International Journal of Scientific Research in Mathematical and Statistical Sciences
Research Paper
Vol.6, Issue.2, pp.108-112, April (2019)
E-ISSN: 2348-4519
DOI: https://doi.org/10.26438/ijsrmss/v6i2.108112

Block Related Indices and Coindices of a Graph

B. Basavanagoud

Department of Mathematics, Karnatak University, Dharwad, India

Corresponding Author: b.basavanagoud@gmail.com, Tel.: +91-9449177029

Available online at: www.isroset.org

Received: 29/Mar/2019, Accepted: 14/Apr/2019, Online: 30/Apr/2019

***Abstract*—** A nontrivial connected graph with no cutvertices is called a *block*. A *block* of a graph is a subgraph of a graph which itself is a block and which is maximal with respect to this property. So far we have seen the graph invariants which are defined on vertices and edges of a graph. In this paper, we introduce new indices and coindices related to blocks of a graph.

Keywords *— block, block indices, block coindices.*

I. INTRODUCTION

Let $G = (V, E)$ be a simple (molecular) graph with vertex set $V(G)$, edge set $E(G)$ and block set $U(G)$. The vertices, edges and blocks of a graph are called elements of G. The *degree of a vertex* $v \in V(G)$ is the number of vertices adjacent to v in G. It will be denoted by $d_G(v)$. If u and v are two adjacent vertices of G, then the edge connecting them will be denoted by uv. *Degree of an edge* $e = uv$ is denoted by $d_G(e)$ and is defined as $d_G(e) = d_G(u) + d_G(v) - 2$. If a block $B \in U(G)$ with the edge set $\{e_1, e_2, \ldots, e_s; s \geq 1\}$, then we say that the edge e_i and block B are incident with each other, where $1 \leq i \leq s$. If a block $B \in U(G)$ with the vertex set $\{v_1, v_2, \ldots, v_t; s \geq 2\}$, then we say that the vertex v_i and block B are incident with each other, where $1 \leq i \leq t$. If two distinct blocks are incident with a common cutvertex, then they are adjacent blocks. The *degree of a block* B in G, denoted by $d_G(B)$, is the number of blocks adjacent to B in G. We denote the number of edges incident with B in G by $D_G(B)$. The *block graph* $B(G)$ of a graph G is the graph whose vertices are the blocks of G and in which two vertices are adjacent whenever the corresponding blocks are adjacent [12]. The *point-block graph* $bp(G)$ of a graph G is the graph whose vertices can be put in one to one correspondence with the set of vertices and blocks of G in such a way that two vertices of $bp(G)$ are adjacent if and only if one corresponds to a block B of G and the other to a vertex v of G and v is incident with B[13]. The *line-block graph* $bq(G)$ of a graph G is the graph whose vertices can be put in one to one correspondence with the set of edges and blocks of G in such a way that two vertices of $bq(G)$ are adjacent if and only if one corresponds to a block B of G and the other to an edge e of G and e is in B[1]. The *line graph* $L(G)$ of G is the graph whose vertex set is $E(G)$ in which two vertices are adjacent if and only if they are adjacent in G. In this paper, we denote the adjacency (or incidence) of elements of graphs by the symbol $\sim$ and nonadjacency by $\nsim$. For terminology not defined here we refer the reader to [12].

A graph invariant is a number related to a graph which is independent of the structure. In chemical graph theory, one such graph invariant is topological index. The first and second Zagreb indices of a graph G, denoted by $M_1(G)$ and $M_2(G)$, are among the oldest, most popular and extremely studied vertex degree based topological indices and are defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v),$$

respectively. Their mathematical theory is nowadays well elaborated. For details, see the papers [6, 11, 15]. For historical data on the Zagreb indices see [10]. For surveys on degree-based topological indices see [9].

The first Zagreb index can also be written as [7, 8]

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

Noticing that contribution of nonadjacent vertex pairs should be taken into account when computing the weighted Wiener polynomials of certain composite graphs, authors in [7] defined first Zagreb coindex and second Zagreb coindex as

$$\overline{M_1}(G) = \sum_{uv \notin E(G)} [d_G(u) + d_G(v)]$$

and

$$\overline{M}_2(G) = \sum_{uv \notin E(G)} d_G(u) d_G(v),$$

respectively.

