^{M}\left(t_j - \bar{Y}\right)^2/(M-1)$$

Table II. RB (%) and RE (%)

Estimator	**Relative Bias (%)**			**Efficiency (%)Relative**		
	Population			**Population**		
	1	2	3	1	2	3
$\hat{\hat{Y}}_{reg}$	0.02	-2.67	0.37	100.00	100.00	100.00
$\hat{\hat{Y}}_{regd}$	0.02	0.59	0.97	100.87	272.31	143.70
$t_{R1} = \hat{\hat{Y}}_C$	0.09	-1.78	0.48	51.35	63.15	109.68
$\hat{\hat{Y}}_{k1}$	0.03	-2.07	0.39	133.46	90.28	96.68
$t_{Reg1} = \hat{\hat{Y}}_{k2}$	0.04	0.89	0.14	67.59	625.29	86.39
$\hat{\hat{Y}}_{k3}$	0.02	0.78	1.08	**134.06**	**726.46**	**169.14**
t_{R2}	0.09	-1.79	0.48	52.85	63.51	109.42
t_{R3}	0.09	-1.79	0.48	51.94	63.25	109.55
t_{R4}	0.06	-1.81	0.41	70.80	64.61	102.99
t_{R5}	0.03	-2.49	0.47	90.09	125.49	108.36
t_{R6}	0.04	-2.43	0.44	92.05	119.27	106.16
t_{R7}	0.08	-1.79	0.47	52.94	63.44	108.81
t_{Reg2}	0.04	0.89	0.17	69.29	628.06	87.53
t_{Reg3}	0.04	0.89	0.16	68.26	626.07	86.96
t_{Reg4}	0.02	0.88	0.73	88.46	636.08	122.65
t_{Reg5}	0.01	0.62	0.27	106.47	615.85	92.43
t_{Reg6}	0.01	0.64	0.46	108.14	660.56	103.70
t_{Reg7}	0.04	0.88	0.22	69.39	627.46	90.33
t_{opt}	-0.01	0.78	1.87	**134.13**	**736.78**	**219.18**

Tables I and II prompt the following comments:

(a) The absolute relative bias of each estimator is within reasonable range $\pm 3\%$.

(b) The suggested class of estimators t_{Regcd} (with optimum values of c and d) and in particularly the optimal estimator t_{opt} given in (15) have performed very well followed by Kiregyera [12] estimator $\hat{\hat{Y}}_{k3}$.

V. CONCLUSION

When partial information on the main auxiliary variable x and complete auxiliary information on the additional auxiliary variable z, which is highly positive correlated with x, is available and the relation between y and x is a straight line not passing through the origin in this situations our optimal estimator may perform very well. This is reflected in Tables I and II. This is supported by the empirical study presented above. Estimators listed in this paper can be further extended in many ways, e.g., using exponential type estimators in ratio method of estimation, using ratio in regression method of estimation, estimation of ratio of two or more study variables, incorporating non-response (Kumar and Kumar [28]) etc.

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