Milic′evic′ et al. [14] in 2004 *reformulated the Zagreb indices* in terms of edge-degrees instead of vertex-degrees. The first and second reformulated Zagreb indices are defined respectively, as

$$EM_1(G) = \sum_{e \in E(G)} d_G(e)^2 = \sum_{e \sim f \in E(G)} [d_G(e) + d_G(f)]$$

and

$$EM_2(G) = \sum_{e \sim f \in E(G)} d_G(e) d_G(f),$$

where $e \sim f$ means that the edges e and f are adjacent in G.
In this paper, we introduce the block related new indices and coindices of a graph. The rest of the paper is organised as follows. In section 2, we introduce block indices and coindices of a graph. In section 3, we compute block indices and coindices of a graph as an example.

II. BLOCK INDICES AND COINDICES

It is important to note that, in case of Zagreb indices, the transformation $G \to L(G)$ yields the " reformulated Zagreb indices". Similarly, the transformations $G \to B(G)$, $G \to bp(G)$ and $G \to bq(G)$ yields the " block indices and coindices" as follows.
Let G be a (molecular) graph, and let $B_1 \sim B_2$ ($B_1 \nsim B_2$) be the blocks B_1 and B_2 are adjacent (resp., not adjacent). Let $e \sim B$ ($e \nsim B$) be the edge e is incident (resp., not incident) with block B.

(i) The B −indices and coindices are:

$$BB_1(G) = \sum_{B \in U(G)} d_G^2(B) = \sum_{B_i \sim B_j} [d_G(B_i) + d_G(B_j)],$$

$$BB_2(G) = \sum_{B_i \sim B_j} d_G(B_i) d_G(B_j) \text{ and}$$

$$\overline{BB}_1(G) = \sum_{B_i \nsim B_j} [d_G(B_i) + d_G(B_j)],$$

$$\overline{BB}_2(\mathrm{G}) = \sum_{B_i \nsim B_j} d_G(B_i) d_G(B_j).$$

(ii) The C −indices and coindices are:

$$BC_1(G) = \sum_{B_i \sim B_j} [D_G(B_i) + D_G(B_j)],$$

$$BC_2(G) = \sum_{B_i \sim B_j} D_G(B_i) D_G(B_j) \text{ and}$$

$$\overline{BC}_1(G) = \sum_{B_i \nsim B_j} [D_G(B_i) + D_G(B_j)],$$

$$\overline{BC}_2(G) = \sum_{B_i \nsim B_j} D_G(B_i) D_G(B_j).$$

(iii) The V −index and coindex are:

$$BV(G) = \sum_{e \sim B} d_G(e) D_G(B) \text{ and}$$

$$\overline{BV}(G) = \sum_{e \nsim B} d_G(e) D_G(B).$$

(iv) The V^* −index and coindex are:

$$BV^*(G) = \sum_{v \sim B} d_G(v) D_G(B) \text{ and}$$

$$\overline{BV}^*(G) = \sum_{v \nsim B} d_G(v) D_G(B).$$

(v) The P −index and coindex are:

$$BP(G) = \sum_{e \sim B} d_G(e) d_G(B) \text{ and}$$

$$\overline{BP}(G) = \sum_{e \nsim B} d_G(e) d_G(B).$$

(vi) The P^* −index and coindex are:

$$BP^*(G) = \sum_{v \sim B} d_G(v) d_G(B) \text{ and}$$

$$\overline{BP}^*(G) = \sum_{v \nsim B} d_G(v) d_G(B).$$

(vii) The K^* −index and coindex are:

$$BK^*(G) = \sum_{B_i \sim B_j} d_G(B_i) D_G(B_j) \text{ and}$$

$$\overline{BK^*}(G) = \sum_{B_i \nsim B_j} d_G(B_i) D_G(B_j)$$

$(viii)$ The other auxiliary indices are:

$$\xi(G) = \sum_{B \in U(G)} D_G^2(B)$$

$$\eta(G) = \sum_{B \in U(G)} d_G(B)$$

$$\chi(G) = \sum_{B \in U(G)} d_G(B) D_G(B).$$

In literature, so many indices are introduced and their properties are studied [2, 3, 4, 5 9].

III. COMPUTATION OF BLOCK INDICES AND COINDICES

For example: Consider a graph G with vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}$, edges e_1, e_2, e_3, $e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}$ and blocks B_1, B_2, B_3, B_4 as labeled in Fig 1.
Here,

1. $d_G(e_1) = 3$, $d_G(e_2) = 3$, $d_G(e_3) = 4$, $d_G(e_4) = 4$, $d_G(e_5) = 3$, $d_G(e_6) = 3$, $d_G(e_7) = 6$, $d_G(e_8) = 5$, $d_G(e_9) = 2$, $d_G(e_{10}) = 5$, $d_G(e_{11}) = 5$, $d_G(e_{12}) = 2$ and $d_G(e_{13}) = 5$.
2. $d_G(B_1) = 1$, $d_G(B_2) = 3$, $d_G(B_3) = 2$ and $d_G(B_4) = 2$.
3. $D_G(B_1) = 6$, $D_G(B_2) = 1$, $D_G(B_3) = 3$ and $D_G(B_4) = 3$.

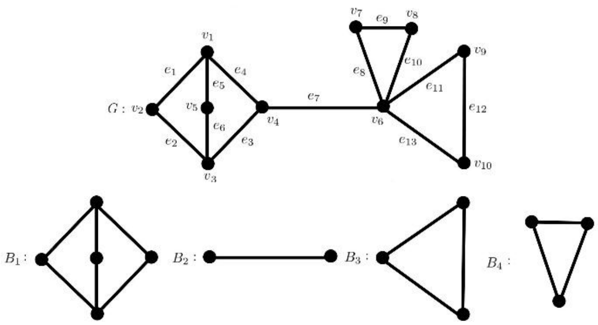

Figure 1: A Graph G and its blocks

(i) The B −indices of G are:

$$BB_1(G) = \sum_{B\in U(G)} d_G^2(B)$$
$$= d_G^2(B_1) + d_G^2(B_2) + d_G^2(B_3) + d_G^2(B_4)$$
$$= 1 + 9 + 4 + 4 = 18$$

and

$$BB_2(G) = \sum_{B_i \sim B_j} d_G(B_i) d_G(B_j)$$
$$= d_G(B_1)d_G(B_2) + d_G(B_2)d_G(B_3)$$
$$+d_G(B_2)d_G(B_4) + d_G(B_3)d_G(B_4)$$
$$= 1\cdot 3 + 3\cdot 2 + 3\cdot 2 + 2\cdot 2 = 19.$$

and

The B −coindices of G are:

$$\overline{BB}_1(G) = \sum_{B_i \nsim B_j} [d_G(B_i) + d_G(B_j)]$$
$$= [d_G(B_1) + d_G(B_3)] + [d_G(B_1) + d_G(B_4)]$$
$$= 3 + 3 = 6$$

and

$$\overline{BB}_2(G) = \sum_{B_i \nsim B_j} d_G(B_i) d_G(B_j)$$
$$= d_G(B_1)d_G(B_3) + d_G(B_1)d_G(B_4)$$
$$= 2 + 2 = 4.$$

(ii) The C −indices of G are:

$$BC_1(G) = \sum_{B_i \sim B_j} [D_G(B_i) + D_G(B_j)]$$
$$= [D_G(B_1) + D_G(B_2)] + [D_G(B_2) + D_G(B_3)]$$
$$+[D_G(B_2) + D_G(B_4)] + [D_G(B_3) + D_G(B_4)]$$
$$= 7 + 4 + 4 + 6 = 21$$

and

$$BC_2(G) = \sum_{B_i \sim B_j} D_G(B_i) D_G(B_j)$$
$$= D_G(B_1)D_G(B_2) + D_G(B_2)D_G(B_3) + D_G(B_2)D_G(B_4)$$
$$+D_G(B_3)D_G(B_4)$$
$$= 6 + 3 + 3 + 9 = 21.$$

The C −coindices of G are:

$$\overline{BC}_1(G) = \sum_{B_i \nsim B_j} [D_G(B_i) + D_G(B_j)]$$
$$= [D_G(B_1) + D_G(B_3)] + [D_G(B_1) + D_G(B_4)]$$
$$= 9 + 9 = 18 \text{ and}$$
$$\overline{BC}_2(G) = \sum_{B_i \nsim B_j} D_G(B_i) D_G(B_j)$$
$$= D_G(B_1)D_G(B_3) + D_G(B_1)D_G(B_4)$$
$$= 18 + 18 = 36.$$

(iii) The V −index and coindex of G are:

$$BV(G) = \sum_{e\sim B} d_G(e) D_G(B)$$
$$= d_G(e_1)D_G(B_1) + d_G(e_2)D_G(B_1) + d_G(e_3)D_G(B_1)$$
$$+d_G(e_4)D_G(B_1) + d_G(e_5)D_G(B_1) + d_G(e_6)D_G(B_1)$$
$$+d_G(e_7)D_G(B_2) + d_G(e_8)D_G(B_4) + d_G(e_9)D_G(B_4)$$
$$+d_G(e_{10})D_G(B_4) + d_G(e_{11})D_G(B_3) + d_G(e_{12})D_G(B_3)$$
$$+d_G(e_{13})D_G(B_3)$$
$$= 18 + 18 + 24 + 24 + 18 + 18 + 6 + 15 + 6 + 15 + 15 + 6$$
$$+ 15 = 198$$

and

$$\overline{BV}(G) = \sum_{e\nsim B} d_G(e) D_G(B)$$
$$= d_G(e_1)D_G(B_2) + d_G(e_1)D_G(B_3) + d_G(e_1)D_G(B_4)$$
$$+d_G(e_2)D_G(B_2) + d_G(e_2)D_G(B_3) + d_G(e_2)D_G(B_4)$$
$$+d_G(e_3)D_G(B_2) + d_G(e_3)D_G(B_3) + d_G(e_3)D_G(B_4)$$
$$+d_G(e_4)D_G(B_2) + d_G(e_4)D_G(B_3) + d_G(e_4)D_G(B_4)$$
$$+d_G(e_5)D_G(B_2) + d_G(e_5)D_G(B_3) + d_G(e_5)D_G(B_4)$$
$$+d_G(e_6)D_G(B_2) + d_G(e_6)D_G(B_3) + d_G(e_6)D_G(B_4)$$
$$+d_G(e_7)D_G(B_1) + d_G(e_7)D_G(B_3) + d_G(e_7)D_G(B_4)$$
$$+d_G(e_8)D_G(B_1) + d_G(e_8)D_G(B_2) + d_G(e_8)D_G(B_3)$$
$$+d_G(e_9)D_G(B_1) + d_G(e_9)D_G(B_2) + d_G(e_9)D_G(B_3)$$
$$+d_G(e_{10})D_G(B_1) + d_G(e_{10})D_G(B_2) + d_G(e_{10})D_G(B_3)$$
$$+d_G(e_{11})D_G(B_1) + d_G(e_{11})D_G(B_2) + d_G(e_{11})D_G(B_4)$$
$$+d_G(e_{12})D_G(B_1) + d_G(e_{12})D_G(B_2) + d_G(e_{12})D_G(B_4)$$
$$+d_G(e_{13})D_G(B_1) + d_G(e_{13})D_G(B_2) + d_G(e_{13})D_G(B_4)$$
$$= 3 + 9 + 9 + 3 + 9 + 9 + 4 + 12 + 12 + 4 + 12 + 12$$
$$+3 + 4 + 4 + 3 + 4 + 4 + 36 + 18 + 18 + 30 + 5 + 15$$
$$+12 + 2 + 6 + 30 + 5 + 15 = 312.$$

(iv) The V^* −index and coindex of G are:

$$BV^*(G) = \sum_{v\sim B} d_G(v)D_G(B)$$
$$= d_G(v_1)D_G(B_1) + d_G(v_2)D_G(B_1) + d_G(v_3)D_G(B_1)$$
$$+d_G(v_4)D_G(B_1) + d_G(v_5)D_G(B_1) + d_G(v_5)D_G(B_2)$$
$$+d_G(v_7)D_G(B_4) + d_G(v_8)D_G(B_4) + d_G(v_6)D_G(B_2)$$
$$+d_G(v_6)D_G(B_4) + d_G(v_9)D_G(B_3) + d_G(v_{10})D_G(B_3)$$
$$+d_G(v_{16})D_G(B_3)$$
$$= 18 + 12 + 18 + 18 + 12 + 3 + 5 + 9 + 6 + 6 + 15 + 6 + 6 = 134$$

and

$$\overline{BV}^*(G) = \sum_{v\nsim B} d_G(v)D_G(B)$$
$$= d_G(v_1)D_G(B_2) + d_G(v_1)D_G(B_3) + d_G(v_1)D_G(B_4)$$
$$+d_G(v_2)D_G(B_2) + d_G(v_2)D_G(B_3) + d_G(v_2)D_G(B_4)$$
$$+d_G(v_3)D_G(B_2) + d_G(v_3)D_G(B_3) + d_G(v_3)D_G(B_4)$$
$$+d_G(v_4)D_G(B_3) + d_G(v_4)D_G(B_4) + d_G(v_6)D_G(B_1)$$
$$+d_G(v_5)D_G(B_2) + d_G(v_5)D_G(B_3) + d_G(v_5)D_G(B_4)$$
$$+d_G(v_7)D_G(B_1) + d_G(v_7)D_G(B_2) + d_G(v_7)D_G(B_3)$$
$$+d_G(v_8)D_G(B_1) + d_G(v_8)D_G(B_2) + d_G(v_8)D_G(B_3)$$
$$+d_G(v_9)D_G(B_1) + d_G(v_9)D_G(B_2) + d_G(v_9)D_G(B_4)$$
$$+d_G(v_{10})D_G(B_1) + d_G(v_{10})D_G(B_4) + d_G(v_{10})D_G(B_2)$$
$$= 3 + 9 + 9 + 2 + 6 + 6 + 3 + 9 + 9 + 9$$
$$+9 + 2 + 6 + 6 + 30 + 12 + 2 + 6 + 12$$
$$+2 + 6 + 12 + 2 + 6 + 12 + 2 + 6 = 196.$$

(v) The P −index and coindex of G are:

$$BP(G) = \sum_{e\sim B} d_G(e)d_G(B)$$
$$= d_G(e_1)d_G(B_1) + d_G(e_2)d_G(B_1) + d_G(e_3)d_G(B_1)$$
$$+d_G(e_4)d_G(B_1) + d_G(e_5)d_G(B_1) + d_G(e_6)d_G(B_1)$$
$$+d_G(e_7)d_G(B_2) + d_G(e_8)d_G(B_4) + d_G(e_9)d_G(B_4)$$
$$+d_G(e_{10})d_G(B_4) + d_G(e_{11})d_G(B_3) + d_G(e_{12})d_G(B_3)$$
$$+d_G(e_{13})d_G(B_3)$$
$$= 3 + 3 + 4 + 4 + 3 + 3 + 18 + 10 + 4 + 10 + 10 + 4 + 10 = 86$$

and

$$\overline{BP}(G) = \sum_{e\nsim B} d_G(e)d_G(B)$$
$$= d_G(e_1)d_G(B_2) + d_G(e_1)d_G(B_3) + d_G(e_1)d_G(B_4)$$
$$+d_G(e_2)d_G(B_2) + d_G(e_2)d_G(B_3) + d_G(e_2)d_G(B_4)$$
$$+d_G(e_3)d_G(B_2) + d_G(e_3)d_G(B_3) + d_G(e_3)d_G(B_4)$$
$$+d_G(e_4)d_G(B_2) + d_G(e_4)d_G(B_3) + d_G(e_4)d_G(B_4)$$
$$+d_G(e_5)d_G(B_2) + d_G(e_5)d_G(B_3) + d_G(e_5)d_G(B_4)$$
$$+d_G(e_6)d_G(B_2) + d_G(e_6)d_G(B_3) + d_G(e_6)d_G(B_4)$$
$$+d_G(e_7)d_G(B_1) + d_G(e_7)d_G(B_3) + d_G(e_7)d_G(B_4)$$
$$+d_G(e_8)d_G(B_1) + d_G(e_8)d_G(B_2) + d_G(e_8)d_G(B_3)$$
$$+d_G(e_9)d_G(B_1) + d_G(e_9)d_G(B_2) + d_G(e_9)d_G(B_3)$$
$$+d_G(e_{10})d_G(B_1) + d_G(e_{10})d_G(B_2) + d_G(e_{10})d_G(B_3)$$
$$+d_G(e_{11})d_G(B_1) + d_G(e_{11})d_G(B_2) + d_G(e_{11})d_G(B_4).$$
$$+d_G(e_{12})d_G(B_1) + d_G(e_{12})d_G(B_2) + d_G(e_{12})d_G(B_4)$$
$$+d_G(e_{13})d_G(B_1) + d_G(e_{13})d_G(B_2) + d_G(e_{13})d_G(B_4)$$
$$= 9 + 6 + 6 + 9 + 6 + 6 + 12 + 8 + 8 + 12 + 8 + 8$$
$$+9 + 6 + 6 + 9 + 6 + 6 + 6 + 12 + 12 + 5 + 15 + 10$$
$$+2 + 6 + 4 + 5 + 15 + 10 + 5 + 15 + 10 + 2 + 6 + 4$$
$$+5 + 15 + 10 = 314.$$

(vi) The P^* −index and coindex of G are:

$$BP^*(G) = \sum_{v\sim B} d_G(v)d_G(B)$$
$$= d_G(v_1)d_G(B_1) + d_G(v_2)d_G(B_1) + d_G(v_3)d_G(B_1)$$
$$+d_G(v_4)d_G(B_1) + d_G(v_5)d_G(B_1) + d_G(v_5)d_G(B_2)$$
$$+d_G(v_6)d_G(B_2) + d_G(v_6)d_G(B_4) + d_G(v_7)d_G(B_4)$$
$$+d_G(v_8)d_G(B_4) + d_G(v_{16})d_G(B_3) + d_G(v_9)d_G(B_3)$$
$$+d_G(v_{10})d_G(B_3)$$
$$= 3 + 2 + 3 + 3 + 2 + 9 + 15 + 10 + 4 + 4 + 10 + 4 + 4 = 73$$

and

$$\overline{BP}^*(G) = \sum_{v\nsim B} d_G(v)d_G(B)$$
$$= d_G(v_1)d_G(B_2) + d_G(v_1)d_G(B_3) + d_G(v_1)d_G(B_4)$$
$$+d_G(v_2)d_G(B_2) + d_G(v_2)d_G(B_3) + d_G(v_2)d_G(B_4)$$
$$+d_G(v_3)d_G(B_2) + d_G(v_3)d_G(B_3) + d_G(v_3)d_G(B_4)$$
$$+d_G(v_4)d_G(B_3) + d_G(v_4)d_G(B_4) + d_G(v_5)d_G(B_2)$$
$$+d_G(v_5)d_G(B_3) + d_G(v_5)d_G(B_4) + d_G(v_6)d_G(B_1)$$
$$+d_G(v_7)d_G(B_1) + d_G(v_7)d_G(B_2) + d_G(v_7)d_G(B_3)$$
$$+d_G(v_8)d_G(B_1) + d_G(v_8)d_G(B_2) + d_G(v_8)d_G(B_3)$$
$$+d_G(v_9)d_G(B_1) + d_G(v_9)d_G(B_2) + d_G(v_9)d_G(B_4)$$
$$+d_G(v_{10})d_G(B_1) + d_G(v_{10})d_G(B_2) + d_G(v_{10})d_G(B_4)$$
$$= 9 + 6 + 6 + 6 + 4 + 4 + 9 + 6 + 6 + 6 + 6 + 5$$
$$+2 + 6 + 2 + 6 + 2 + 6 + 2 + 6$$
$$= 105.$$

(vii) The K^* −index and coindex are:

$$BK^*(G) = \sum_{B_i\sim B_j} d_G(B_i)D_G(B_j)$$
$$= d_G(B_1)D_G(B_2) + d_G(B_2)D_G(B_1) + d_G(B_2)D_G(B_3)$$
$$+d_G(B_3)D_G(B_2) + d_G(B_2)D_G(B_4) + d_G(B_4)D_G(B_2)$$
$$+d_G(B_3)D_G(B_4) + d_G(B_4)D_G(B_3)$$
$$= 1 + 18 + 9 + 2 + 9 + 2 + 6 + 6 = 53$$

and

$$\overline{BK^*}(G) = \sum_{B_i\nsim B_j} d_G(B_i)D_G(B_j)$$
$$= d_G(B_1)D_G(B_3) + d_G(B_3)D_G(B_1) + d_G(B_1)D_G(B_4)$$
$$+d_G(B_4)D_G(B_1)$$
$$= 3 + 12 + 3 + 12 = 30.$$

$(viii)$ The other auxiliary indices are

$$\xi(G) = \sum_{B\in U(\mathrm{G})} D_G^2(B)$$
$$= D_G^2(B_1) + D_G^2(B_2) + D_G^2(B_3) + D_G^2(B_4)$$
$$= 36 + 1 + 9 + 9 = 55$$

and

$$\eta(G) = \sum_{B \in U(G)} d_G(B)$$
$$= d_G(B_1) + d_G(B_2) + d_G(B_3) + d_G(B_4)$$
$$= 1 + 3 + 2 + 2 = 8$$

and

$$\chi(G) = \sum_{B \in U(G)} d_G(B) D_G(B)$$
$$= d_G(B_1)D_G(B_1) + d_G(B_2)D_G(B_2) + d_G(B_3)D_G(B_3)$$
$$+ d_G(B_4)D_G(B_4)$$
$$= 6 + 3 + 6 + 6 = 21.$$

IV. CONCLUSION

In this paper, we have introduced some important block indices which play a key role in finding topological indices related to blocks in a graph.

ACKNOWLEDGEMENT

The author is thankful to the referee for useful suggestions. The author is thankful to University Grants Commission (UGC), Government of India, New Delhi, for the financial support through UGC-SAP DRS-III for 2016-2021: F.510/3/DRS-III/2016(SAP-I) dated: 29th Feb. 2016.

REFERENCES

[1] B. Basavanagoud, S. Patil, Line-block graph of a graph,J. Karnatak Univ. Sci., 50 (2015) 14-18.

[2] B. Basavanagoud, S. Patil, A note on hyper-Zagreb index of graph operations, Iranian J. Math. Chem., 7(1) (2016) 89-92.

[3] B. Basavanagoud, S. Patil, A note on hyper-Zagreb coindex of graph operations, J. Appl. Math. Comput., 53 (2017) 647-655.

[4] B. Basavanagoud, S. Patil, H. Y. Deng, On the second order first Zagreb Index, Iranian J. Math. Chem., 8(3) (2017) 299-311.

[5] B. Basavanagoud, V. R. Desai, S. Patil, (β,α)-Connectivity index of graphs, Appl. Math. Nonlinear Sci., 2(1), (2017) 21-30.

[6] K. C. Das, I. Gutman, Some properties of the second Zagreb index, MATCH Commun. Math. Comput. Chem.,52 (2004) 103-112.

[7] T. Dosˇlicˇ, Vertex-weighted Wiener polynomials for composite graphs, Ars Math. Contemp., 1 (2008) 66-80.

[8] T. Dosˇlicˇ, B. Furtula, A. Graovac, I. Gutman, S. Moradi, Z. Yarahmadi, On vertex-degree-based molecular structure descriptors, MATCH Commun. Math. Comput.Chem., 66 (2011) 613-626.

[9] I. Gutman, Degree-based topological indices, Croat. Chem. Acta, 86 (2013) 351-361.

[10] I. Gutman, On the origin of two degree-based topological indices, Bull. Acad. Serbe Sci. Arts (Cl. Sci. Math. Natur.), 146 (2014) 39-52.

[11] I. Gutman, B. Furtula, Z. Kovijanic' Vukic'evic', G. Popivoda, Zagreb indices and coindices, MATCH Commun. Math. Comput. Chem., 74 (2015) 5-16.

[12] F. Harary, Graph Theory, Addison-Wesley, Reading, 1969.

[13] V. R. Kulli, The block-point tree of a graph, Indian J. pure appl. Math, 7 (1976) 10-14.

[14] A. Milic'evic', S. Nikolic', N. Trinajstic', On reformulated Zagreb indices, Mol. Divers., 8 (2004) 393-399.

[15] S. Nikolic', G. Kovacˇevic', A. Milicˇevic', N. Trinajstic', The Zagreb indices 30 years after, Croat. Chem. Acta, 76 (2003) 113-124.

AUTHORS PROFILE

Dr. B. Basavanagoud is a professor in the department of Mathematics, Karnatak University, Dharwad, Karnataka state, India. He was chairman of the department for two terms, i.e., 2010-2012 and 2016-2018. He obtained his Ph. D degree from Gulbarga University, Kalaburgi, Karnataka, India, under the supervision of Prof. V. R. Kulli. He visited Taiwan on invitation to deliver an invited talk in Indo-Taiwan conference on Discrete Mathematics. He chaired a session and delivered a contributed talk in prestigious International Congress of Mathematicians (ICM) held at Hyderabad in the year 2010. He has more than 30 years of teaching experience, completed 7 research projects and organized 3 international conferences/ workshops. He has guided 12 students for their M. Phil, 10 students for their Ph. D. At present 5 students are working for their Ph. D. He has delivered more than 40 invited /contributed talks and has more than 140 research publications in reputed national/international journals. At present he is Academic Council member of Karnatak University Dharwad (2017-2019). He is also life member for several academic bodies.

